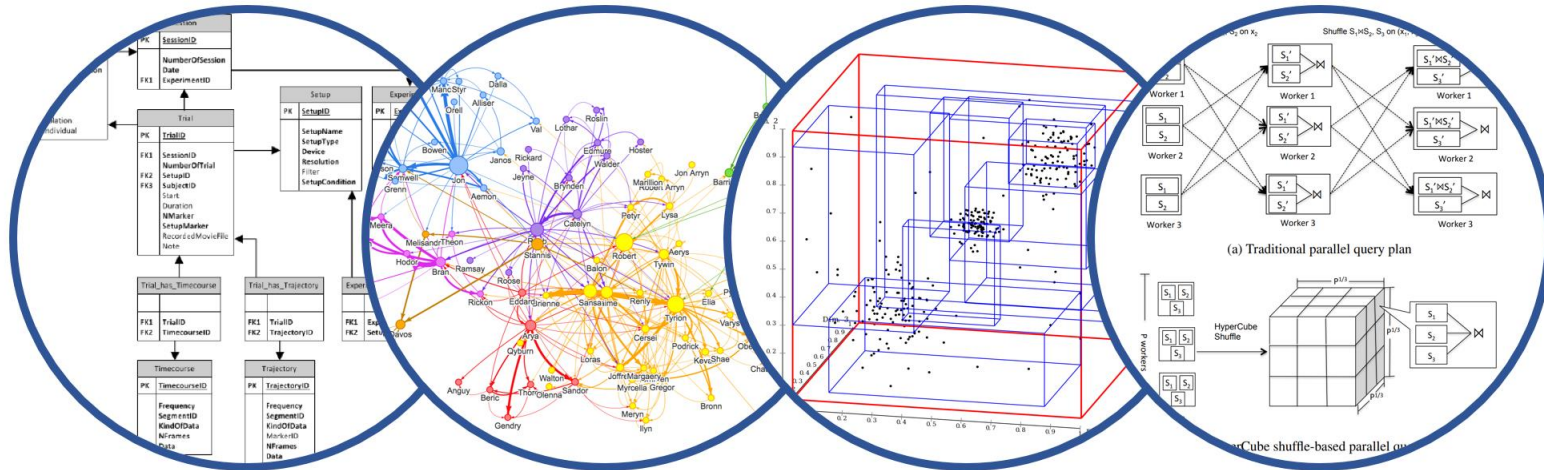




Introduction to Data Management Indexes

Paul G. Allen School of Computer Science and Engineering
University of Washington, Seattle

Announcements

HW6: Part 2 due 11/27. More work than part 1

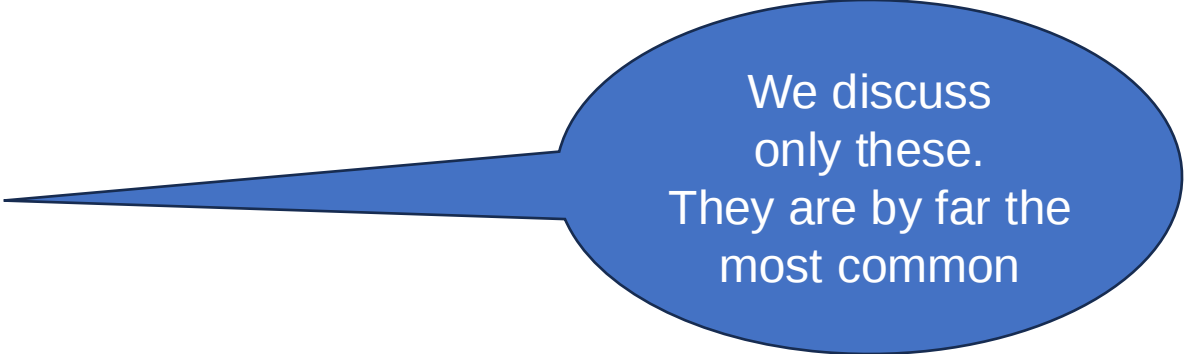
Announcements

- Lecture on Wednesday, Nov. 27:
 - No in-person lecture
 - Recording only
- Monday, Dec. 9, Final Review session
 - CSE2 G10
 - 10:30 – 12:20 (we may finish earlier)

B+ Trees

Index Types

- B+ trees



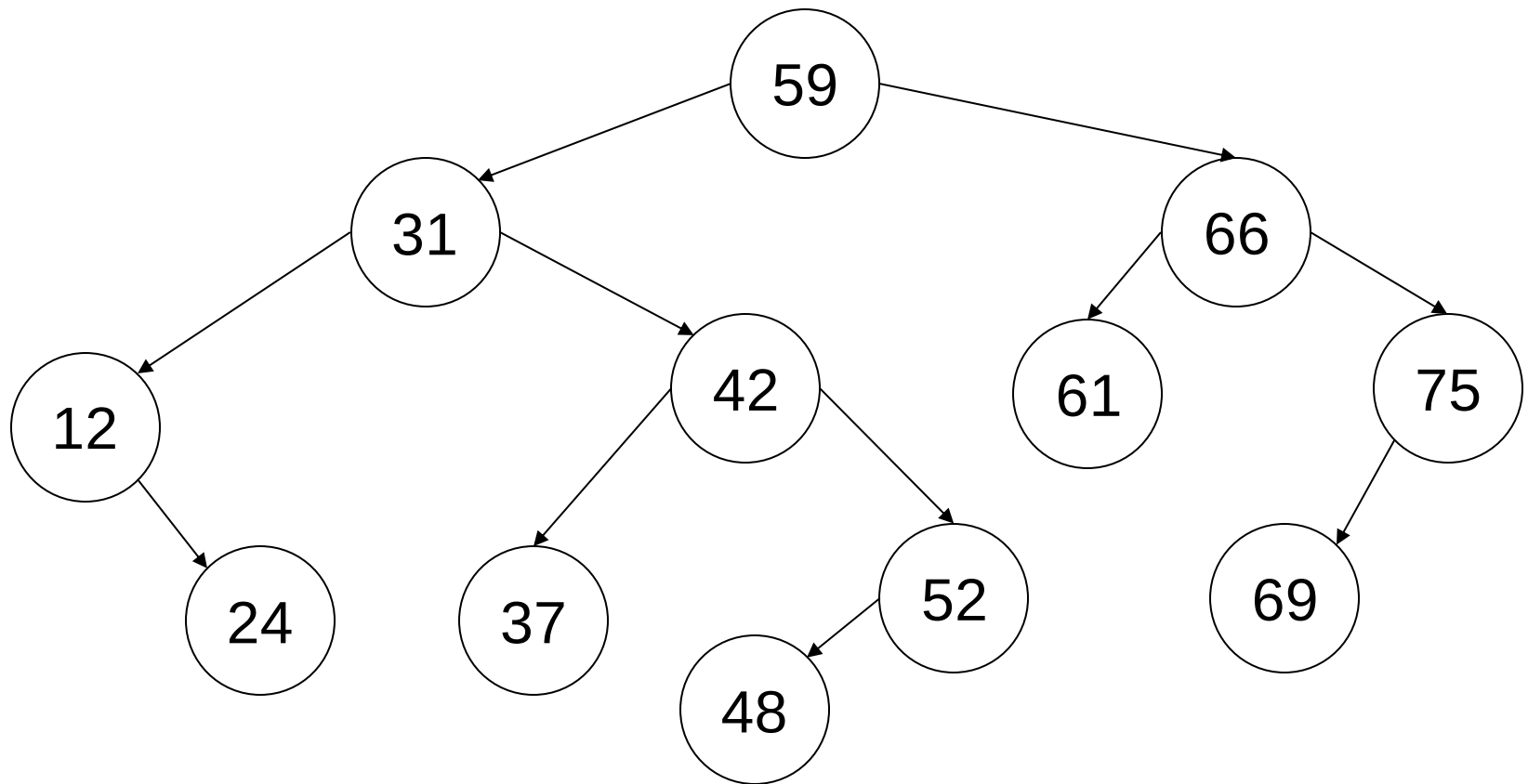
We discuss only these. They are by far the most common

- Hash tables

- Bitmaps (for attributes with few values)

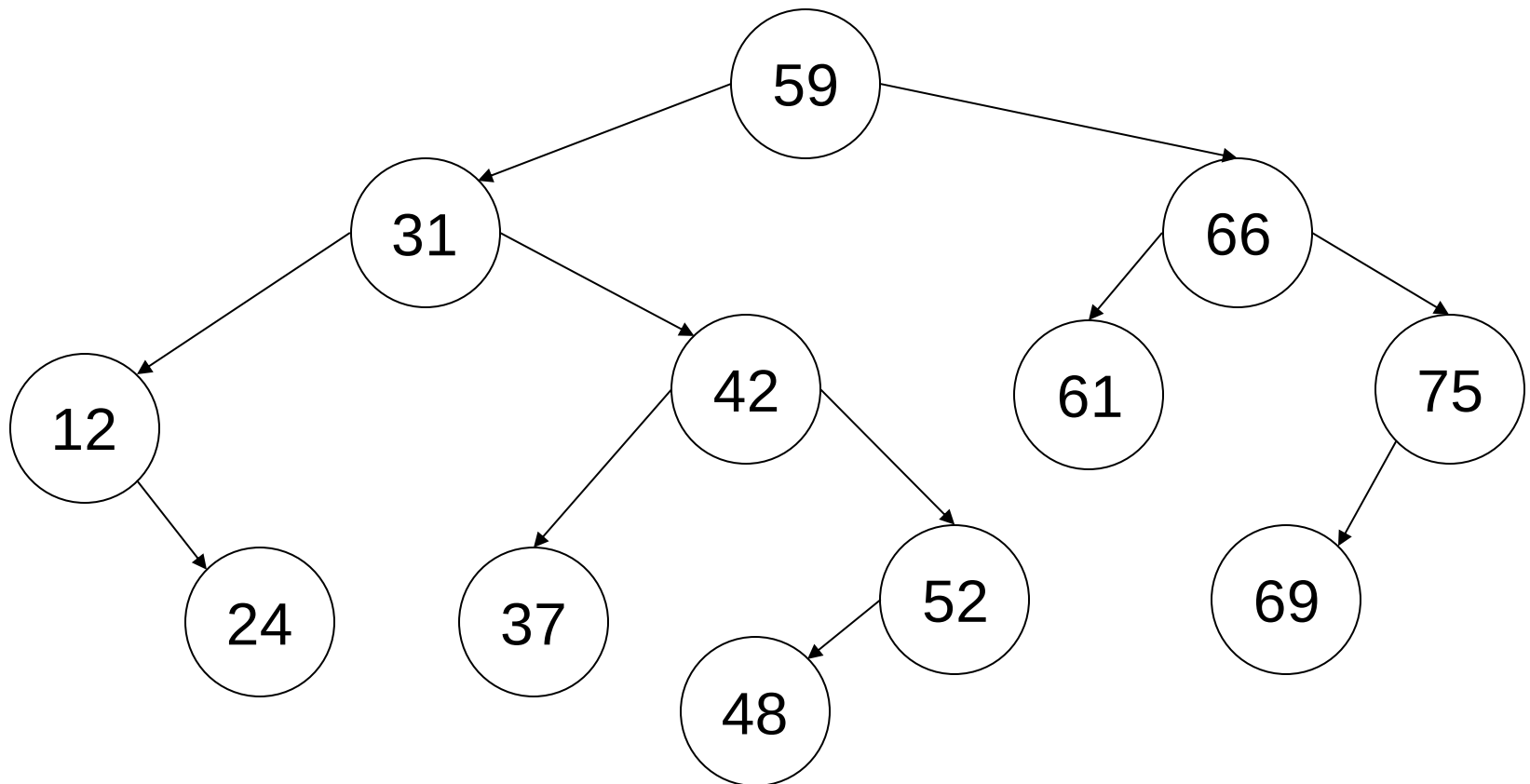
- R+ tree (for 2D data)

