CSE 351: The Hardware/Software Interface

Section 2

Integer representations, two's complement, and bitwise operators

Integer representations

* In addition to decimal notation, it's important to be able to understand binary and hexadecimal representations of integers
* Decimal: 3735928559
  * No prefix, just the number
* Binary: 0b11011110101011011011111011101111
  * "0b" prefix denotes binary notation
* Hexadecimal: 0xDEADBEEF
  * "0x" prefix denotes hexadecimal notation
* Which notation is the most compact of the three? Why use one over another?

Binary scale

- Each digit in binary notation is either 0b0 (zero) or 0b1 (one)
- To convert from (unsigned) binary to decimal notation, take the sum of the *n*th digit multiplied by 2^{n-1}
  - As an example, 0b1101 = $1 * 2^3 + 1 * 2^2 + 0 * 2^1 + 1 * 2^0 = 8 + 4 + 0 + 1 = 13$

Binary scale

* To convert from decimal to binary, use a combination of division and modulus to get each digit, tracking the remainder
* As an example, let's convert 11 to binary
  * $(11 / 2^0) \% 2 = 1$, so the first digit is 0b1. Remainder is $11 - 1 * 2^0 = 10$
  * $(10 / 2^1) \% 2 = 5 \% 2 = 1$, so the second digit is 0b1. Remainder is $10 - 1 * 2^1 = 8$
  * $(8 / 2^2) \% 2 = 4 \% 2 = 0$, so the third digit is 0b0. Remainder is $8 - 0 * 2^2 = 8$
  * $(8 / 2^3) \% 2 = 1 \% 2 = 1$, so the fourth digit is 0b1
  * Finally, we have that 11 is 0b1011 in binary

Hexadecimal scale

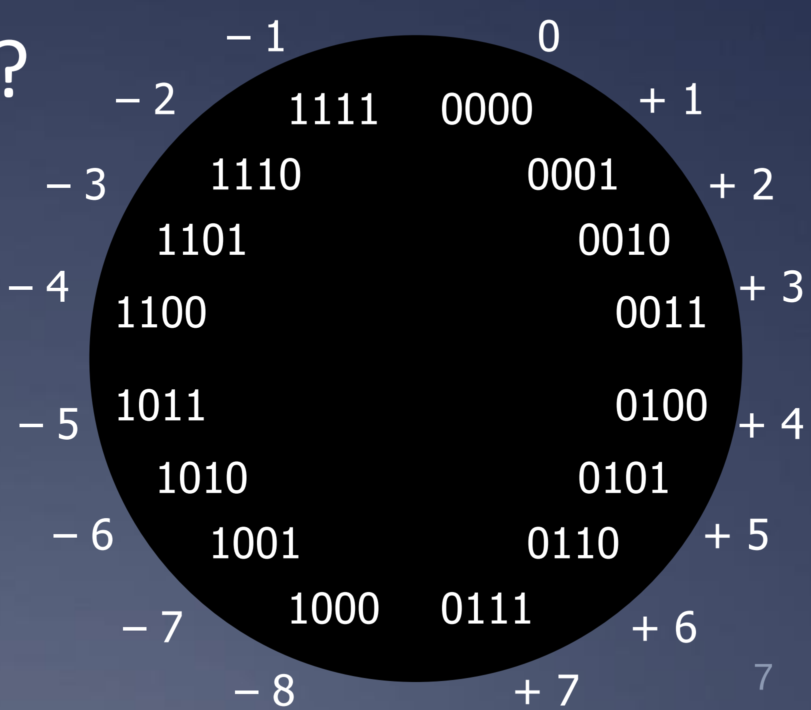
- Each digit ranges in value from 0x0 (zero) to 0xF (fifteen)
  - A => ten, B => eleven, C => twelve, D => thirteen, E => fourteen, F => fifteen
- To convert from (unsigned) hexadecimal to decimal notation, take the sum of the *n*th digit multiplied by 16^{n-1}
  - As an example, 0xACE = 0xA * 16^2 + 0xC * 16^1 + 0xE * 16^0 = 10 * 256 + 12 * 16 + 14 = 2766

Hexadecimal scale

- The decimal to hexadecimal conversion is the same process as decimal to binary except with 2 instead of 16
- As an example, let's convert 3254 to hexadecimal
  - $(3254 / 16^0) \% 16 = 6$, so first digit is 0x6. Remainder is $3254 - 0x6 * 16^0 = 3248$
  - $(3248 / 16^1) \% 16 = 203 \% 16 = 11 = 0xB$, so second digit is 0xB. Remainder is $3248 - 0xB * 16^1 = 3248 - 176 = 3072$
  - $(3072 / 16^2) \% 16 = 12 \% 16 = 12 = 0xC$, so third digit is 0xC
  - Finally, we have that 3254 is 0xCB6 in hexadecimal
- If we were to write a program to convert from decimal to binary or to hexadecimal, how could we compute the *n*th digit efficiently using bitwise operators and modulus (%)?

