CSE 351

Number Representation & Operators
Section 2
October 8, 2015

Number Bases

- Any numerical value can be represented as a linear combination of powers of n, where n is an integer greater than 1
- Example: decimal (n=10)
    - Decimal numbers are just linear combinations of 1, 10, 100, 1000, etc
    - 1234 = 1*1000 + 2*100 + 3*10 + 4*1
- We can also use the base n=2 (binary) or n=16 (hexadecimal)

Binary Numbers

- Each digit is either a 1 or a 0
- Each digit corresponds to a power of 2
- Why use binary?
    - Easy to physically represent two states in memory, registers, across wires, etc
    - High/Low voltage levels
    - This can scale to much larger numbers by using more hardware to store more bits

Converting Binary Numbers

- To convert the decimal number *d* to binary, do the following:
- Compute (*d* % 2). This will give you the lowest-order bit
- Continue to divide *d* by 2, round down to the nearest integer, and compute (*d % 2)* for successive bits
- Example: Convert 25 to binary
  - First bit: (25 % 2) = 1
  - Second bit: (12 % 2) = 0
  - Third bit: 6 % 2 = 0
  - Fourth bit: 3 % 2 = 1
  - Fifth bit: 1 % 2 = 1
  - Stop because we reached zero

Hexadecimal Numbers

- Same concept as decimal and binary, but the base is 16
- Why use hexadecimal?
  - Easy to convert between hex and binary
  - Much more compact than binary

Converting Hexadecimal Numbers

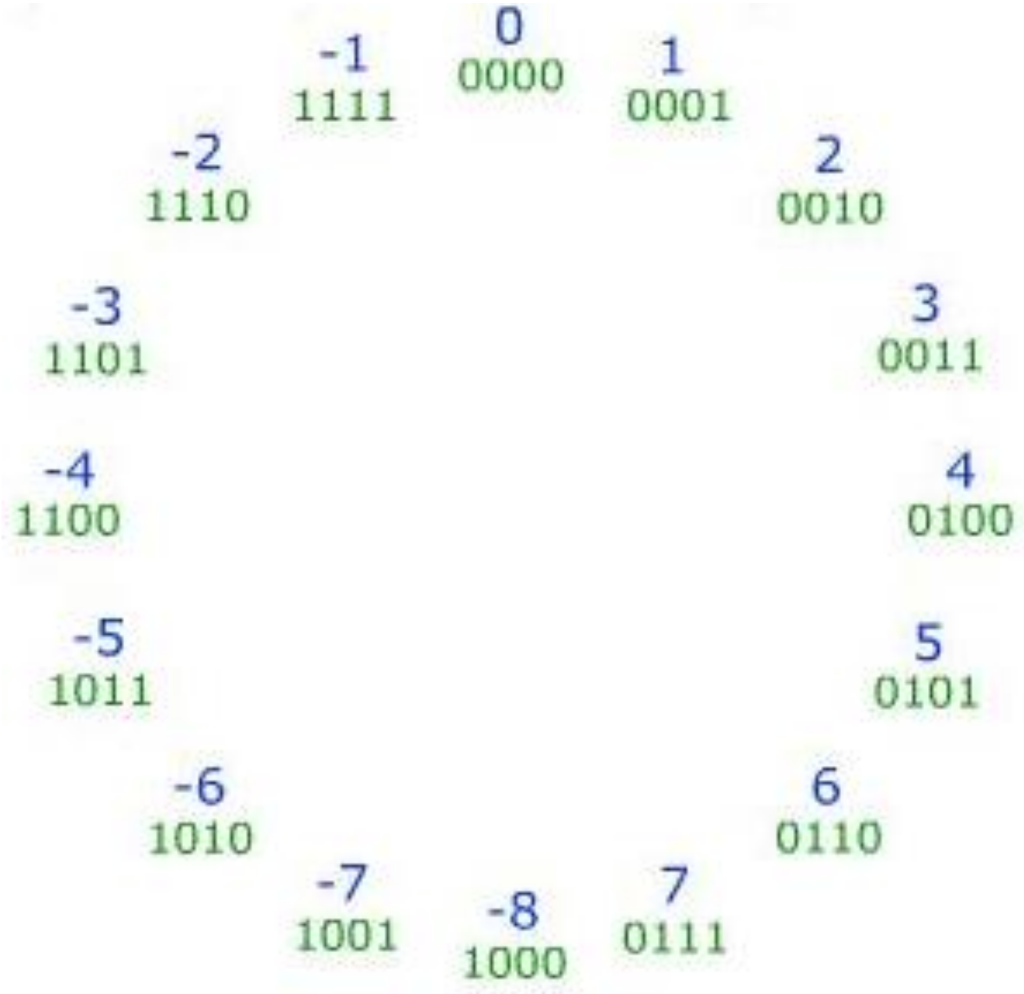
- To convert a decimal number to hexadecimal, use the same technique we used for binary, but divide/mod by 16 instead of 2
- Hexadecimal numbers have a prefix of “0x”
- Example: Convert 1234 to hexadecimal
    - First digit: (1234 % 16) = 2
    - 1234 / 16 = 77
    - Second digit: (77 % 16) = 13 = D
    - 77 / 16 = 4
    - Third digit: 4 % 16 = 4
    - 4 / 16 = 0
    - Stop because we reached zero
    - Result: 0x4D2