Review: Binary Search Tree



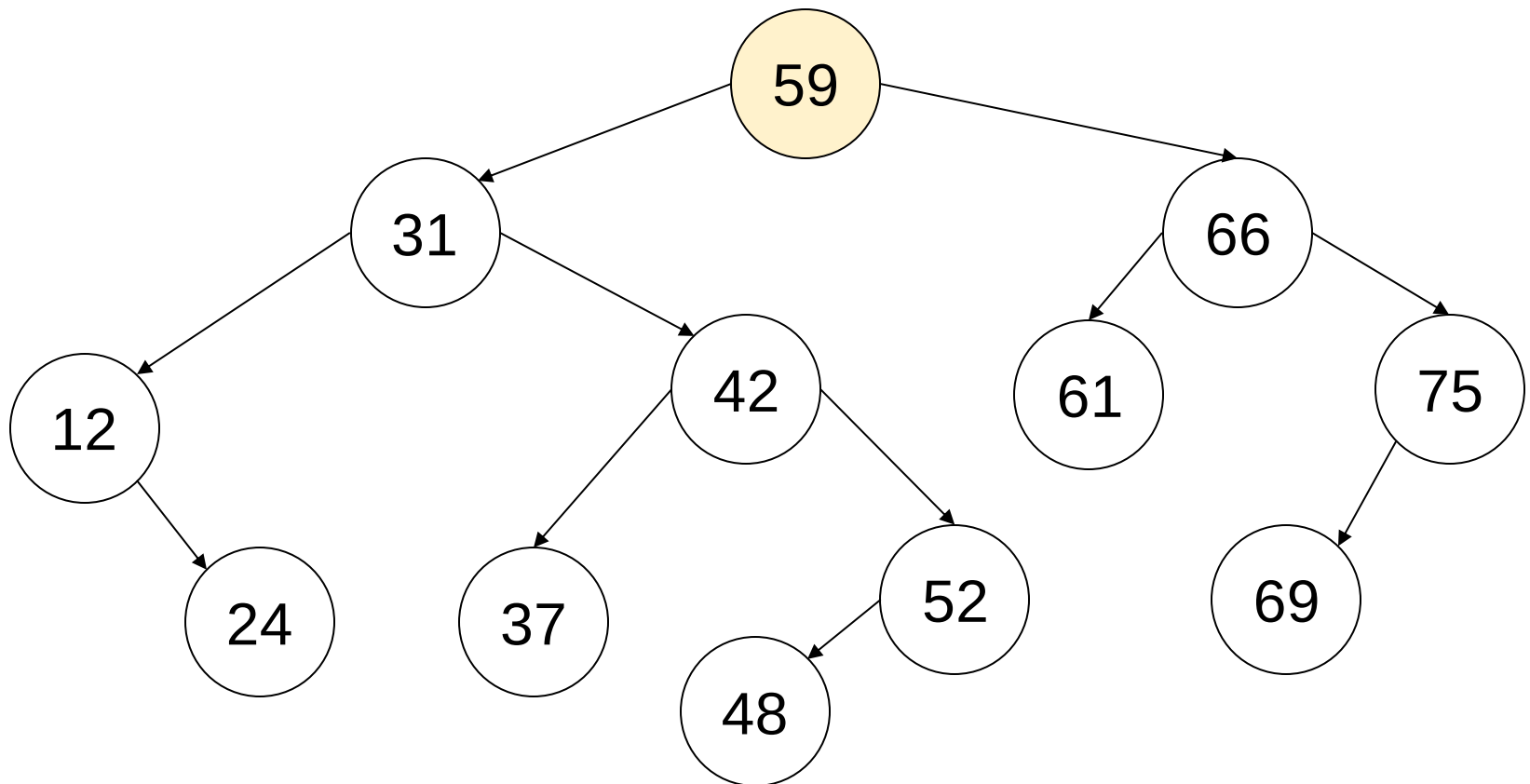
Review: Binary Search Tree

Find 48



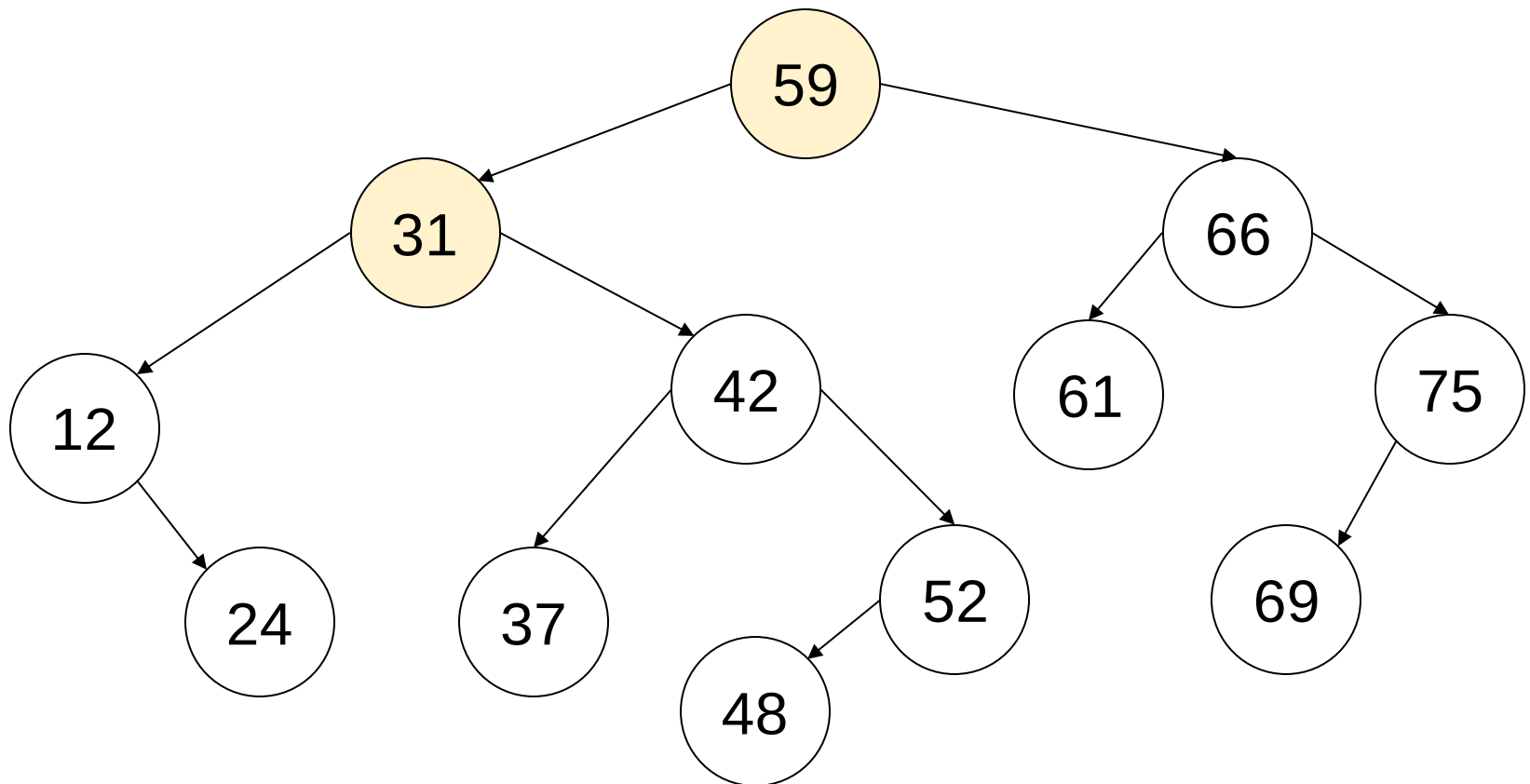
Review: Binary Search Tree

Find 48



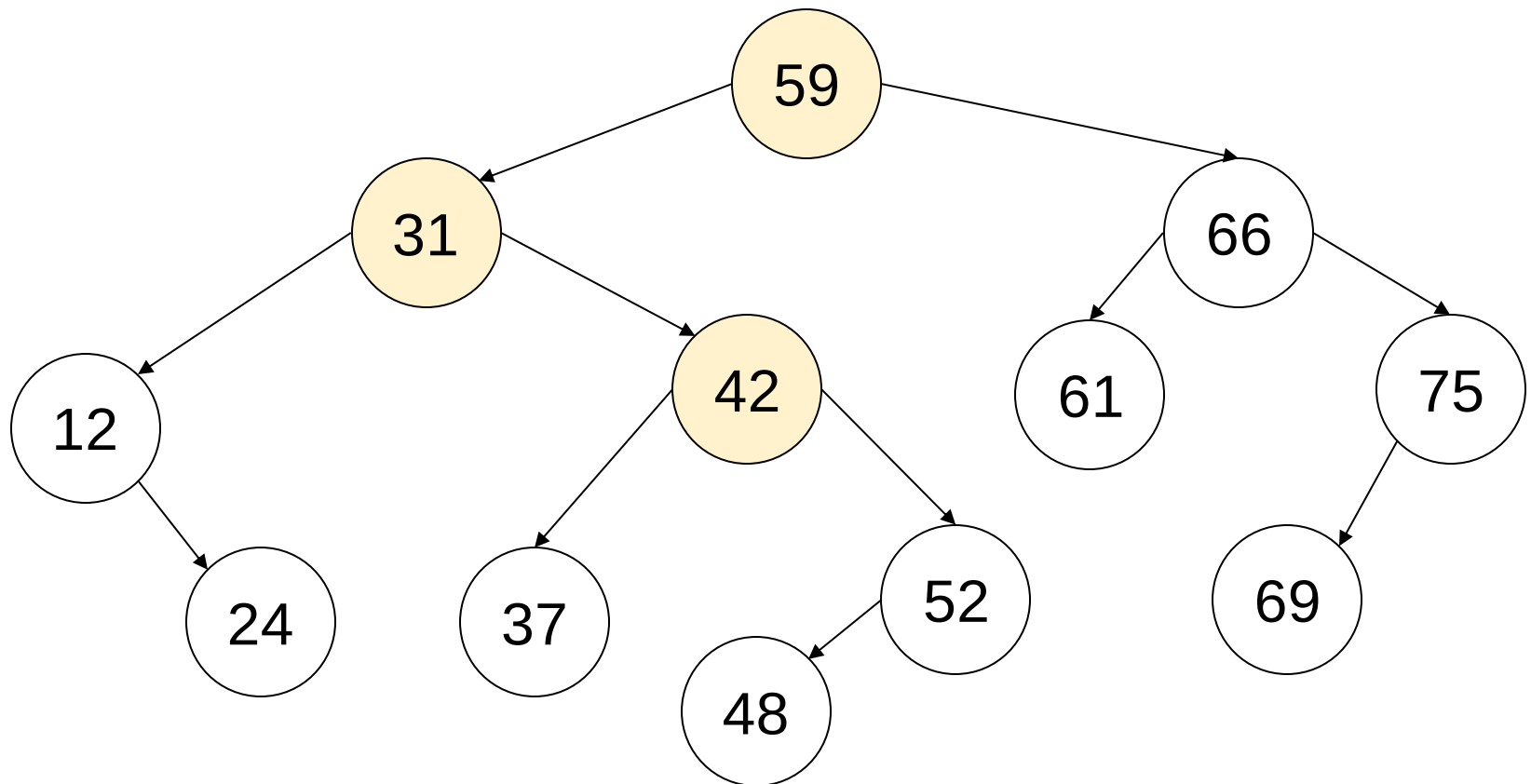
Review: Binary Search Tree

Find 48



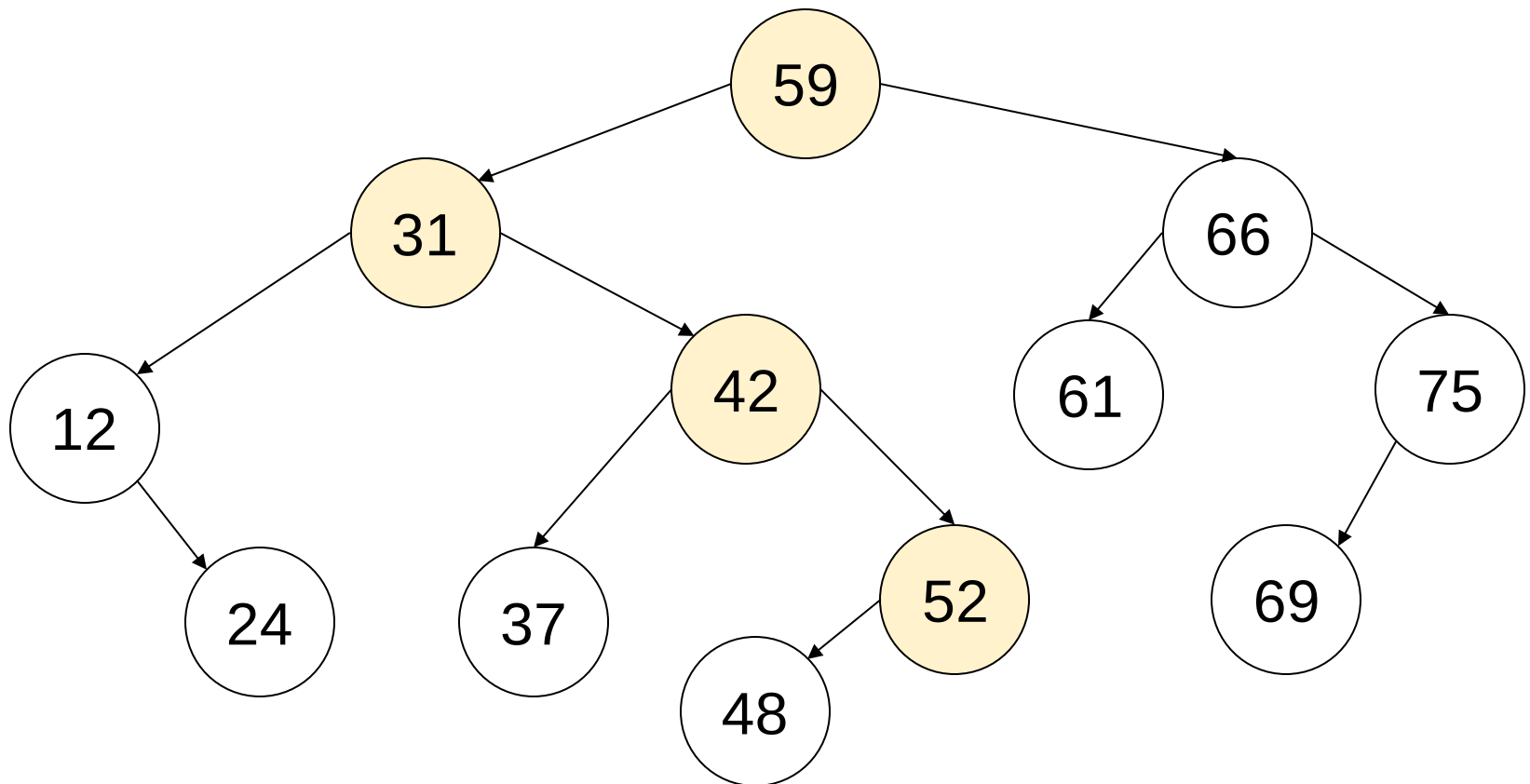
Review: Binary Search Tree

Find 48



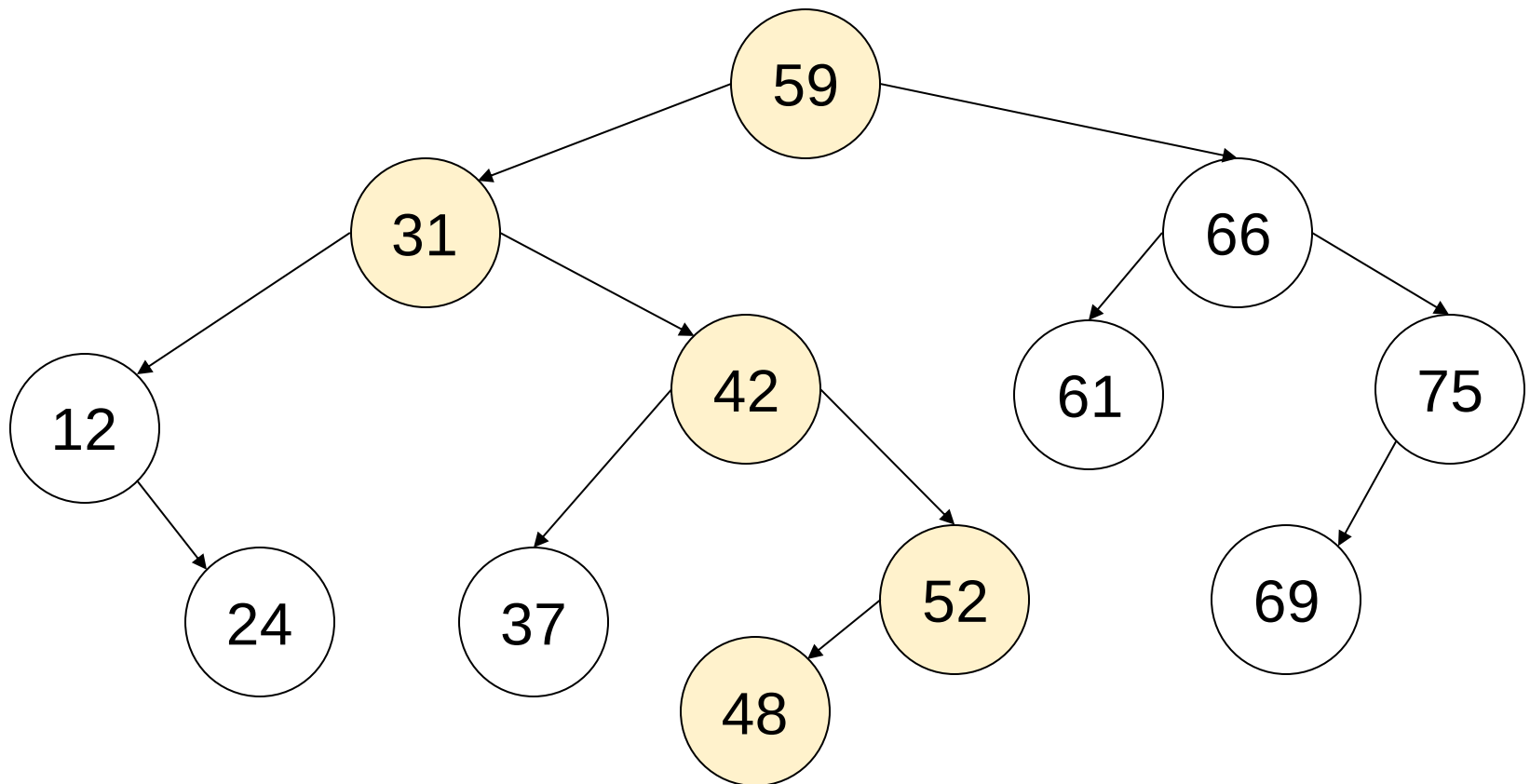
Review: Binary Search Tree

Find 48



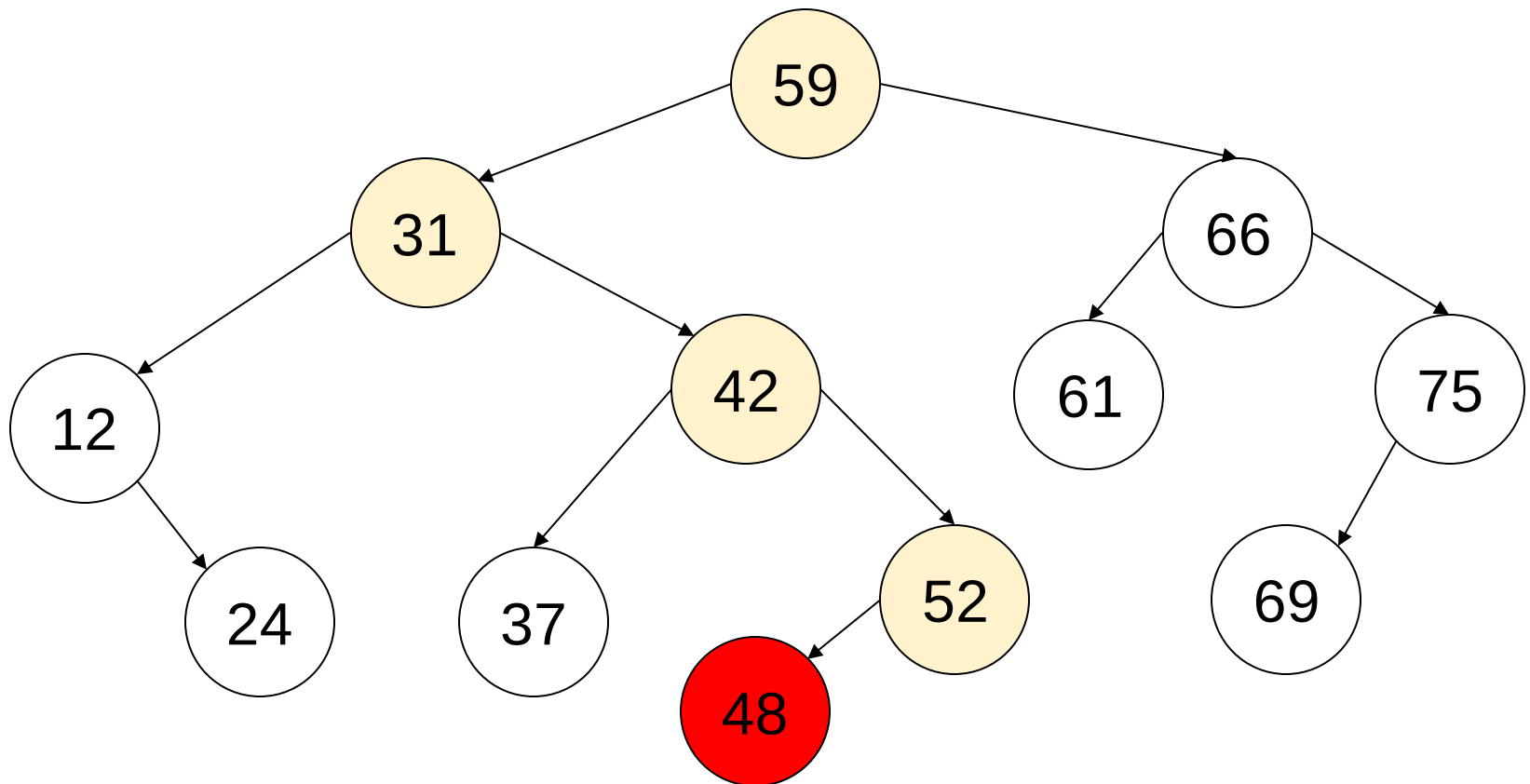
Review: Binary Search Tree

Find 48



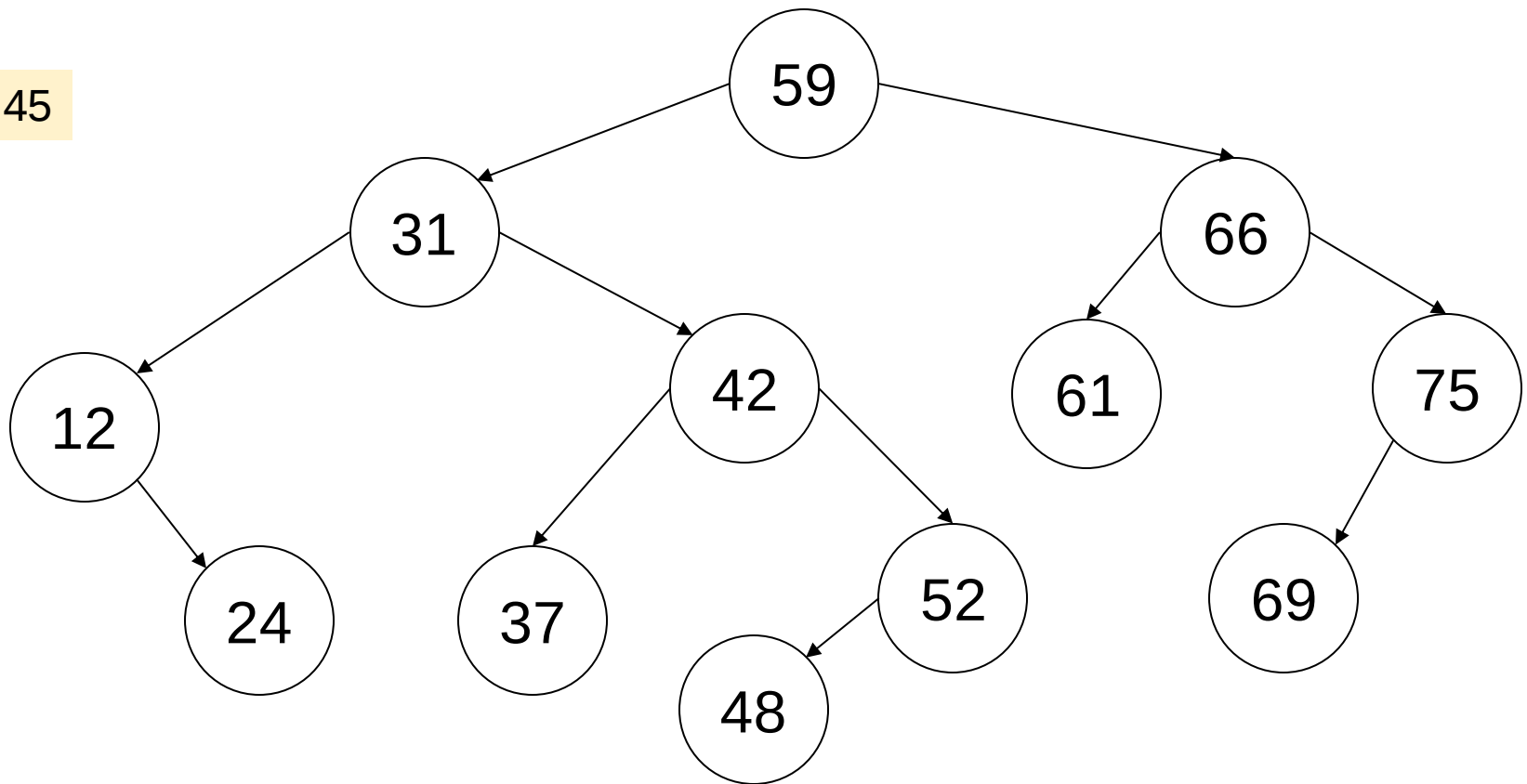
Review: Binary Search Tree

Find 48



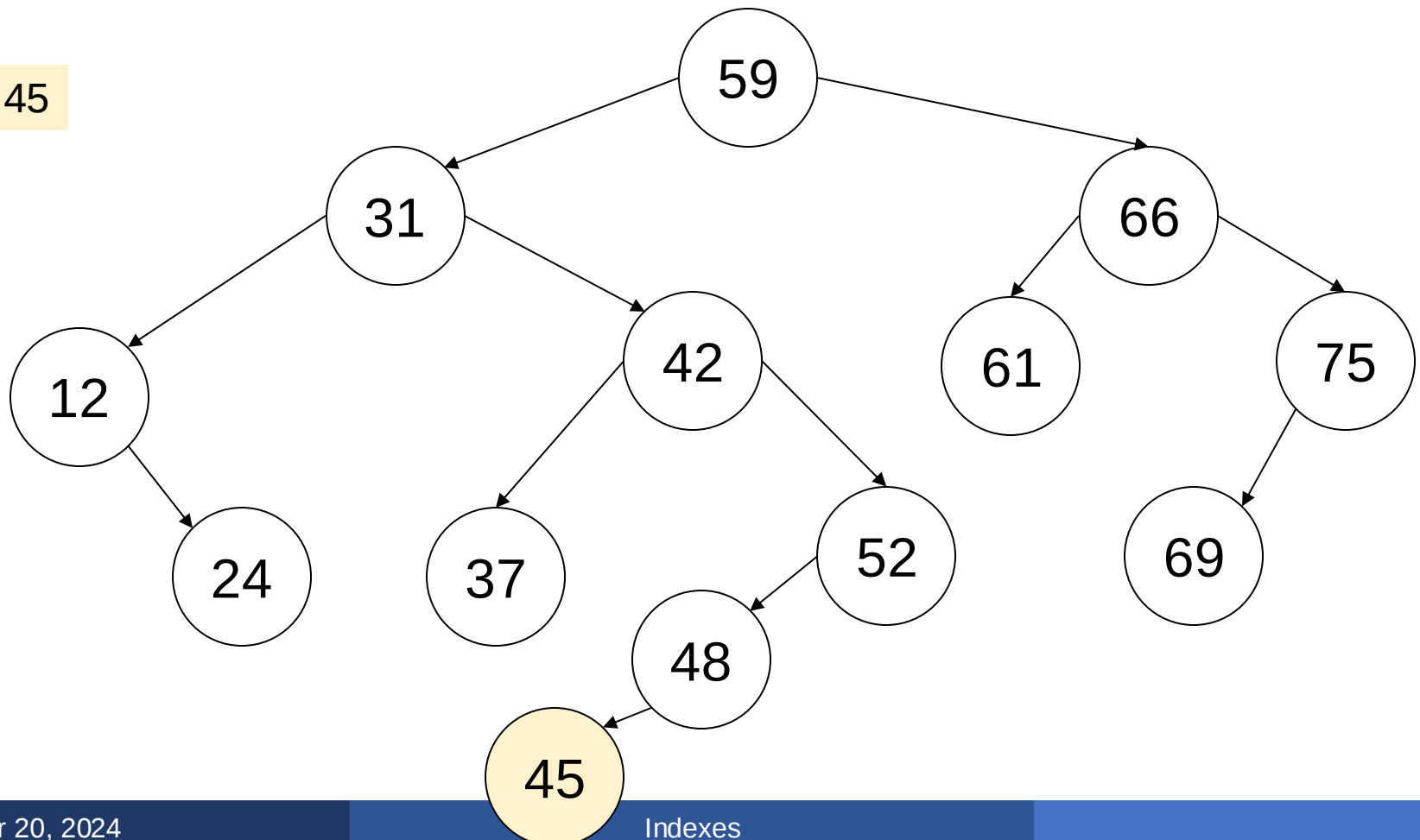
Review: Binary Search Tree

Insert 45



Review: Binary Search Tree

Insert 45



Review: Binary Search Tree

- Need to be balanced: $\text{depth} = O(\log N)$
- Various methods to rebalance:
 - Red/black trees, splay trees, ...
- Time for search/insert/delete = $O(\log N)$

But not good for disk -- why??

B+ Trees

