Data III & Integers I

CSE 351 Autumn 2017

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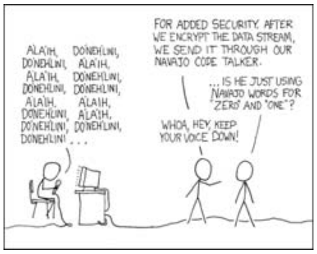


http://xkcd.com/257/

Administrivia

- Homework 1 due tonight
- Lab 1 released
  - *This is considered the hardest assignment of the class by many students*
  - Some progress due Monday 10/9, Lab 1 due Friday 1/13
- Poll Everywhere: you can change your vote

Memory, Data, and Addressing

- Representing information as bits and bytes
- Organizing and addressing data in memory
- Manipulating data in memory using C
- **Boolean algebra and bit-level manipulations**

Boolean Algebra

- Developed by George Boole in 19th Century
  - Algebraic representation of logic (True → 1, False → 0)
  - AND: A&B=1 when both A is 1 and B is 1
  - OR: A|B=1 when either A is 1 or B is 1
  - XOR: A^B=1 when either A is 1 or B is 1, but not both
  - NOT: ~A=1 when A is 0 and vice-versa
  - DeMorgan's Law: ~(A|B) = ~A & ~B
    ~(A&B) = ~A | ~B

AND

&	0	1
0	**0**	**0**
1	**0**	**1**

OR

|	0	1
0	**0**	**1**
1	**1**	**1**

XOR

^	0	1
0	**0**	**1**
1	**1**	**0**

NOT

~	
0	**1**
1	**0**

General Boolean Algebras

- Operate on bit vectors
  - Operations applied bitwise
  - All of the properties of Boolean algebra apply

```
  01101001     01101001     01101001
& 01010101   | 01010101   ^ 01010101   ~ 01010101
```

- Examples of useful operations:

$x \wedge x = 0$

```
  01010101
^ 01010101
  00000000
```

$x \mid 1 = 1, \quad x \mid 0 = x$

```
  01010101
| 11110000
  11110101
```


Bit-Level Operations in C

- & (AND), | (OR), ^ (XOR), ~ (NOT)
  - View arguments as bit vectors, apply operations bitwise
  - Apply to any "integral" data type
    - long, int, short, char, unsigned
- Examples with char a, b, c;
  - ```
    a = (char) 0x41;    // 0x41->0b 0100 0001
    b = ~a;             //      0b           ->0x
    ```
  - ```
    a = (char) 0x69;    // 0x69->0b 0110 1001
    b = (char) 0x55;    // 0x55->0b 0101 0101
    c = a & b;          //      0b           ->0x
    ```
  - ```
    a = (char) 0x41;    // 0x41->0b 0100 0001
    b = a;              //      0b 0100 0001
    c = a ^ b;          //      0b           ->0x
    ```
    ```

Contrast: Logic Operations

- Logical operators in C: && (AND), || (OR), ! (NOT)
  - **0** is False, **anything nonzero** is True
  - **Always** return 0 or 1
  - Early termination (a.k.a. short-circuit evaluation) of &&, ||
- Examples (`char` data type)
  - `!0x41 -> 0x00`
  - `!0x00 -> 0x01`
  - `!!0x41 -> 0x01`
  - `0xCC && 0x33 -> 0x01`
  - `0x00 || 0x33 -> 0x01`
  - `p && *p++`
    - Avoids null pointer (0x0) access via *early termination*
    - Short for: `if (p) { *p++; }`

Roadmap

C:

```
car *c = malloc(sizeof(car));
c->miles = 100;
c->gals = 17;
float mpg = get_mpg(c);
free(c);
```

Java:

```
Car c = new Car();
c.setMiles(100);
c.setGals(17);
float mpg =
    c.getMPG();
```

Memory & data
Integers & floats
Machine code & C
x86 assembly
Procedures & stacks
Arrays & structs
Memory & caches
Processes
Virtual memory
Operating Systems

Assembly language:

```
get_mpg:
    pushq   %rbp
    movq    %rsp, %rbp
    ...
    popq    %rbp
    ret
```

OS:

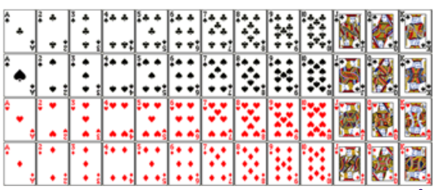
Machine code:

```
0111010000011000
100011010000010000000010
1000100111000010
110000011111101000011111
```

Computer system:

But before we get to integers….

- Encode a standard deck of playing cards
- 52 cards in 4 suits
  - How do we encode suits, face cards?
- What operations do we want to make easy to implement?
  - Which is the higher value card?
  - Are they the same suit?



Two possible representations

1) 1 bit per card (52): bit corresponding to card set to 1

low-order 52 bits of 64-bit word

- "One-hot" encoding (similar to set notation)
- Drawbacks:
  - Hard to compare values and suits
  - Large number of bits required

2) 1 bit per suit (4), 1 bit per number (13): 2 bits set

4 suits 13 numbers

- Pair of one-hot encoded values
- Easier to compare suits and values, but still lots of bits used

- Can we do better?

Two better representations

3) Binary encoding of all 52 cards – only 6 bits needed
  - $2^6 = 64 \geq 52$

low-order 6 bits of a byte

  - Fits in one byte (smaller than one-hot encodings)
  - How can we make value and suit comparisons easier?

4) Separate binary encodings of suit (2 bits) and value (4 bits)

suit value

  - Also fits in one byte, and easy to do comparisons

K	Q	J	. . .	3	2	A
1101	1100	1011	...	0011	0010	0001

♣	00
♦	01
♥	10
♠	11

Compare Card Suits

mask: a bit vector designed to achieve a desired behavior when used with a bitwise operator on another bit vector *v*.
Here we turns all *but* the bits of interest in *v* to 0.

```
char hand[5];        // represents a 5-card hand
char card1, card2;   // two cards to compare
card1 = hand[0];
card2 = hand[1];
...
if ( sameSuitP(card1, card2) ) { ... }
```

```
#define SUIT_MASK  0x30

int sameSuitP(char card1, char card2) {
  return (!((card1 & SUIT_MASK) ^ (card2 & SUIT_MASK)));
  //return (card1 & SUIT_MASK) == (card2 & SUIT_MASK);
}
```

returns `int`

SUIT_MASK = 0x30 = `0 0 1 1 0 0 0 0`

suit value

equivalent

Compare Card Suits

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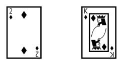


```
0 0 0 1 0 0 1 0          0 0 0 1 1 1 0 1
       &                        &
0 0 1 1 0 0 0 0   SUIT_MASK  0 0 1 1 0 0 0 0
       =                        =
0 0 0 1 0 0 0 0          0 0 0 1 0 0 0 0
                  ^
           0 0 0 0 0 0 0 0
                  !
           0 0 0 0 0 0 0 1
```

!(x^y) equivalent to x==y

Compare Card Values

mask: a bit vector designed to achieve a desired behavior when used with a bitwise operator on another bit vector *v*.

```
char hand[5];       // represents a 5-card hand
char card1, card2;  // two cards to compare
card1 = hand[0];
card2 = hand[1];
...
if ( greaterValue(card1, card2) ) { ... }
```

```
#define VALUE_MASK  0x0F

int greaterValue(char card1, char card2) {
  return ((unsigned int)(card1 & VALUE_MASK) >
          (unsigned int)(card2 & VALUE_MASK));
}
```

VALUE_MASK = 0x0F = 0 0 0 0 1 1 1 1

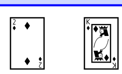
suit value

Compare Card Values

mask: a bit vector designed to achieve a desired behavior when used with a bitwise operator on another bit vector *v*.

```
#define VALUE_MASK  0x0F

int greaterValue(char card1, char card2) {
  return ((unsigned int)(card1 & VALUE_MASK) >
          (unsigned int)(card2 & VALUE_MASK));
}
```