Representing Signed Integers

- There are several ways to represent signed integers
- Sign & Magnitude
  - Use 1 bit for the sign, remaining bits for magnitude
  - Works OK, but there are 2 ways to represent zero (-0 and 0)
  - Also, arithmetic is tricky
- Two's Complement
  - Similar to regular binary representation
  - Highest bit has negative weight rather than positive
  - Works well with arithmetic, only one way to represent zero

Two's Complement

- This is an example of the range of numbers that can be represented by a 4-bit two's complement number
- An *n* bit, two's complement number can represent the range [-2^(n-1), 2^(n-1)-1]
  - Note the asymmetry of this range about 0
- Note what happens when you overflow
- If you still don't understand it, speak up!
  - Very confusing concept



Bitwise Operators

- NOT: ~
  - This will flip all bits in the operand
- AND: &
  - This will perform a bitwise AND on every pair of bits
- OR: |
  - This will perform a bitwise OR on every pair of bits
- XOR: ^
  - This will perform a bitwise XOR on every pair of bits
- SHIFT: <<,>>
  - This will shift the bits right or left

Logical Operators

- NOT: !
  - Evaluates the entire operand, rather than each bit
  - Produces a 1 if == 0, produces 0 otherwise
- AND: &&
  - Produces 1 if both operands are nonzero
- OR: ||
  - Produces 1 if either operand is nonzero

Common Operator Uses

- A double bang (!!) is useful when normalizing values to 0 or 1
  - Imitates Boolean types
- Shifts are useful for multiplying/dividing quickly
  - Most multiplications are reduced to shifts when possible by GCC already
  - When writing assembly routines, shifts will be more useful
  - Shifts are also consistent for negative numbers (thanks to sign extension)
- DeMorgan's Laws:
  - ~(A|B) == (~A & ~B)
  - ~(A&B) == (~A | ~B)

Masks

- These are usually strings of 1s that are used to isolate a subset of bits in an operand
  - Example: the mask 0xFF will “mask” the first byte of an integer
- Once you have created a mask, you can shift it left or right
  - Example: the mask 0xFF << 8 will “mask” the second byte of an integer
- You can apply a mask in different ways
  - To set bits in *x*, you can do *x* = *x* | MASK
  - To invert bits in *x*, you can do *x* = *x* ^ MASK
  - To erase everything but the desired bits in x, do *x* = *x* & MASK

Application: Symmetric Encryption

- This is an example that shows how XOR can be used to encrypt data
- Say Alice wishes to communicate message *M* to Bob
  - Let *M* be the bit string: 0b11011010
- Both Alice and Bob have a secret cipher key *C*
  - Let *C* be the bit string: 0b01100010
- Alice sends Bob the encrypted message *M'* = *M* ^ *C*
  - *M'* = 0b10111000
- Bob applies *C* to *M'* to retrieve *M*
  - *M'* ^ *C* = 0b11011010
- XOR ciphers are not very secure by themselves, but the XOR operation is used in some modes of AES encryption

Application: Gray Codes

- Gray Codes encode numbers such that consecutive numbers only differ in their representations by 1 bit
  - Useful when trying to transfer counter values across different clock domains (common in FIFOs)
  - If each wire represents one binary digit, we want to ensure that when the counter increments, the voltage level changes only on one wire
- Let *n* be our counter output
  - (*n* >> 1) ^ *n* will produce a gray coded version of *n*
- If we receive the gray code *g*, we need to convert it to *n* :

```
for (int mask = g >> 1; mask != 0; mask >> 1) {
        g = g ^ mask;
}
```

- For an example, compile and run [`gray_code.c`]

Lab 1

- Worksheet in class
- Tips
  - Work on 8-bit versions first, then scale your solution to work for 32-bit inputs
  - Save intermediate results in variables for clarity
  - **SHIFTING BY MORE THAN 31 BITS IS UNDEFINED!** It will not yield 0

Example Problems

- Create 0xFFFFFFFF using only one operator
  - Limited to constants from 0x00 -> 0xFF
  - Naïve approach: 0xFF + (0xFF << 8) + (0xFF << 16) …
  - Smart approach: ~0x00 = 0xFFFFFFFF

Example Problems

- Replace the leftmost byte of a 32-bit integer with 0xAB
  - Let our integer be x
  - First, we want to create a mask for the lower 24 bits of the image
    - ~(0xFF << 24) will do that using just two operations
  - (x & mask) will zero out the leftmost 8 bits
  - Now, we want to OR in 0xAB to those zeroed-out bits
  - (x & mask) | (0xAB << 24) will accomplish this
  - Total operators: 5