■ Idea in B Trees

- Make 1 node = 1 page (= 1 block)
- Store multiple keys and children
- Read 1 page, use many keys

■ Idea in B+ Trees

- Keys are stored only on leaves
- Internal nodes used only for guiding
- Leaves are linked in a list, for range queries

B+ Trees Properties

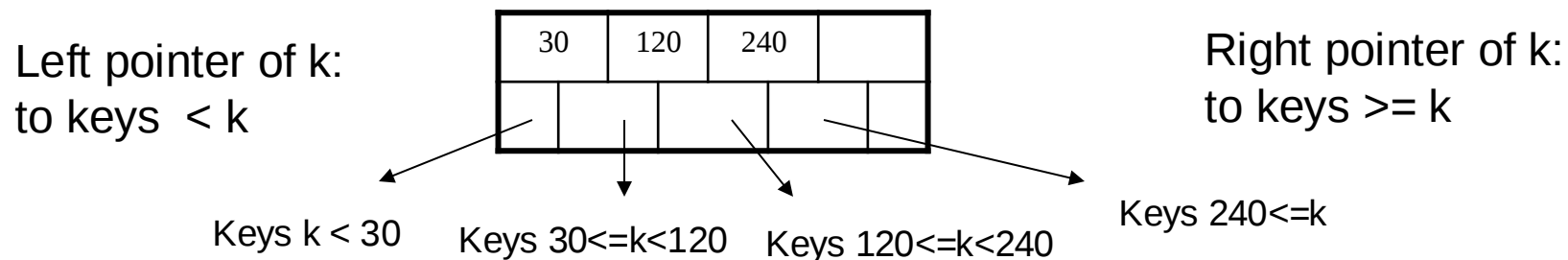
- For each node except the root, maintain 50% occupancy of keys
- Insert and delete must rebalance to maintain constraints

B+ Trees Properties

- Parameter $d = \text{the } \underline{\text{degree}}$
- Each node has $d \leq m \leq 2d$ keys (except root)

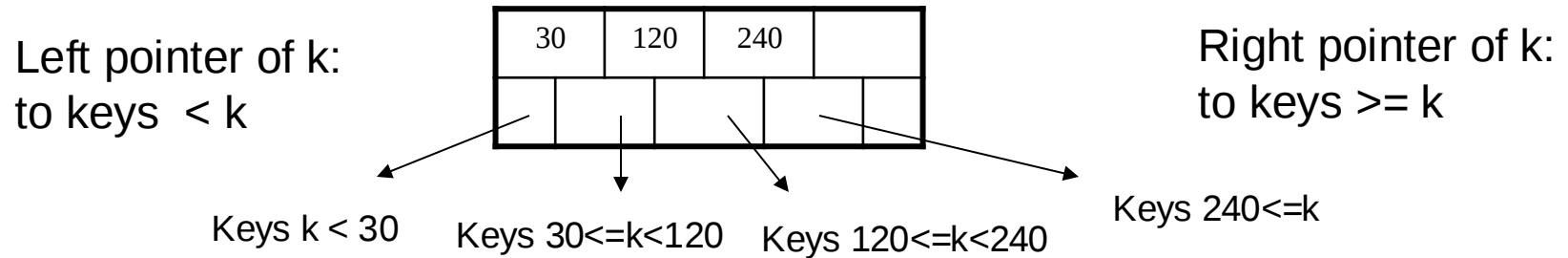
B+ Trees Properties

- Parameter d = the degree
- Each node has $d \leq m \leq 2d$ keys (except root)
- Each node also has $m+1$ pointers

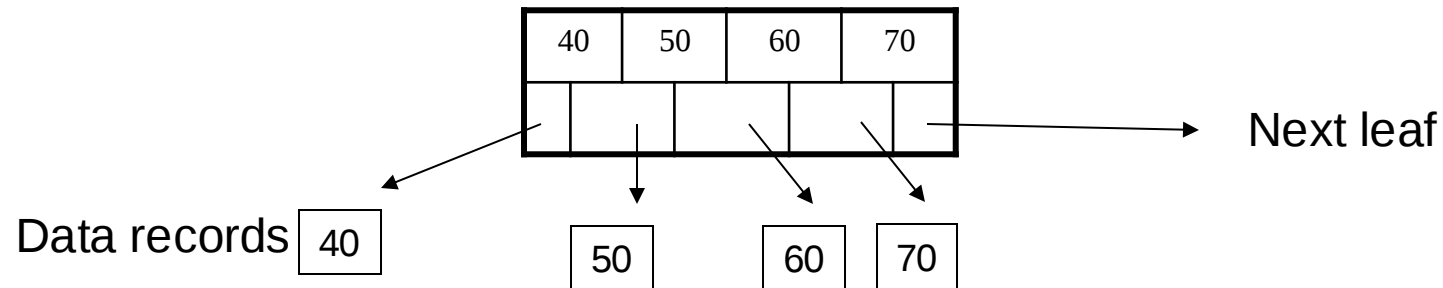


B+ Trees Properties

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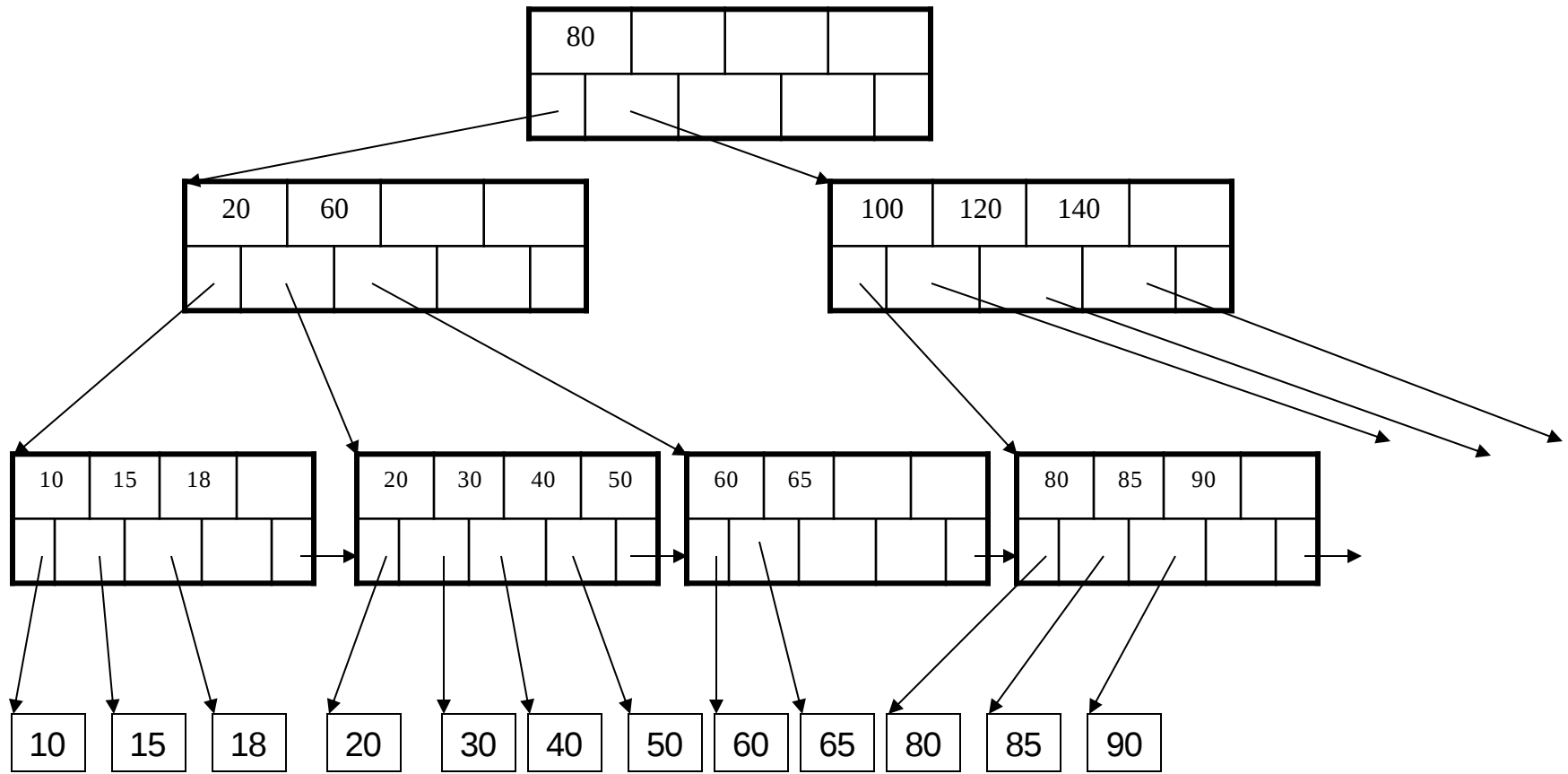


- Each leaf has $d \leq m \leq 2d$ keys:



B+ Tree Example

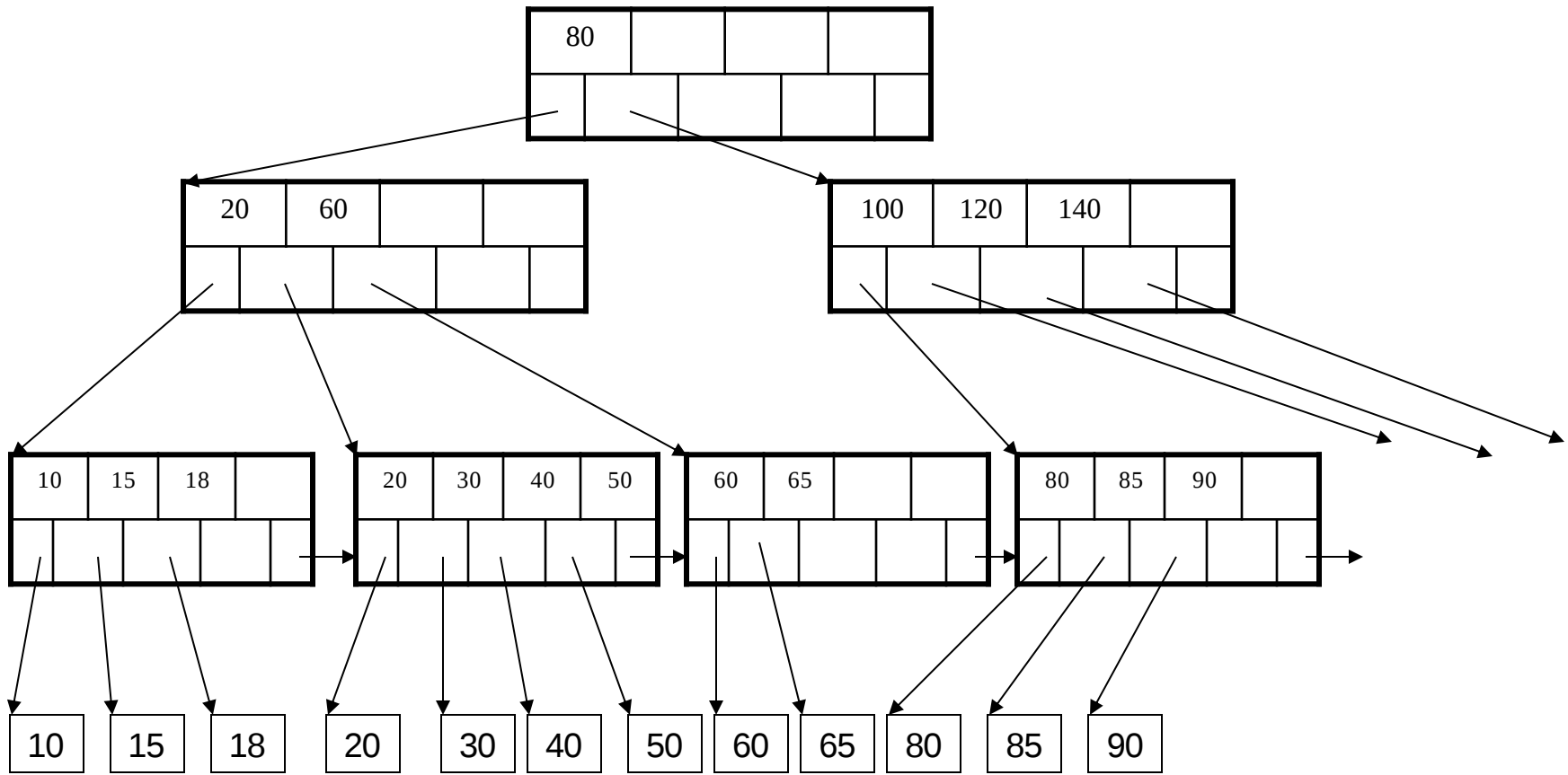
$d = 2$



B+ Tree Example

$d = 2$

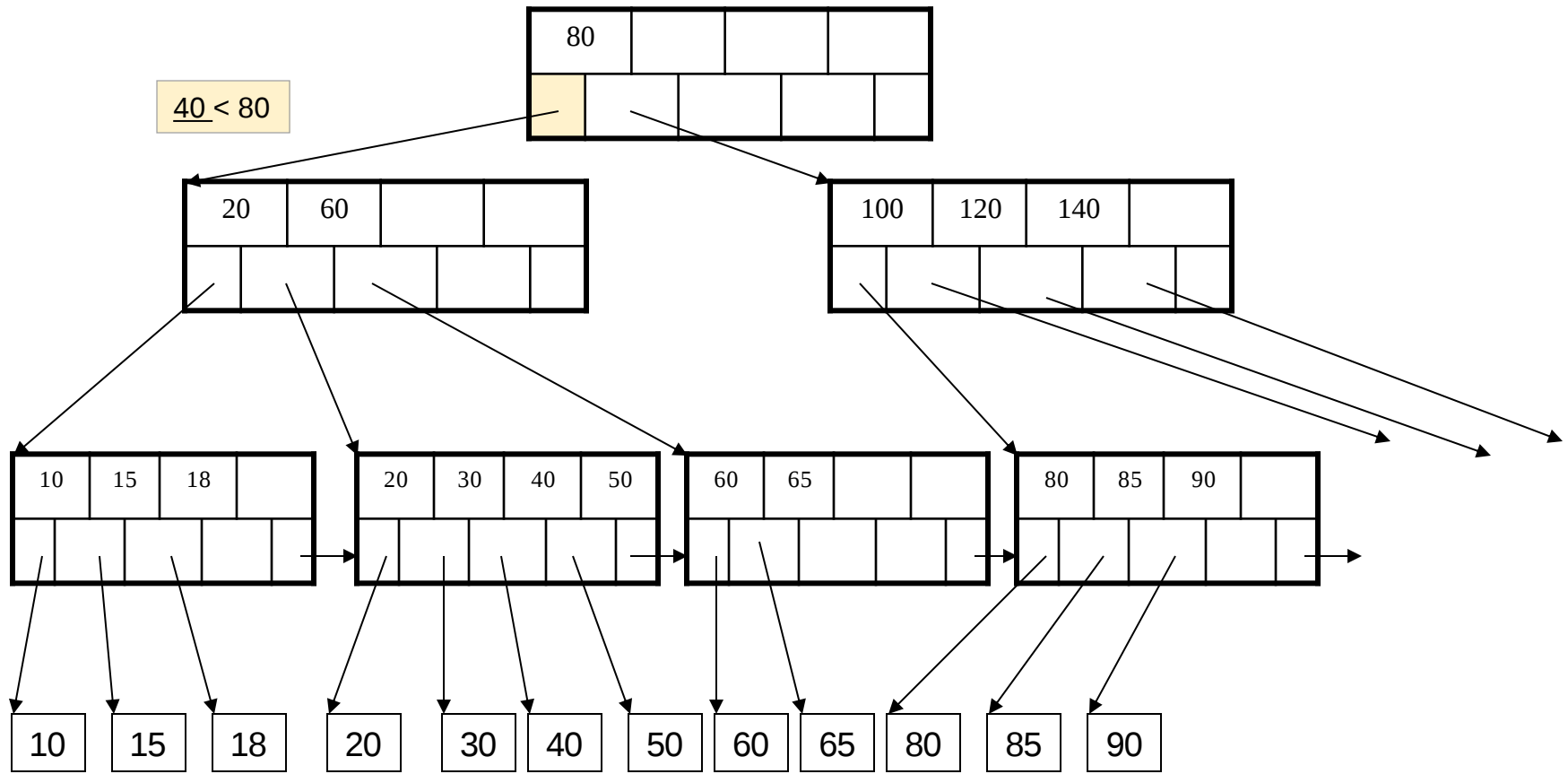
Find the key 40



B+ Tree Example

$d = 2$

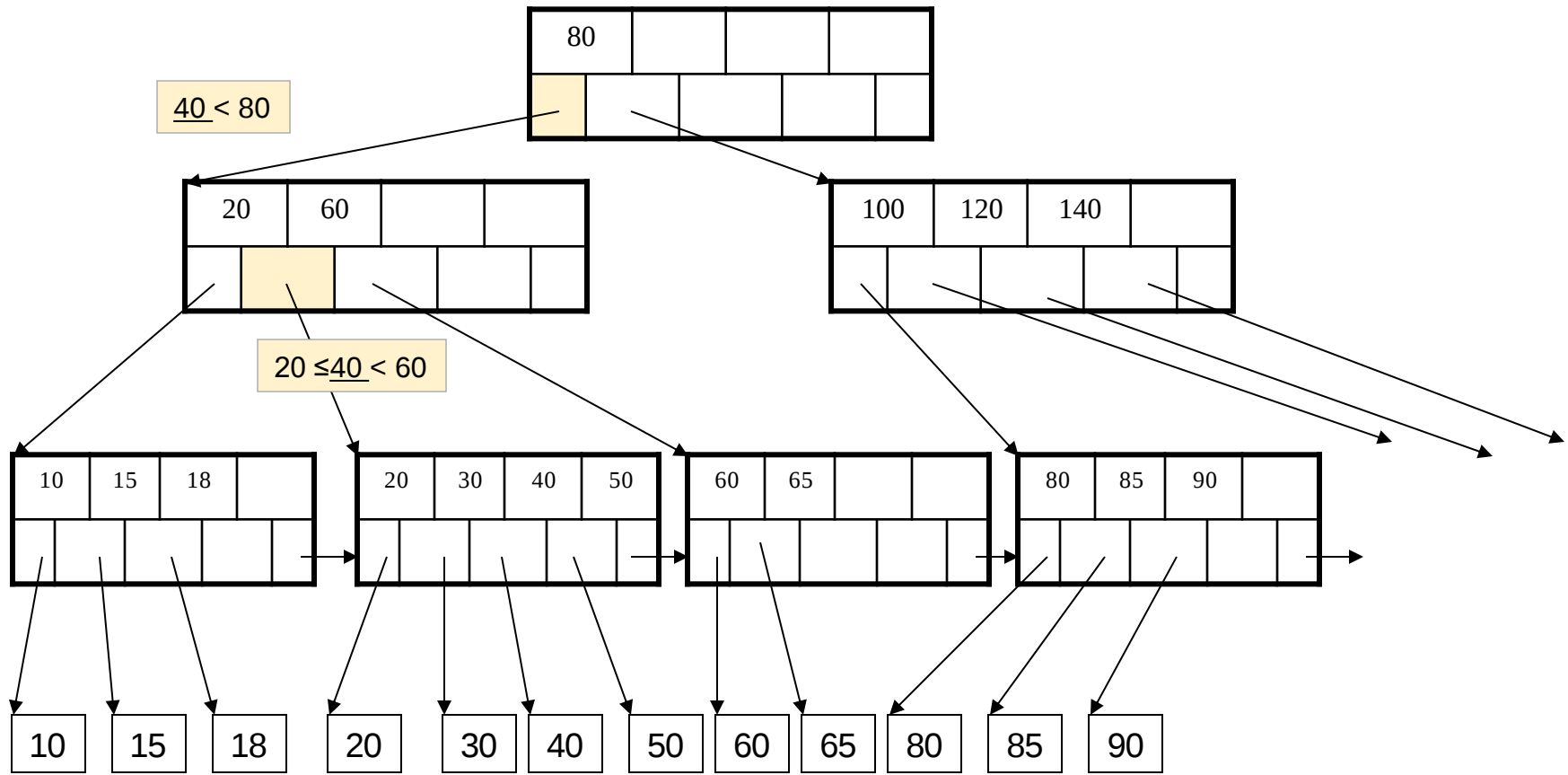
Find the key 40



B+ Tree Example

$d = 2$

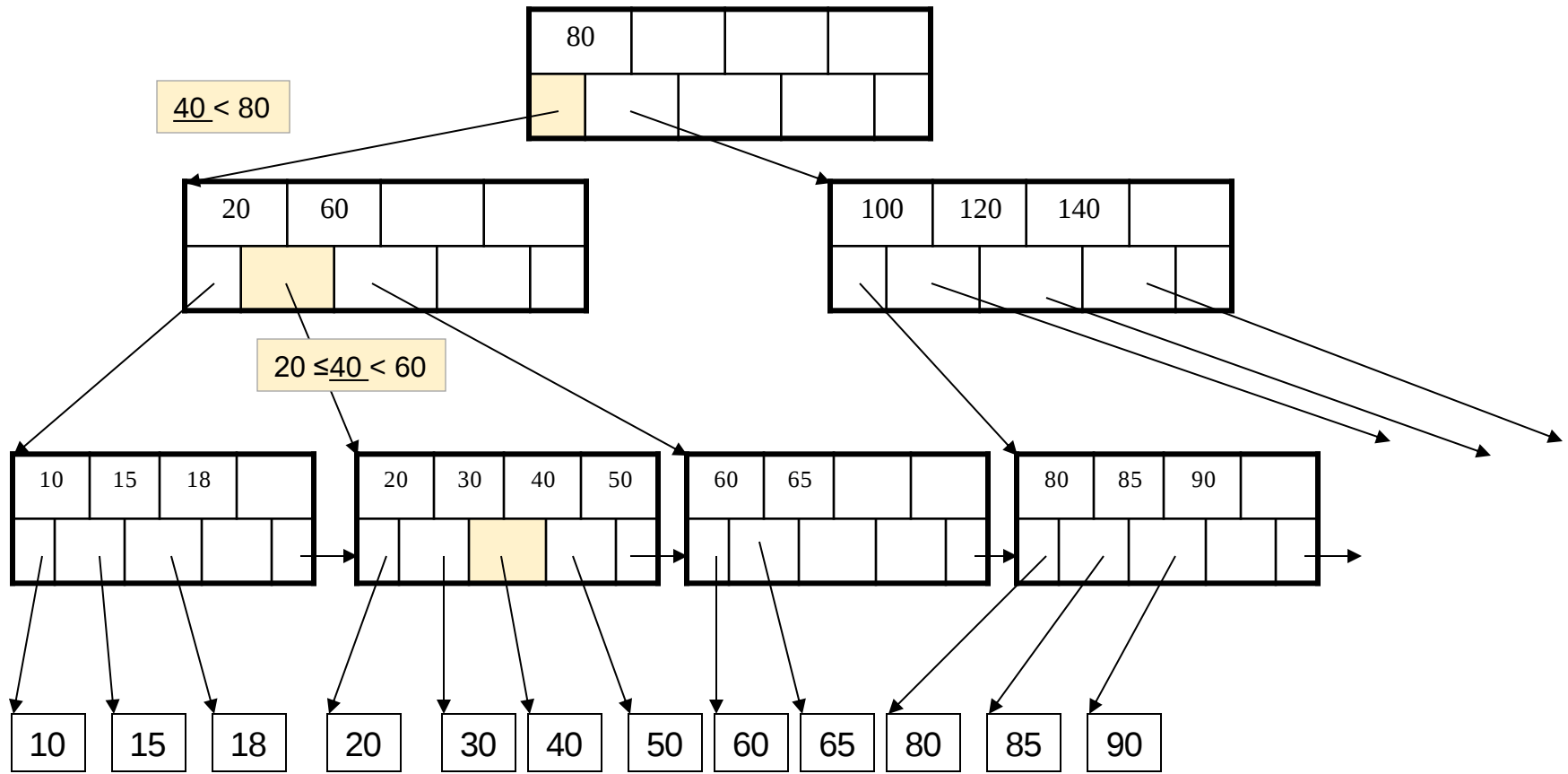
Find the key 40



B+ Tree Example

$d = 2$

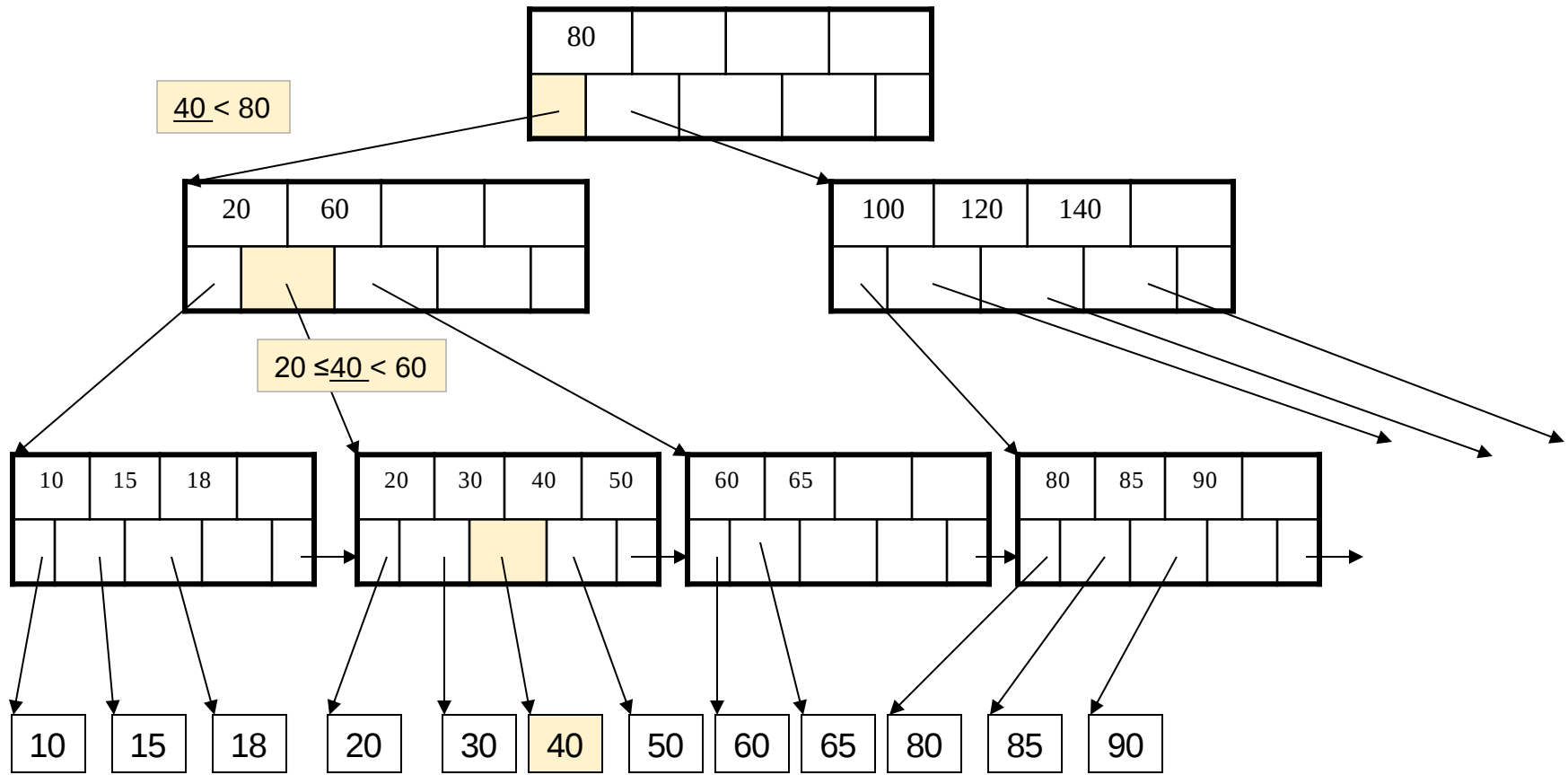
Find the key 40



B+ Tree Example

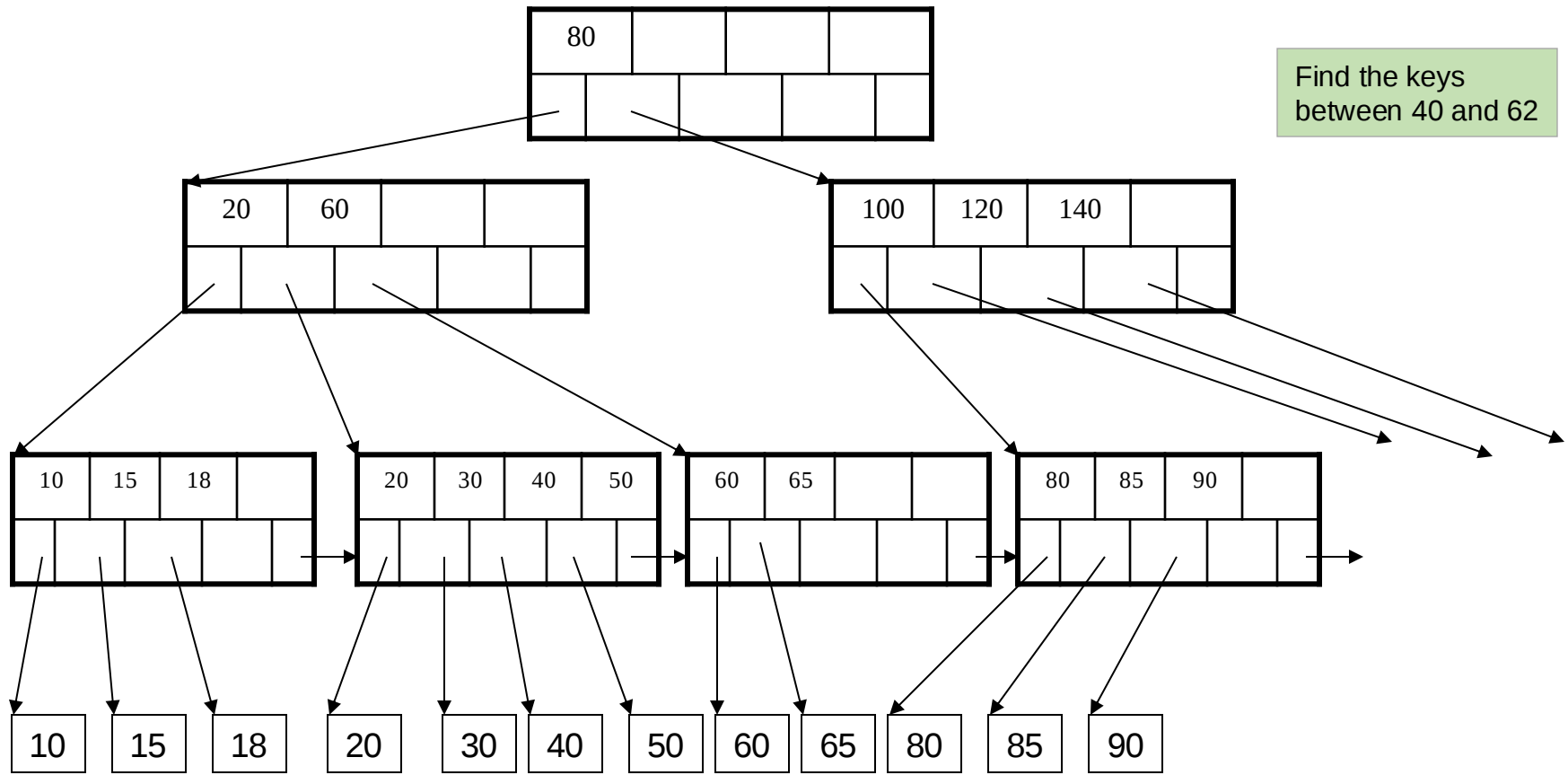
$d = 2$

Find the key 40



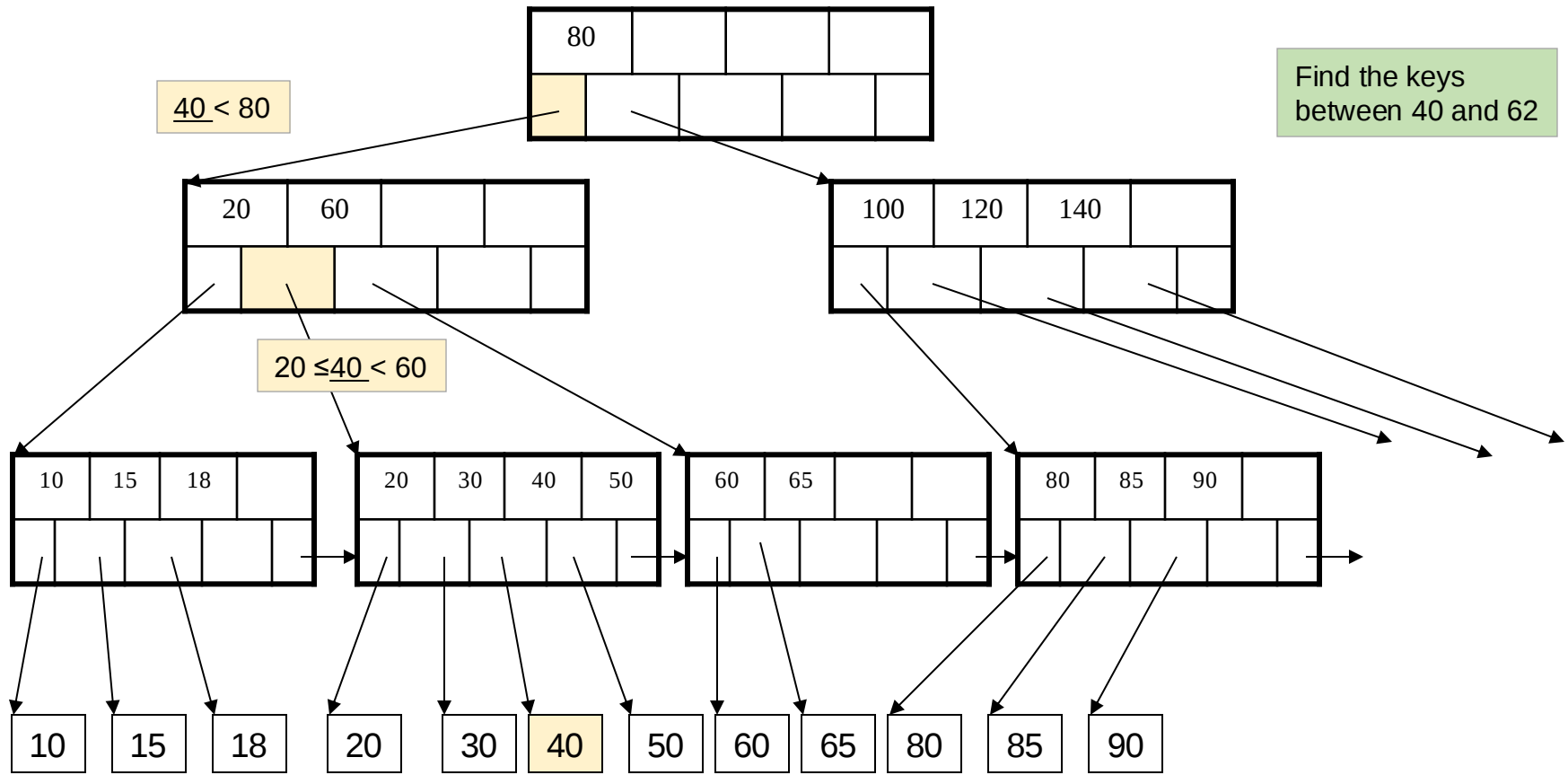
B+ Tree Example

$d = 2$



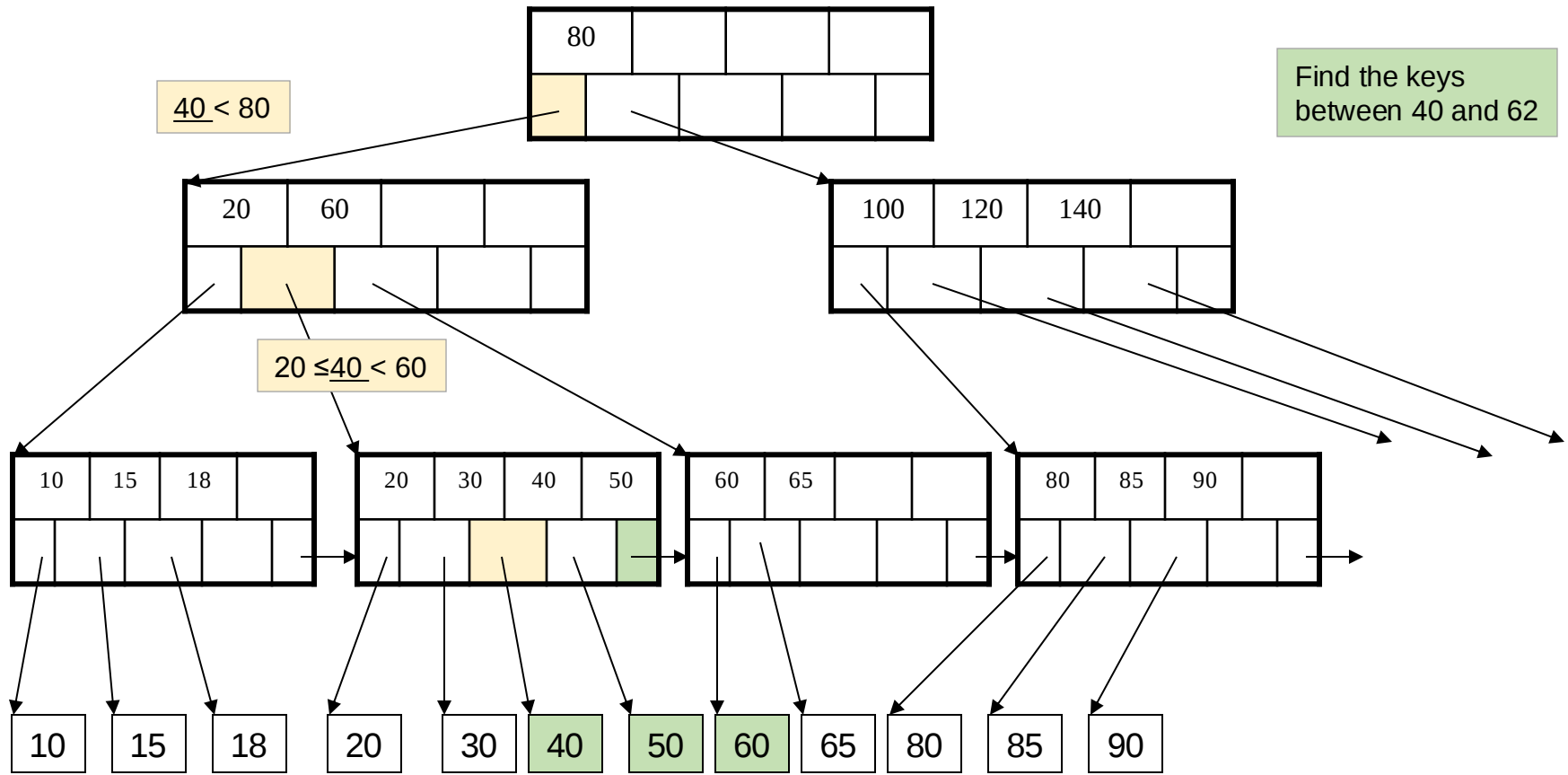
B+ Tree Example

$d = 2$



B+ Tree Example

$d = 2$



Search time

A B+ tree is perfectly balanced

- $\text{Depth} = \log_m N$ where m = avg # of children

Example: $N = 1,000,000,000,000$ data items

- Binary search tree: $\log N = 40$
- B+ tree, assume $m=128$: $\log_m N = 6$ block reads

You can find the data after at most 6 reads!

B+ Trees: Insert and Delete

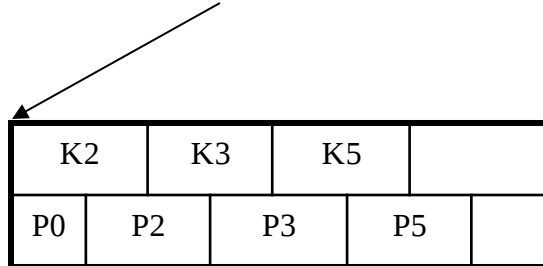
Insertion in a B+ Tree: Summary

Insert (K, P)

- Find leaf where K belongs, insert