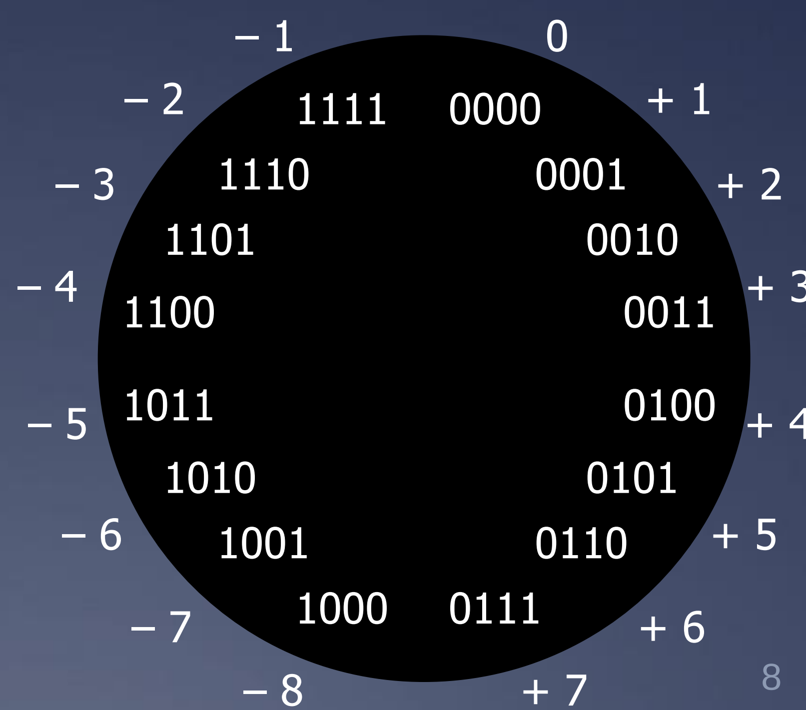
Two's complement review

* In class, we established that two's complement is a nice format for representing signed integers for a couple different reasons. What were they?



Two's complement review

- Let's say that we want to encode -5 in binary using two's complement form and four bits
  - With four bits, the highest bit has a negative weight of 2^3, so 0b1000 = -8
  - -5 = -8 + 2 + 1
    $= 1 * -2^3 + 0 * 2^2 +$
    $1 * 2^1 + 1 * 2^0$
    = 10b1011
  - 5 = 4 + 1
    $= 0 * -2^3 + 1 * 2^2 + 0 * 2^1 +$
    $1 * 2^0$
    = 0b0101



Operator review

- ~ is arithmetic not (flip all bits)
  - Example: ~0b1010 = 0b0101
- ! is logical not (1 if 0b0, else 0)
  - Example: !0b100 = 0, !0b0 = 1
- & is bitwise and
  - Example: 0b101 & 0b110 = 0b100
- | is bitwise or
  - Example: 0b101 | 0b100 = 0b101
- >> is bitwise right shift
  - Example: 0b1010 >> 1 = 0b1101, 0b0101 >> 1 = 0b0010
- << is bitwise left shift
  - Example: 0b1010 << 1 = 0b0100, 0b1000 << 1 = 0b0000

Operator uses

- Can express negation in terms of arithmetic not and addition
  - For example, ~4 + 1 = ~0b0100 + 1 = 0b1011 + 1 = -5 + 1 = -4
- Can use shifting, bitwise and, and logical not to detect if a particular bit is set
  - As a simple example, !!(x & (0x1 << 1)) evaluates to 1 if the second bit it set in x and 0 otherwise
  - Useful for checking if a value is negative
- Can implement ternaries (x = __ ? __ : __) using bitwise and, bitwise or, and arithmetic not
  - This has wide-ranging applications in lab 1

Bitwise operators in practice

- Is what we're learning ever useful in practice?
  - Thankfully (or not, depending on how you look at it), it is
  - Setting bits in permission strings
    - For example, to choose the

Packing and unpacking

- Let’s say that you have values x, y, and z that take 3, 4, and 1 bit to represent, respectively
- Is there a way to store these three values using only eight bits?
- In C, we can define a struct that specifies the width in bits of each value
  - …though the compiler will add padding to make the struct a certain size if you don’t do so yourself
- In Java, there are no structs, and we have to use bitwise operators

Packing and unpacking (C)

```
#include <stdio.h>

typedef struct {
  int x : 3;
  int y : 4;
  int z : 1;
  int padding : 24;
} Flags;

int main(int argc, char* argv[]) {
  Flags flags = {3, 8, 1, 0x8fffff};
  printf("sizeof(flags) is %ju and it stores 0x%x\n",
         sizeof(flags), *(int*) &flags);

  return 0;
}
```

Packing and unpacking (Java)

```
// Pack some values into a byte
byte bitValue = 0;
bitValue |= 3;
bitValue |= 8 << 3;
bitValue |= 1 << 7;

// Unpack the values from the byte
byte x = bitValue & 0x7;
byte y = bitValue & 0x78;
byte z = bitValue & 0x80;

// Alternatively, we could have shifted a particular
// mask instead, e.g. (0x1 << 7) instead of 0x80
```

Lab 1 hints

* Decompose each problem into smaller problems
* If you are stuck on how to solve something, write it as a combination of functions and boolean logic
  * Over time, replace each function or boolean operator with a combination of permitted operators
* Hint for detecting overflow: what is the sign of the integer produced by adding TMax to a positive value? What about when adding negative numbers?
* Hint for counting bits: consider multiple bits at once. 40 operations isn't enough to check each individually