```
0 0 1 0 0 0 1 0          0 0 1 0 1 1 0 1
       &                        &
0 0 0 0 1 1 1 1   VALUE_MASK  0 0 0 0 1 1 1 1
       =                        =
0 0 0 0 0 0 1 0          0 0 0 0 1 1 0 1
```

$2_{10} > 13_{10}$

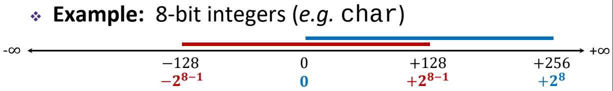
0 (false)

Integers

- **Binary representation of integers**
  - **Unsigned and signed**
  - Casting in C
- Consequences of finite width representation
  - Overflow, sign extension
- Shifting and arithmetic operations

Encoding Integers

- The hardware (and C) supports two flavors of integers
  - *unsigned* – only the non-negatives
  - *signed* – both negatives and non-negatives
- Cannot represent all integers with w bits
  - Only 2^w distinct bit patterns
  - Unsigned values: $0 \ldots 2^w - 1$
  - Signed values: $-2^{w-1} \ldots 2^{w-1} - 1$
- **Example:** 8-bit integers (*e.g.* `char`)



Unsigned Integers

- Unsigned values follow the standard base 2 system
  - $b_7b_6b_5b_4b_3b_2b_1b_0 = b_7 2^7 + b_6 2^6 + \cdots + b_1 2^1 + b_0 2^0$
- Add and subtract using the normal "carry" and "borrow" rules, just in binary

63	00111111
+ 8	+00001000
71	01000111

- Useful formula: $2^{N-1} + 2^{N-2} + \ldots + 2 + 1 = 2^N - 1$
  - *i.e.* N ones in a row = $2^N - 1$
- How would you make *signed* integers?

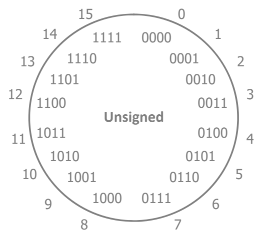
Sign and Magnitude

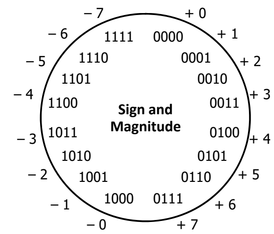
Most Significant Bit

- Designate the high-order bit (MSB) as the "sign bit"
  - `sign=0`: positive numbers; `sign=1`: negative numbers
- Benefits:
  - Using MSB as sign bit matches positive numbers with unsigned
  - All zeros encoding is still = 0
- Examples (8 bits):
  - 0x00 = 00000000_2 is non-negative, because the sign bit is 0
  - 0x7F = 01111111_2 is non-negative ($+127_{10}$)
  - 0x85 = 10000101_2 is negative (-5_{10})
  - 0x80 = 10000000_2 is negative... zero???

Sign and Magnitude

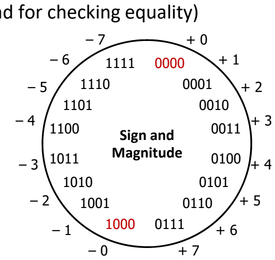
- MSB is the sign bit, rest of the bits are magnitude
- Drawbacks?





Sign and Magnitude

- MSB is the sign bit, rest of the bits are magnitude
- Drawbacks:
  - Two representations of 0 (bad for checking equality)



Sign and Magnitude

- MSB is the sign bit, rest of the bits are magnitude
- Drawbacks:
  - Two representations of 0 (bad for checking equality)
  - Arithmetic is cumbersome
    - Example: `4-3 != 4+(-3)`

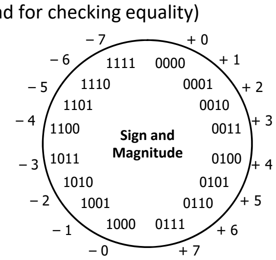
4	0100
- 3	- 0011
1	0001

✓

4	0100
+ -3	+ 1011
-7	1111

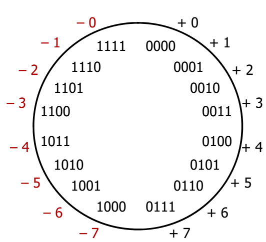
✗

    - Negatives "increment" in wrong direction!



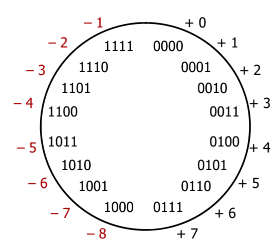
Two's Complement

- Let's fix these problems:
  1) "Flip" negative encodings so incrementing works



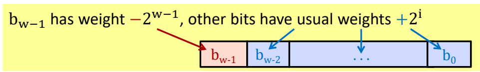
Two's Complement

- Let's fix these problems:
  1) "Flip" negative encodings so incrementing works
  2) "Shift" negative numbers to eliminate –0
- MSB *still* indicates sign!
  - This is why we represent one more negative than positive number (-2^{N-1} to $2^{N-1}-1$)

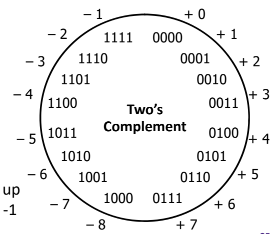


Two's Complement Negatives

- Accomplished with one neat mathematical trick!

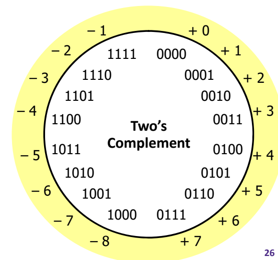


- 4-bit Examples:
  - 1010_2 unsigned: $1*2^3+0*2^2+1*2^1+0*2^0$ = **10**
  - 1010_2 two's complement: $-1*2^3+0*2^2+1*2^1+0*2^0$ = **–6**
- -1 represented as: $1111_2 = -2^3+(2^3 - 1)$
  - MSB makes it super negative, add up all the other bits to get back up to -1



Why Two's Complement is So Great

- Roughly same number of (+) and (–) numbers
- Positive number encodings match unsigned
- Single zero
- All zeros encoding = 0
- Simple negation procedure:
  - Get negative representation of any integer by taking bitwise complement and then adding one!
    `( ~x + 1 == -x )`



Peer Instruction Question

- Take the 4-bit number encoding `x = 0b1011`
- Which of the following numbers is NOT a valid interpretation of x using any of the number representation schemes discussed today?
  - Unsigned, Sign and Magnitude, Two's Complement
  - Vote at http://PollEv.com/justinh

A. -4
B. -5
C. 11
D. -3
E. We're lost…

Summary

- Bit-level operators allow for fine-grained manipulations of data
  - Bitwise AND (&), OR (|), and NOT (~) different than logical AND (&&), OR (||), and NOT (!)
  - Especially useful with bit masks
- Choice of *encoding scheme* is important
  - Tradeoffs based on size requirements and desired operations
- Integers represented using unsigned and two's complement representations
  - Limited by fixed bit width
  - We'll examine arithmetic operations next lecture