- If no overflow (2d keys or less), halt

Insert k1

parent



K2	K3	K5		
P0	P2	P3	P5	

Insertion in a B+ Tree: Summary

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Insertion in a B+ Tree: Summary

Insert (K, P)

- Find leaf where K belongs, insert
- If no overflow (2d keys or less), halt

Insert k4

parent

K1		K2		K3		K5	
P0	P1		P2		P3		P5

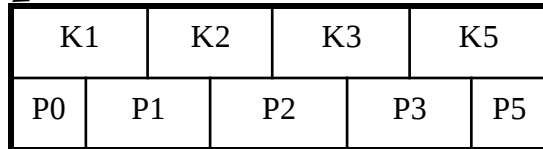
Insertion in a B+ Tree: Summary

Insert (K, P)

- Find leaf where K belongs, insert
- If no overflow ($2d$ keys or less), halt
- If overflow ($2d+1$ keys), split node, insert in parent:

Insert k4

parent



K1	K2	K3	K5	
P0	P1	P2	P3	P5

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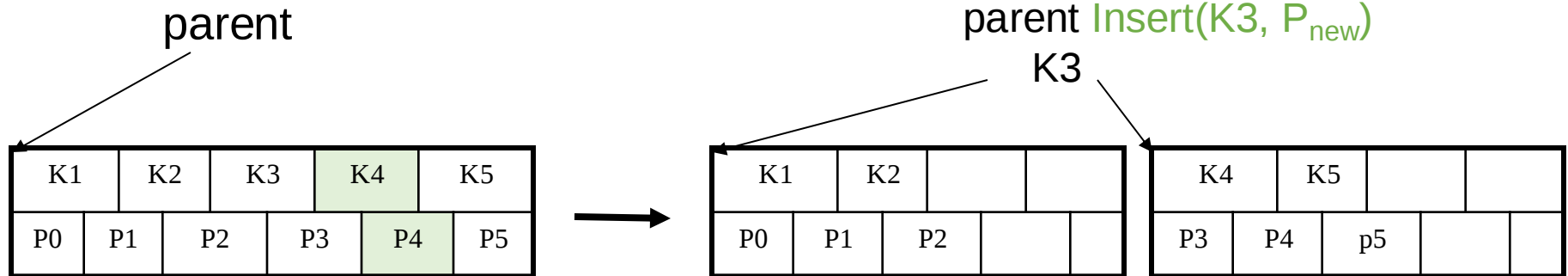
K1		K2		K3		K4		K5	
P0	P1	P2		P3	P4		P5		

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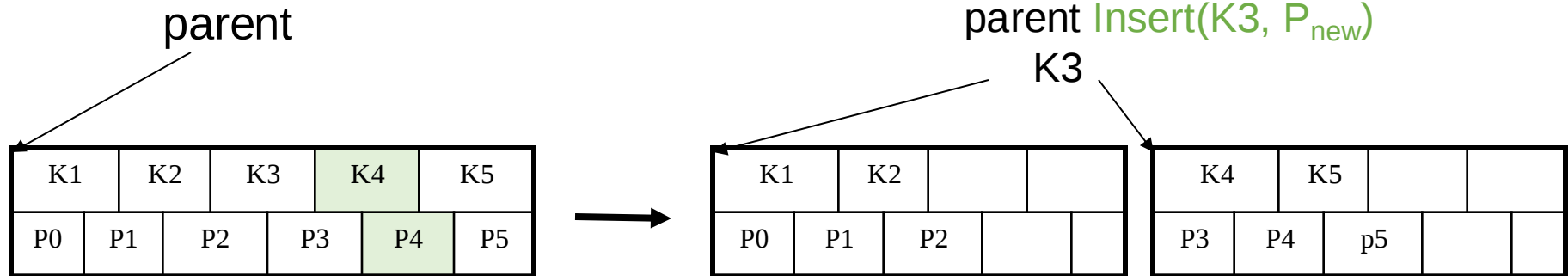


Insertion in a B+ Tree: Summary

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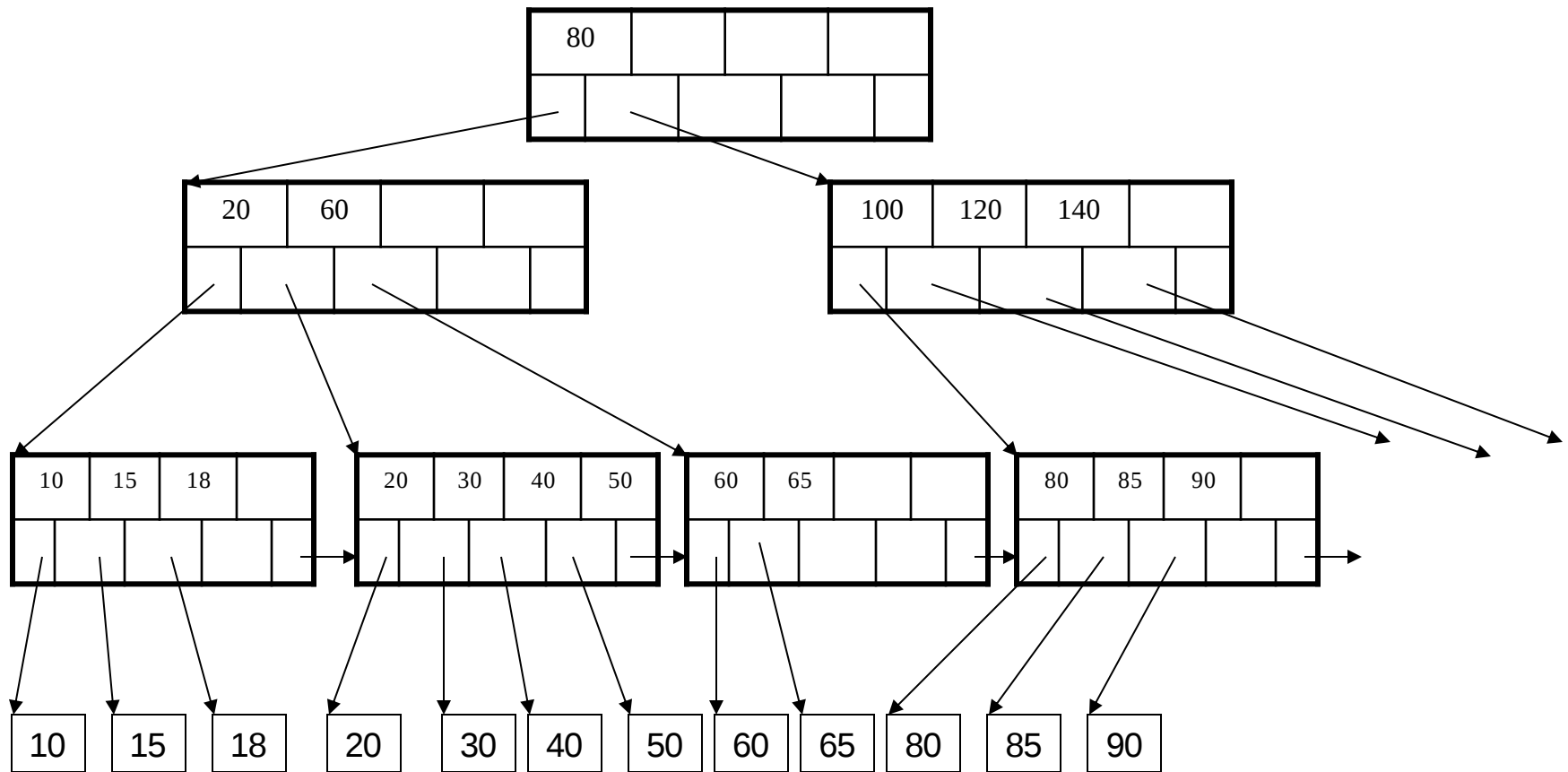
Insert k4



- If leaf, also keep K3 in right node
- When root splits, new root has 1 key only

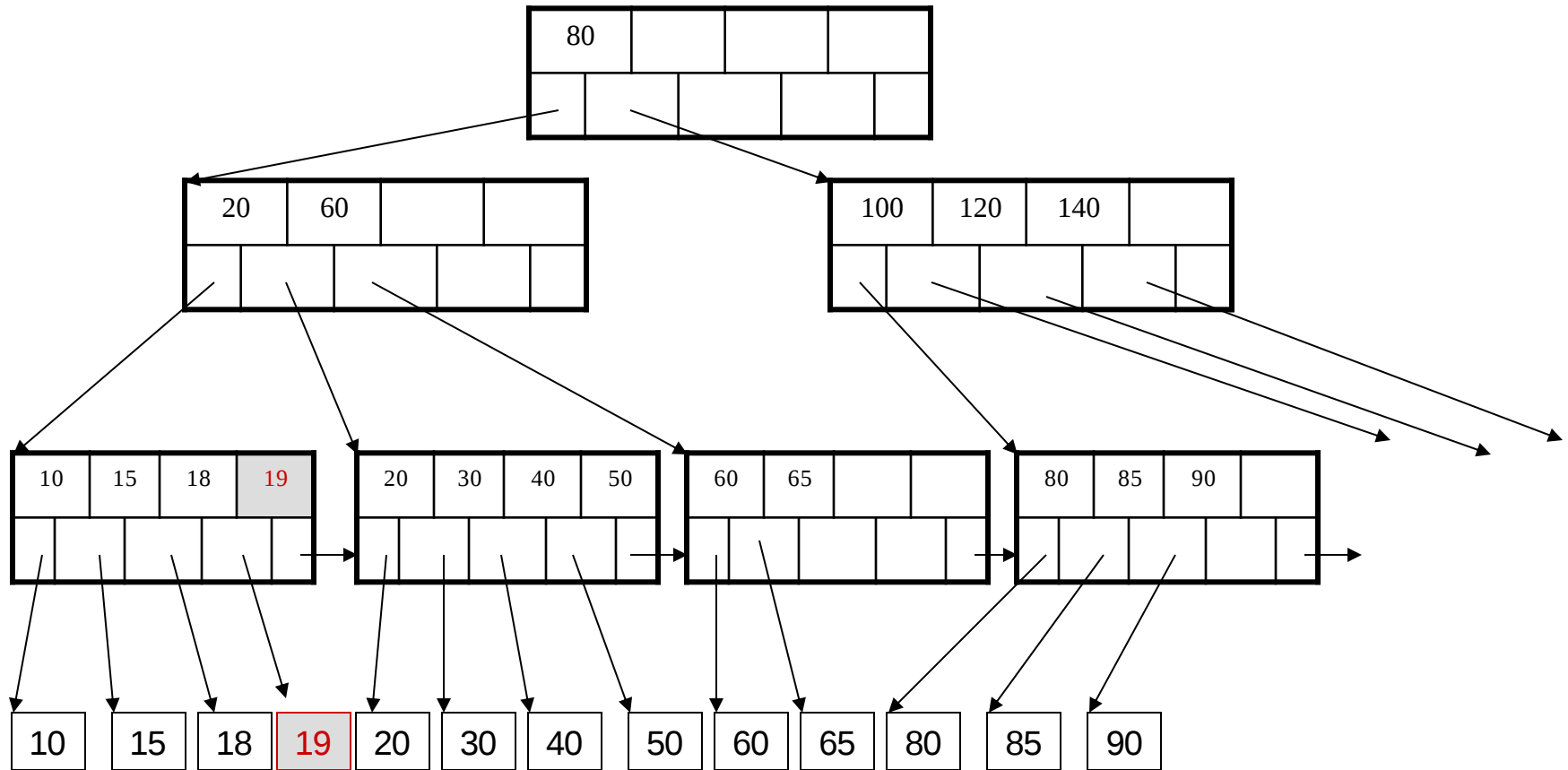
Insertion in a B+ Tree: Example

Insert K=19



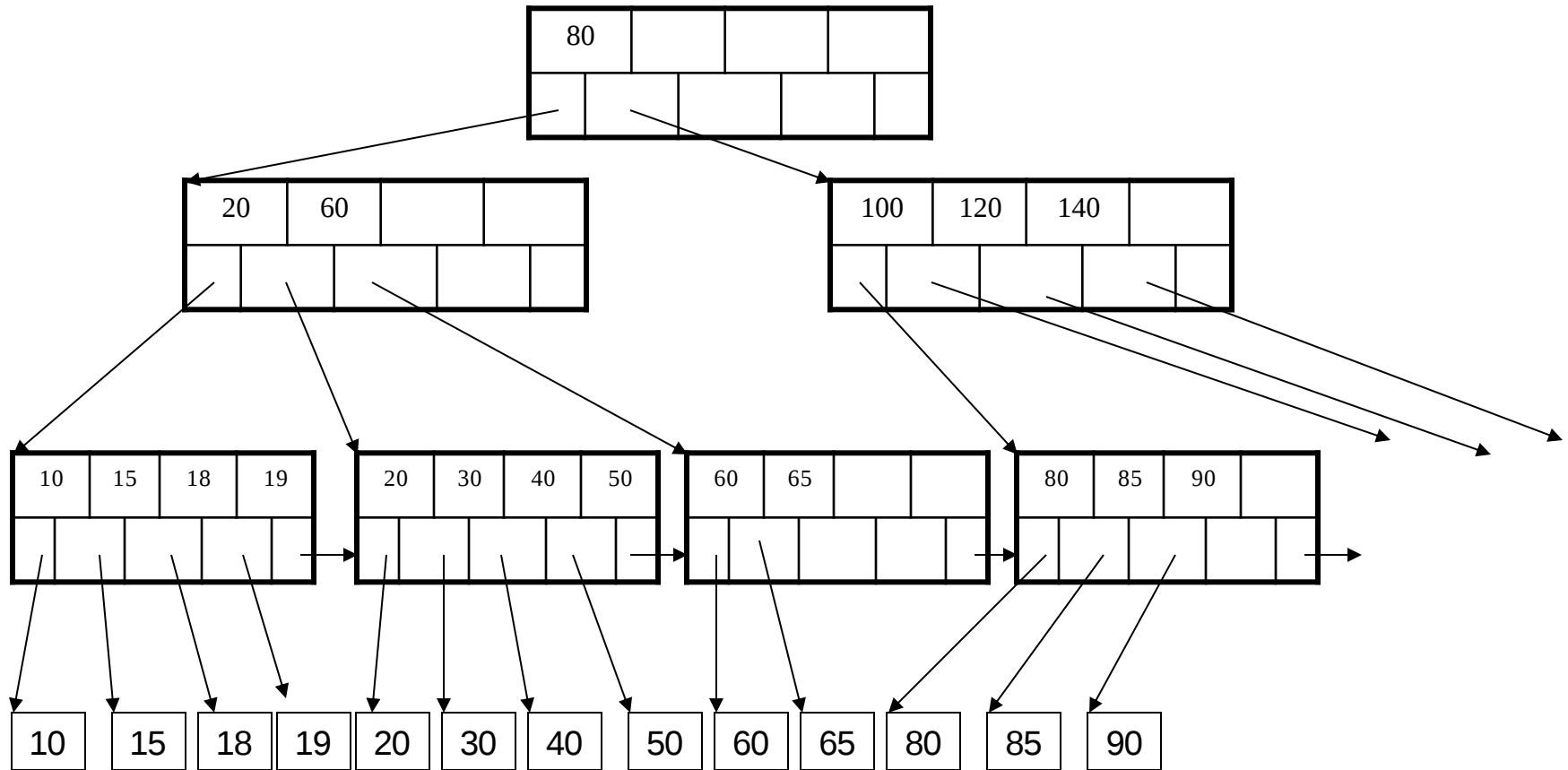
Insertion in a B+ Tree: Example

After insertion



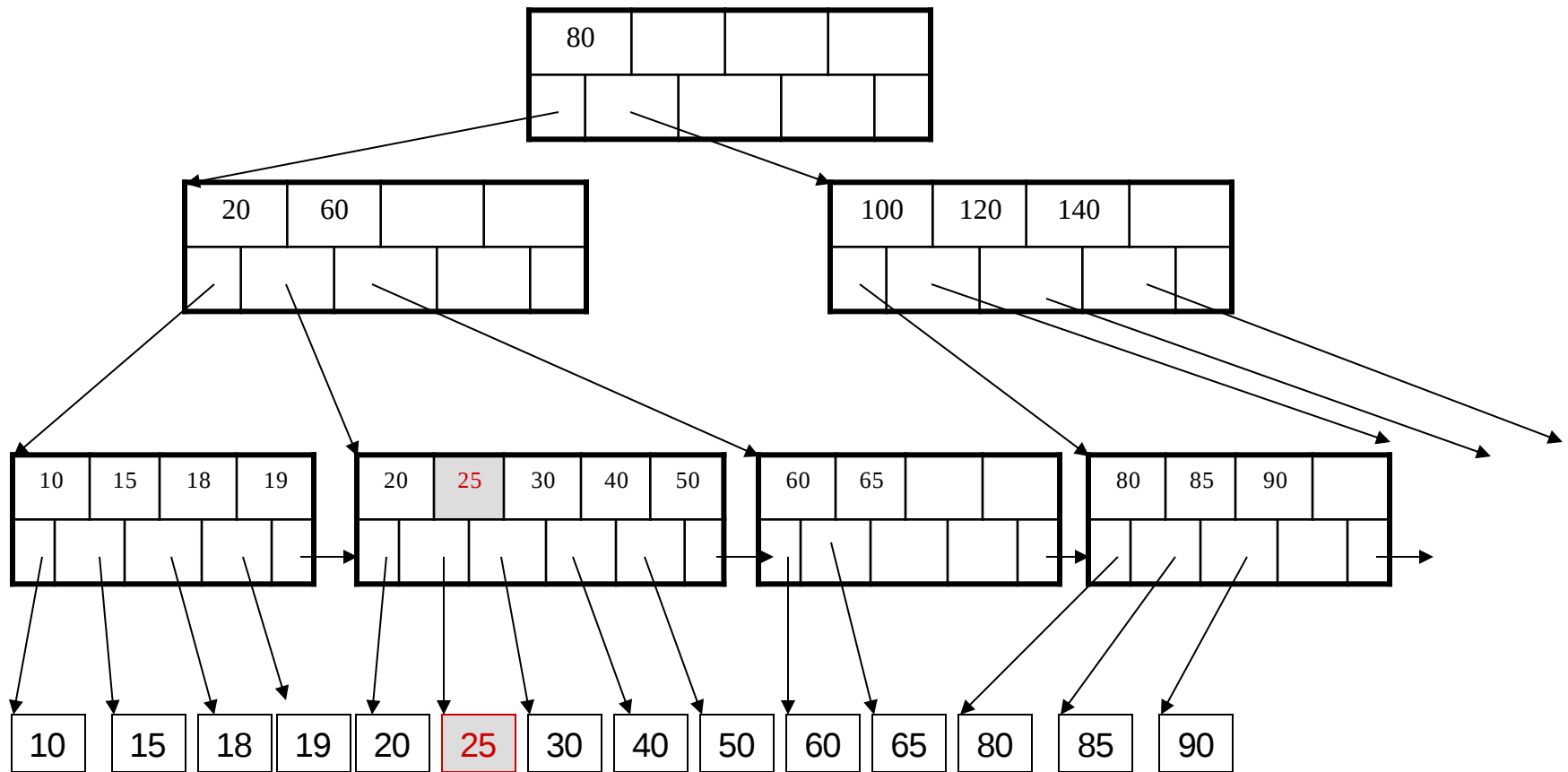
Insertion in a B+ Tree: Example

Now insert 25



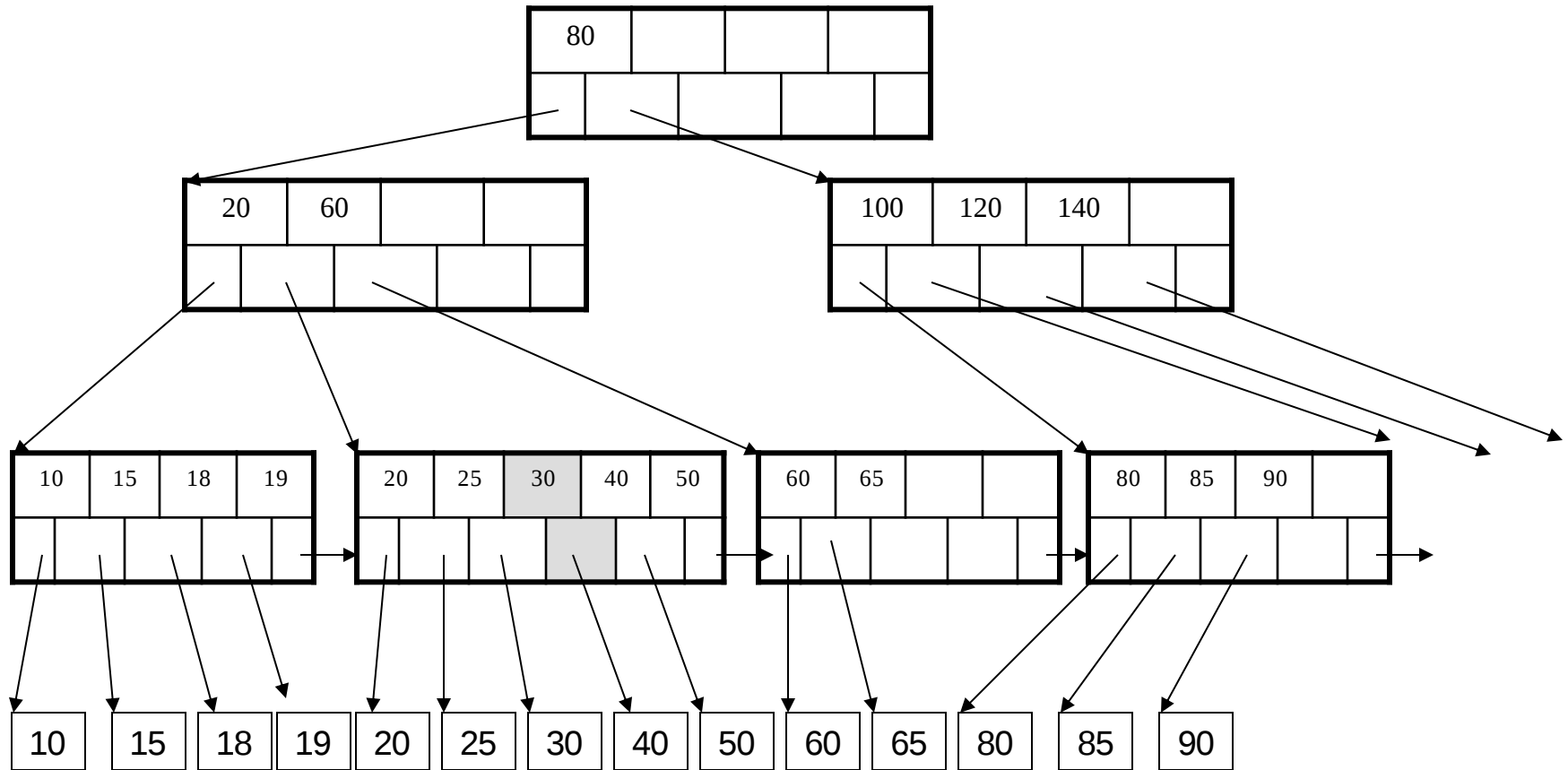
Insertion in a B+ Tree: Example

After insertion



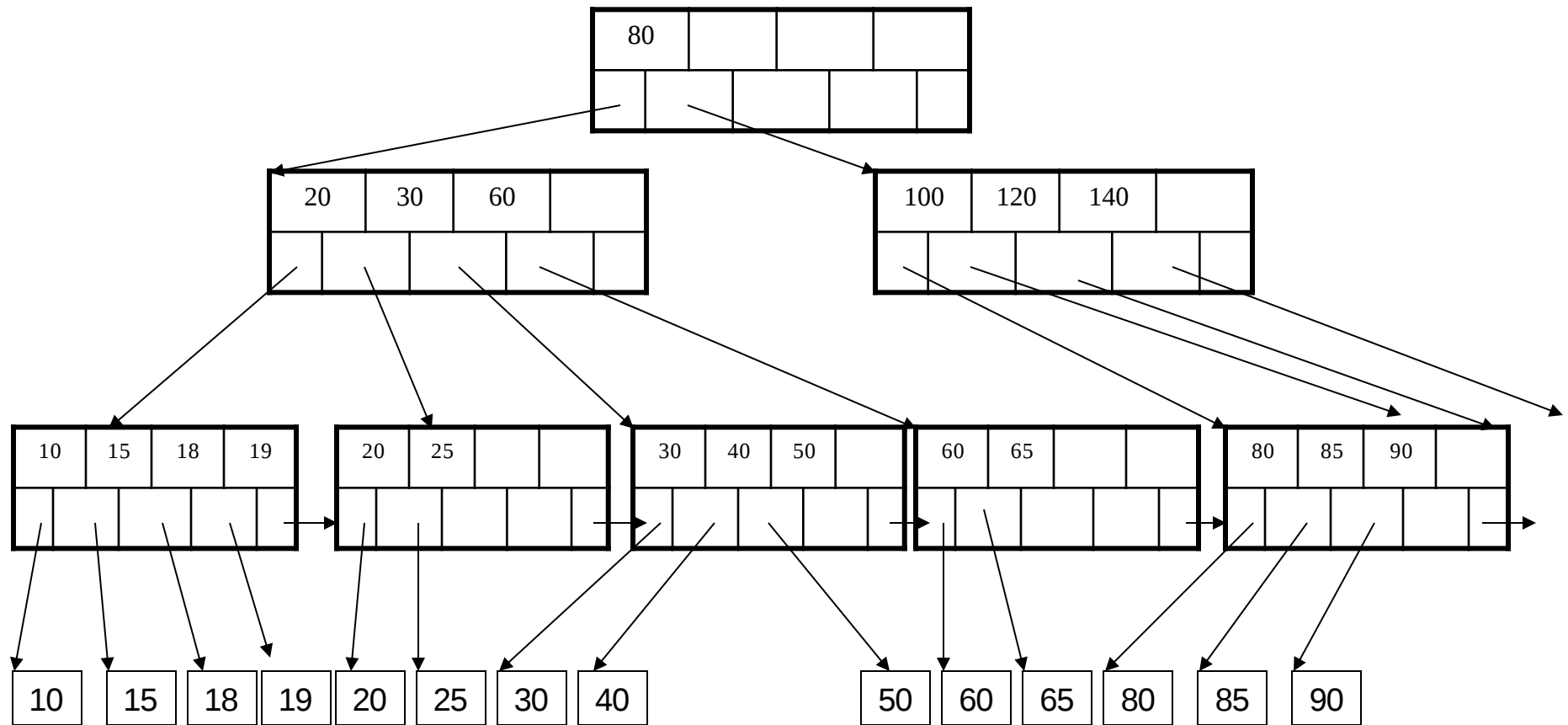
Insertion in a B+ Tree: Example

But now have to split !



Insertion in a B+ Tree: Example

After the split



Deletion: Summary

Delete (K, P)

- Delete K, P from the leaf
- If capacity $\geq$ min: **Stop**

Deletion: Summary

Delete (K, P)

- Delete K, P from the leaf
- If capacity $\geq$ min: **Stop**
- If neighbor capacity $>$ min: rotate and **Stop**

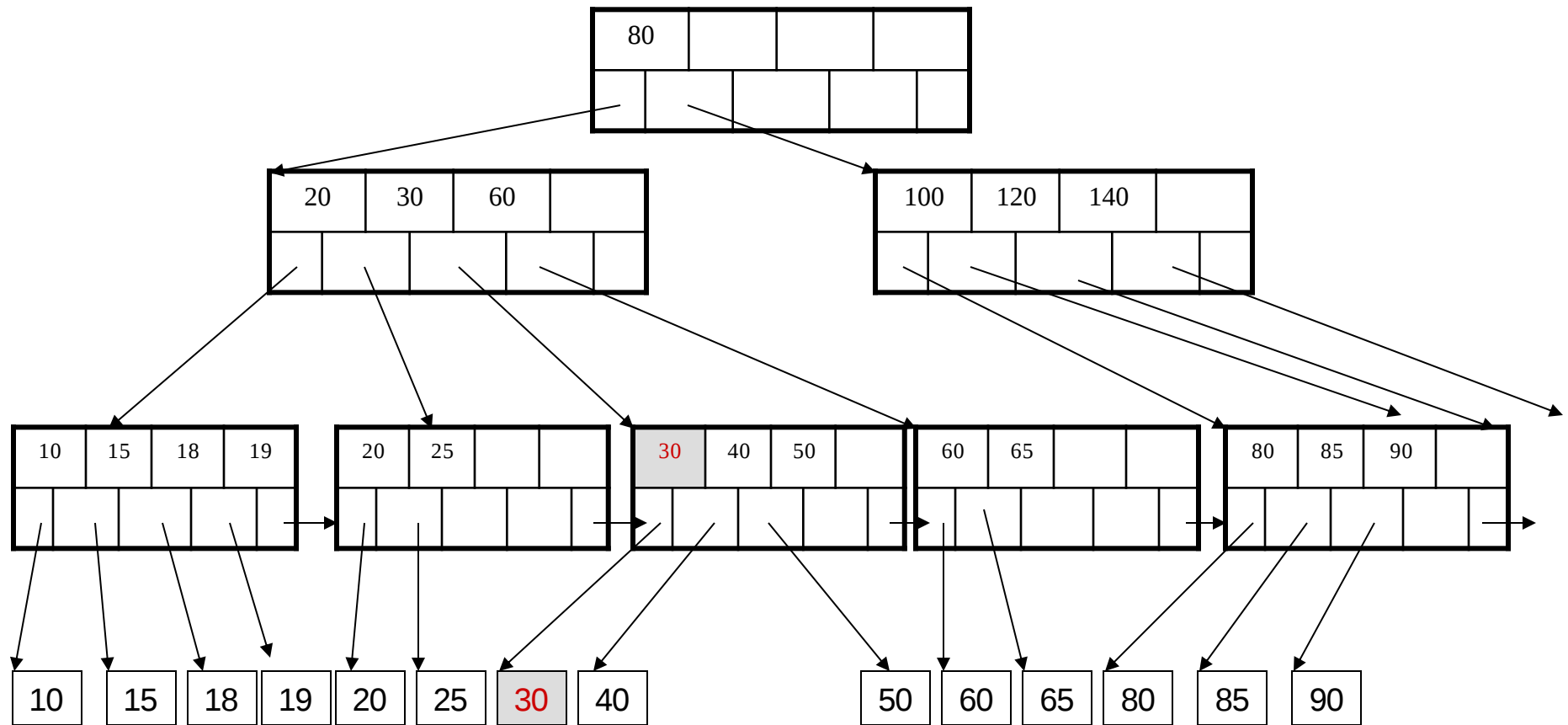
Deletion: Summary

Delete (K, P)

- Delete K, P from the leaf
- If capacity $\geq$ min: **Stop**
- If neighbor capacity $>$ min: rotate and **Stop**
- Merge with a neighbor **P'** (choose right or left) and steal a key **K'** from parent
- **Delete (K', P')** where P'=the parent

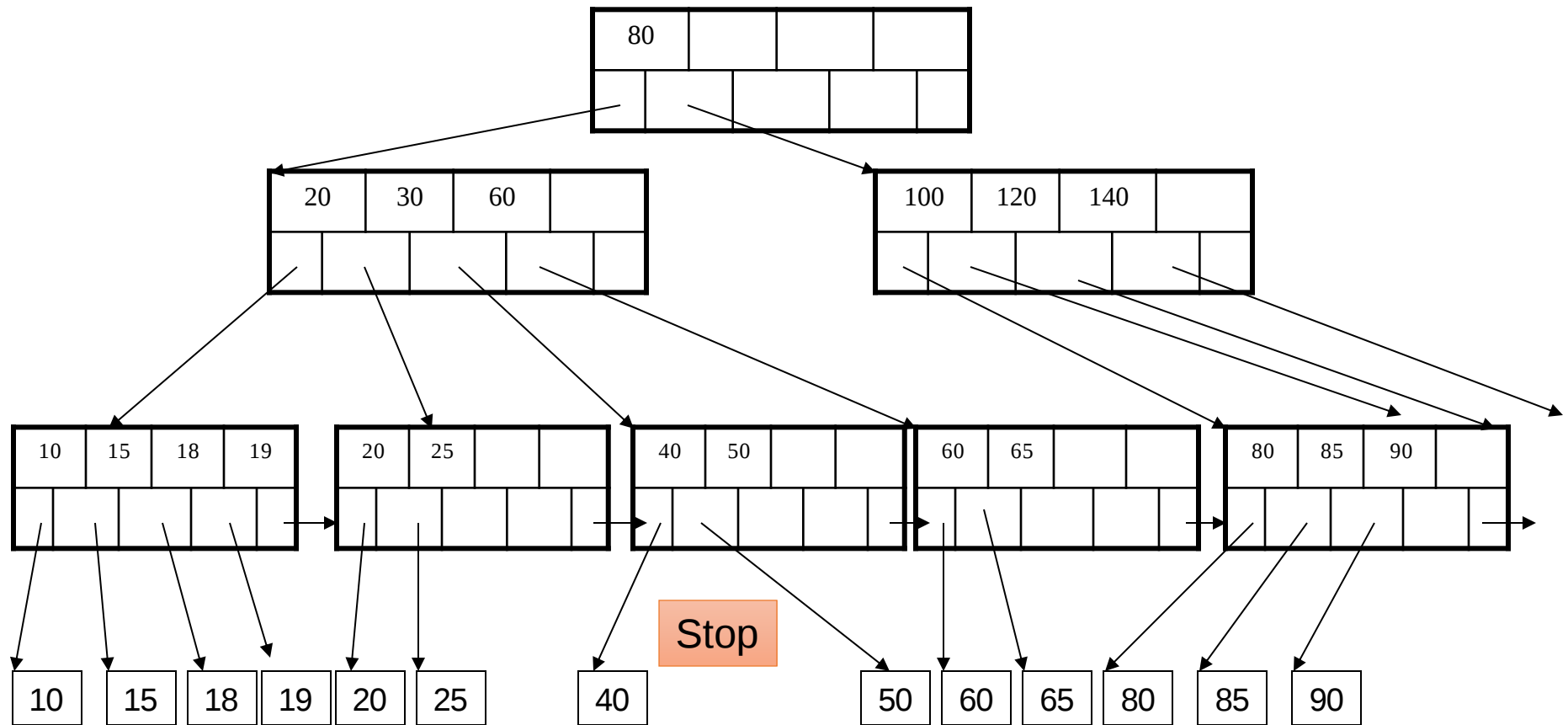
Deletion from a B+ Tree: Example

Delete 30



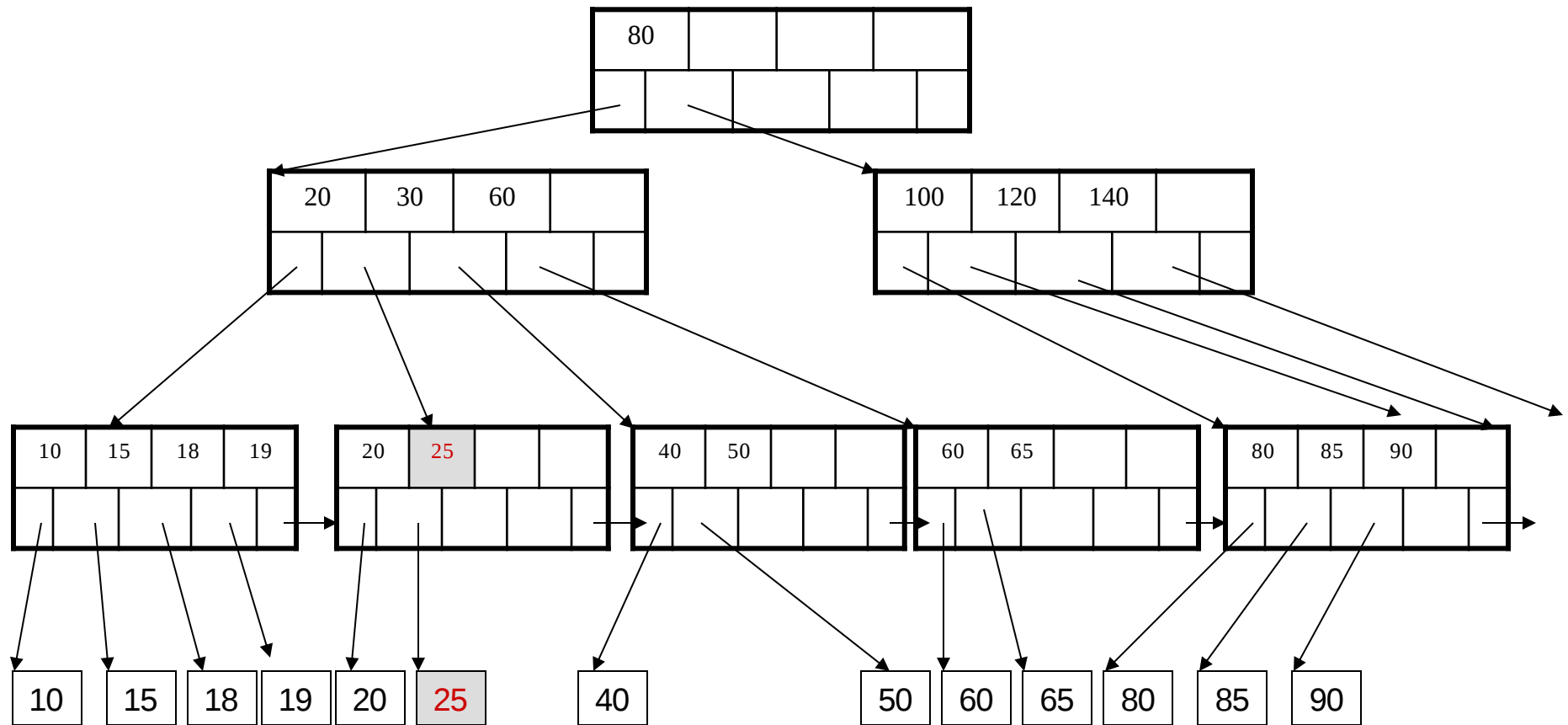
Deletion from a B+ Tree: Example

After deleting 30



Deletion from a B+ Tree: Example

Now delete 25

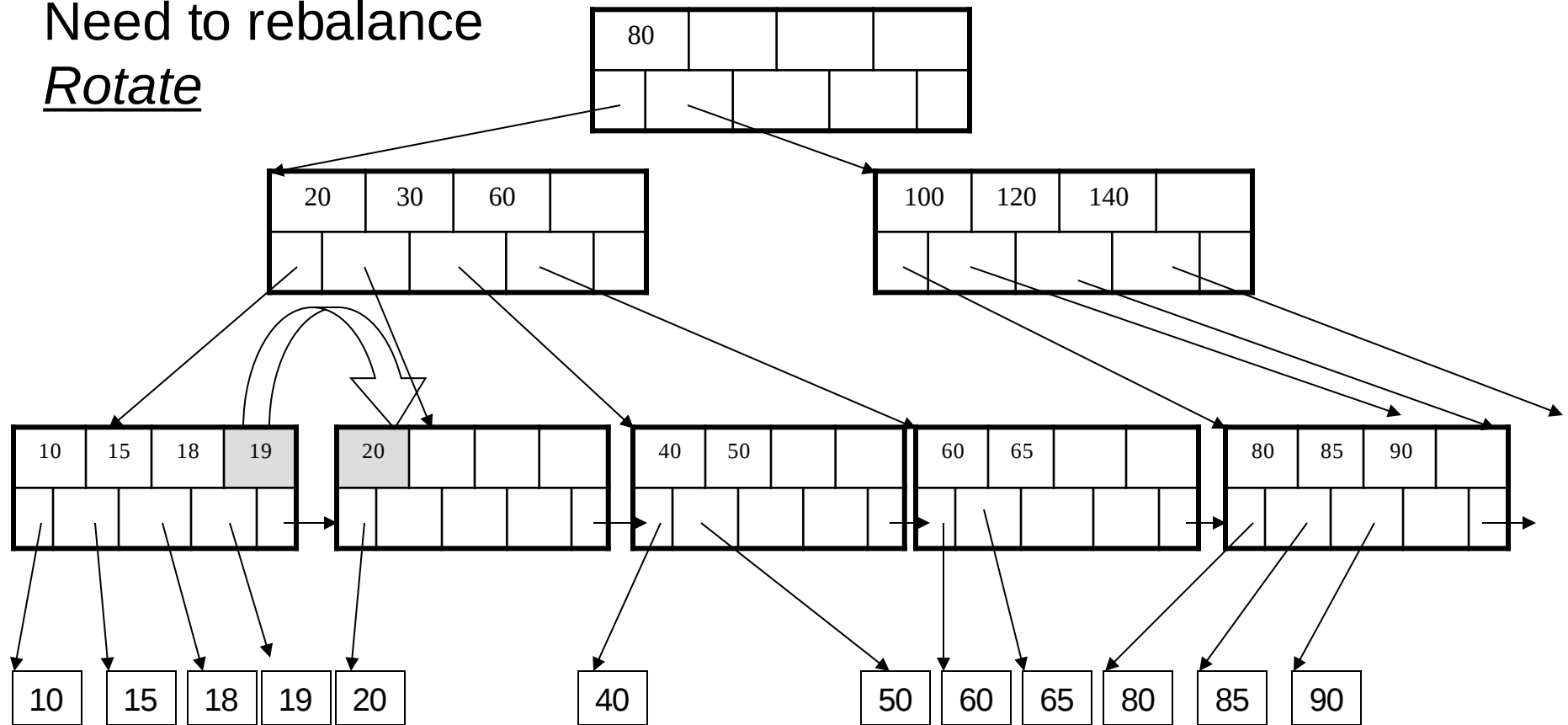


Deletion from a B+ Tree: Example

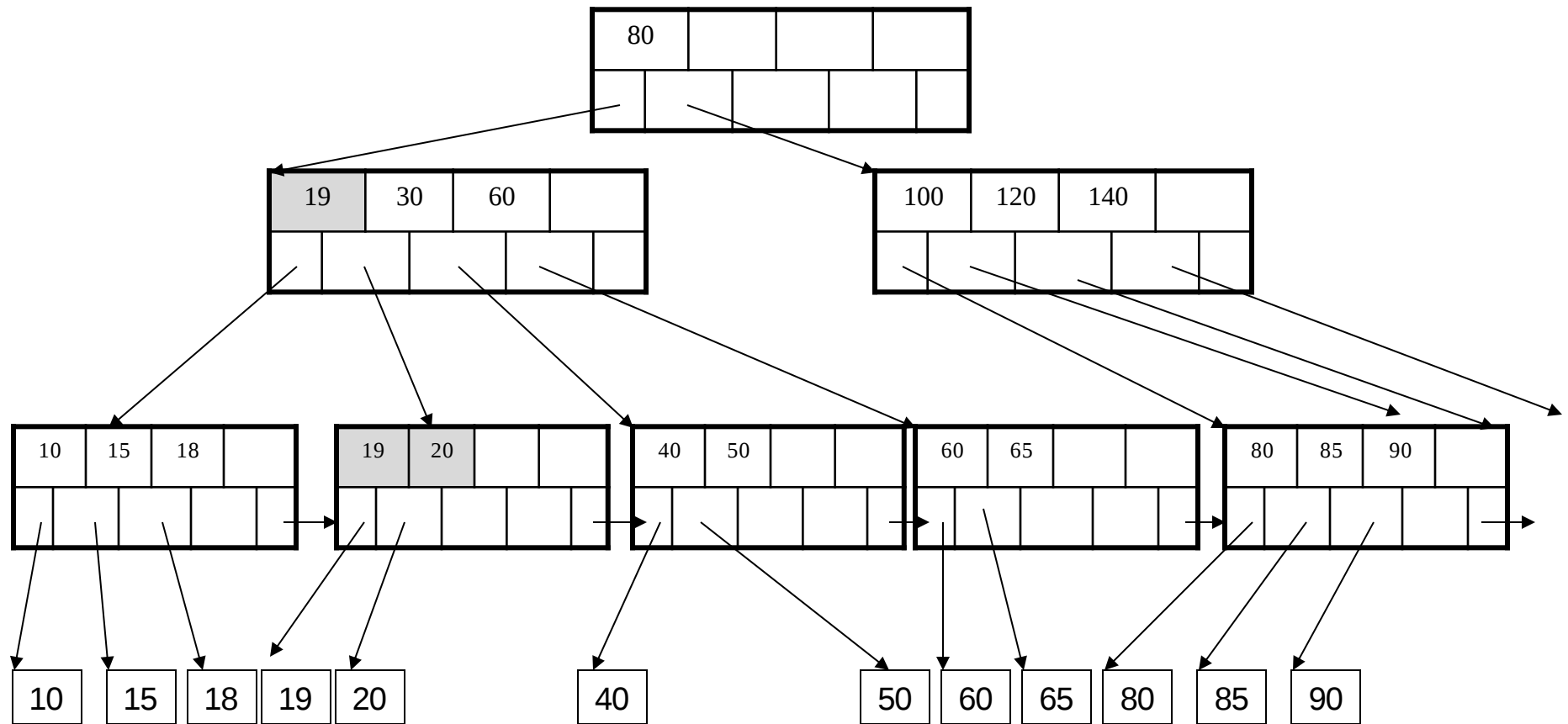
After deleting 25

Need to rebalance

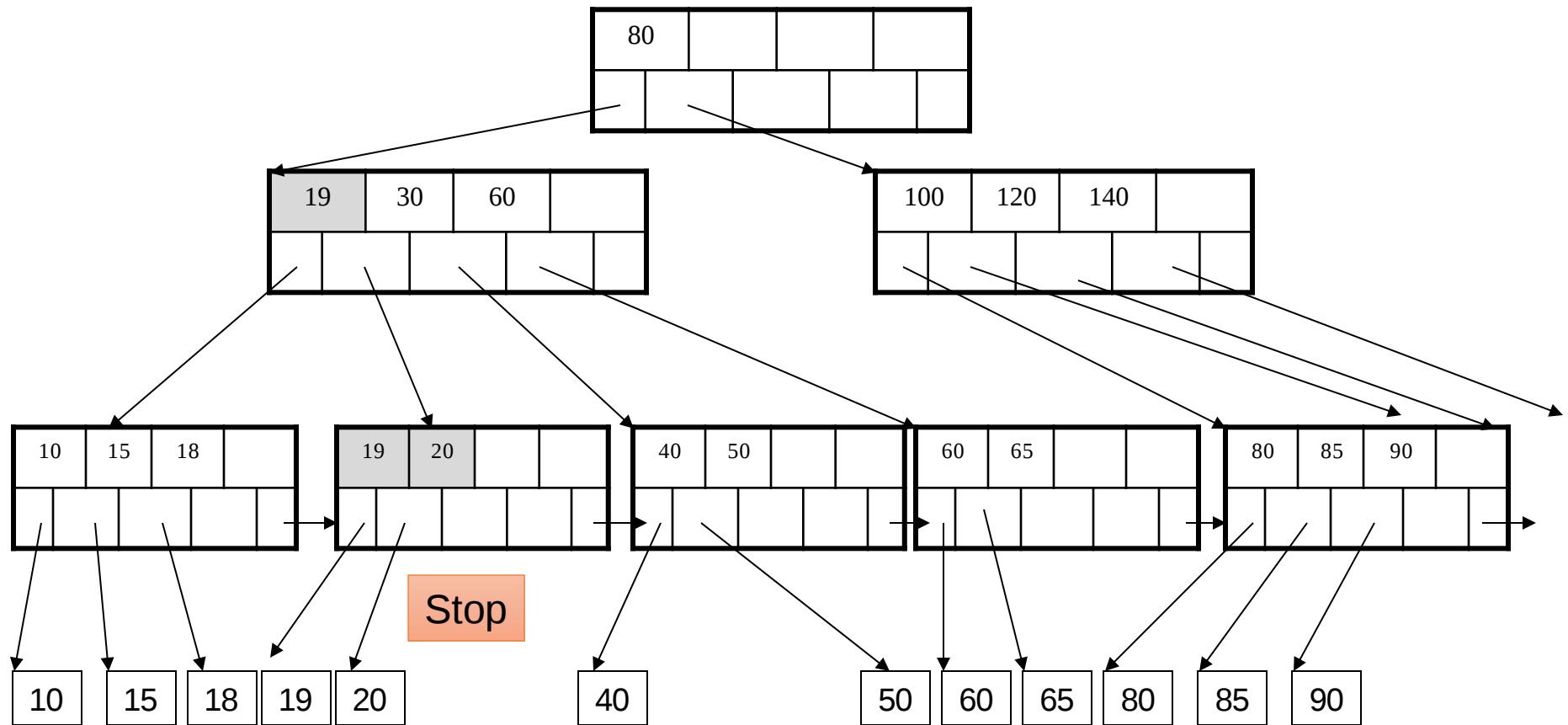
Rotate



Deletion from a B+ Tree: Example

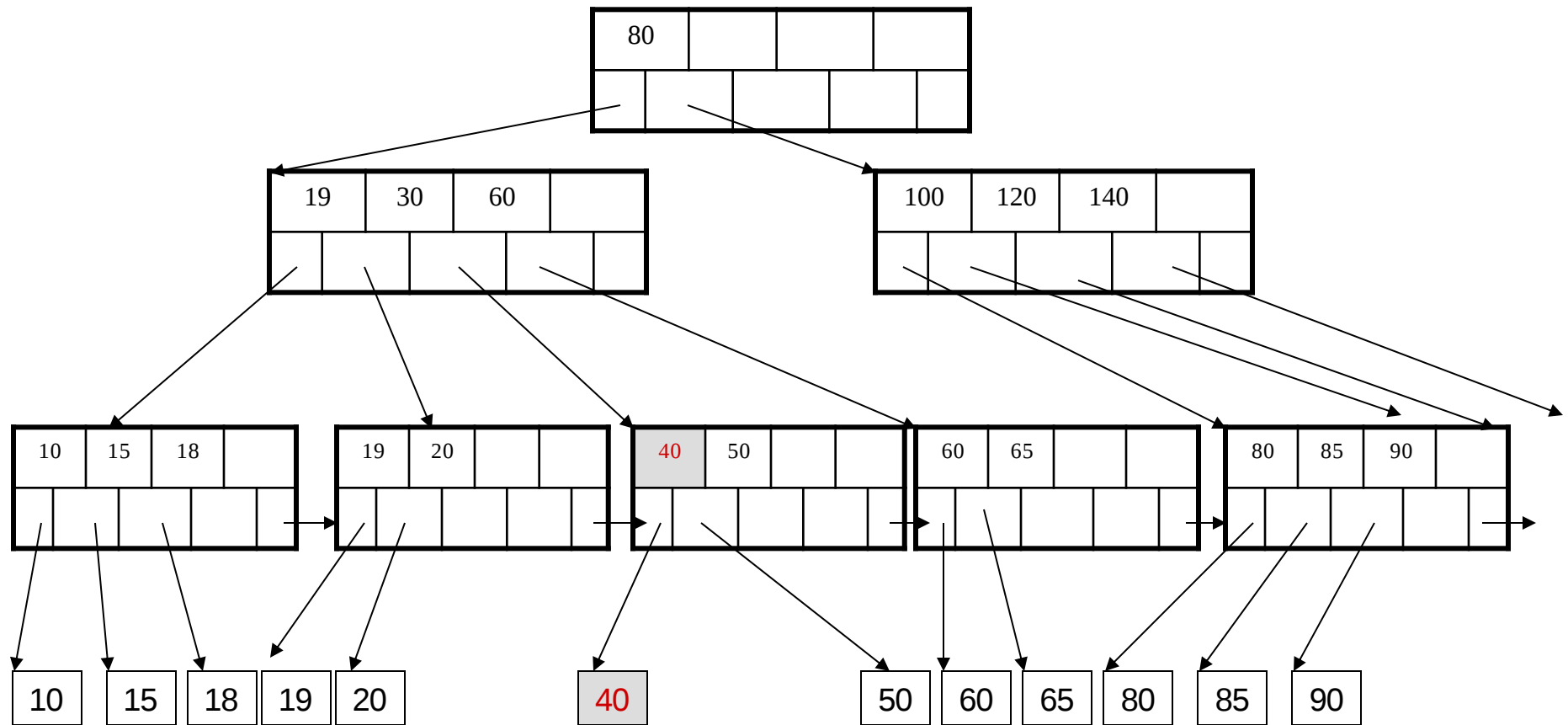


Deletion from a B+ Tree: Example



Deletion from a B+ Tree: Example

Now delete 40

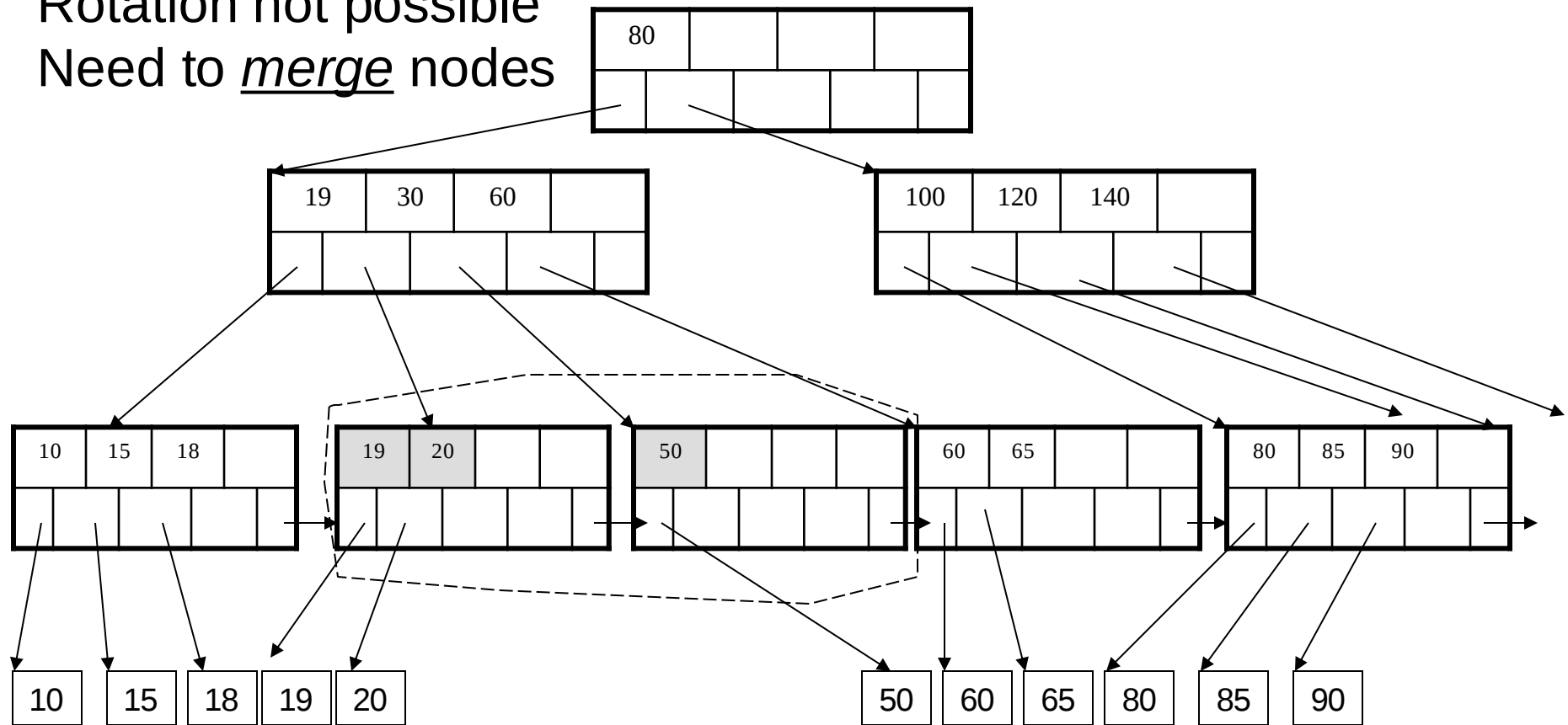


Deletion from a B+ Tree: Example

After deleting 40

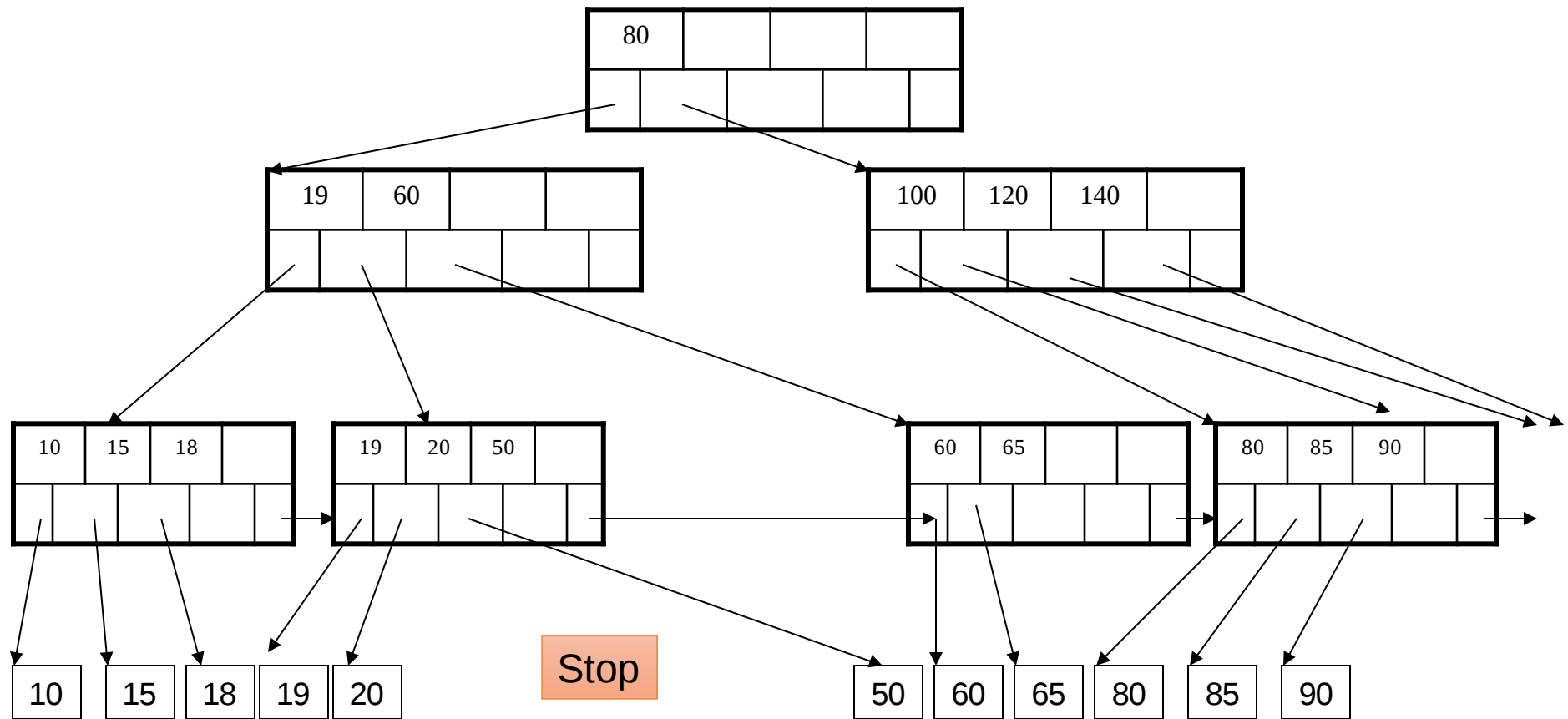
Rotation not possible

Need to merge nodes



Deletion from a B+ Tree: Example

Final tree



Sequential v.s. Random Access

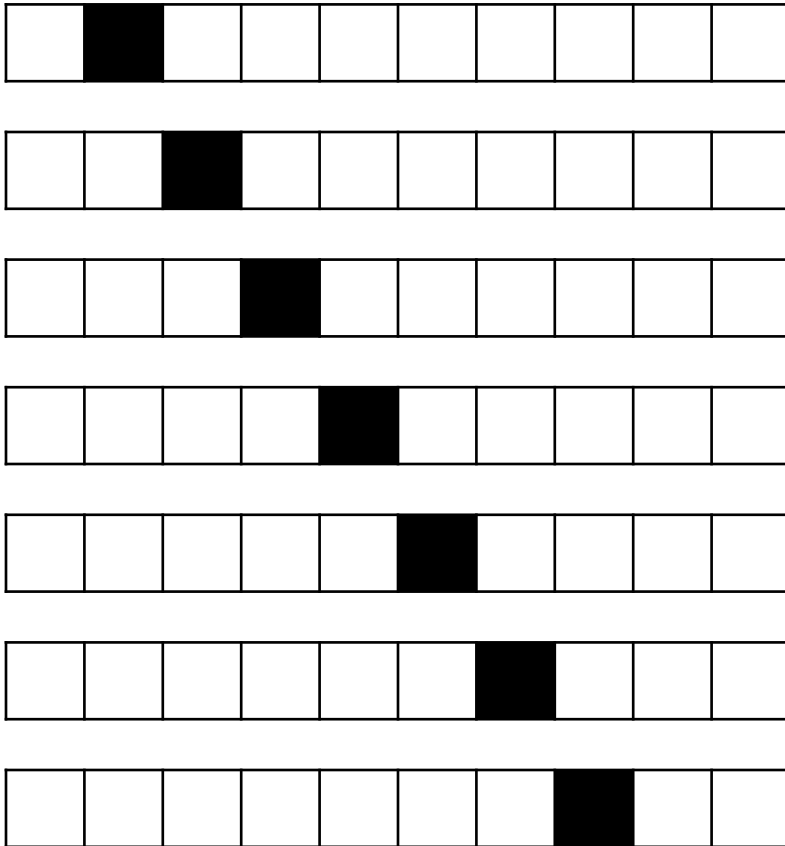
Sequential/Random Access

Recall: the disk always reads one entire block

- **Sequential access**: we read consecutive blocks
1001, 1002, 1003, 1004, ...
- **Random access**: we read them in whatever order
1523, 1003, 2999, 1010, ...

Sequential/Random Access

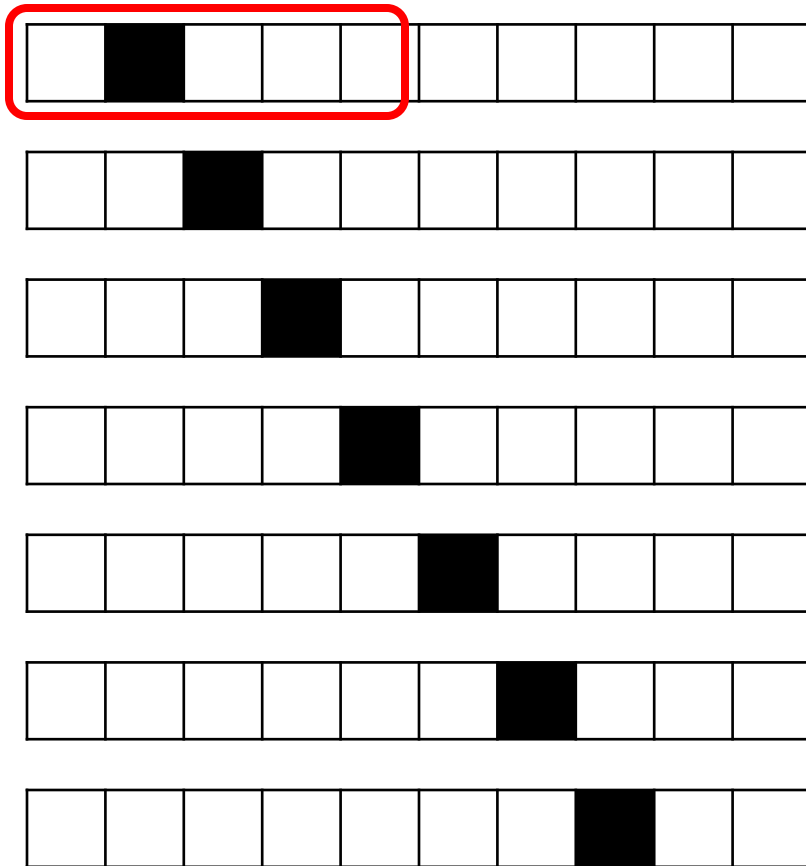
Sequential



Sequential/Random Access

Sequential

Read

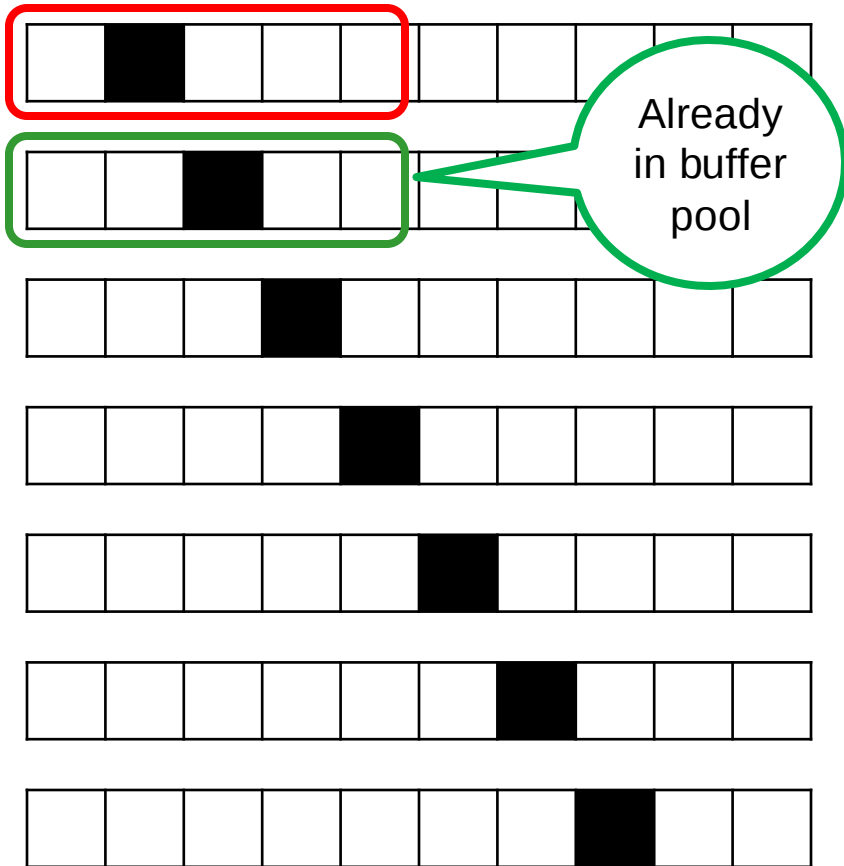


Sequential/Random Access

Sequential

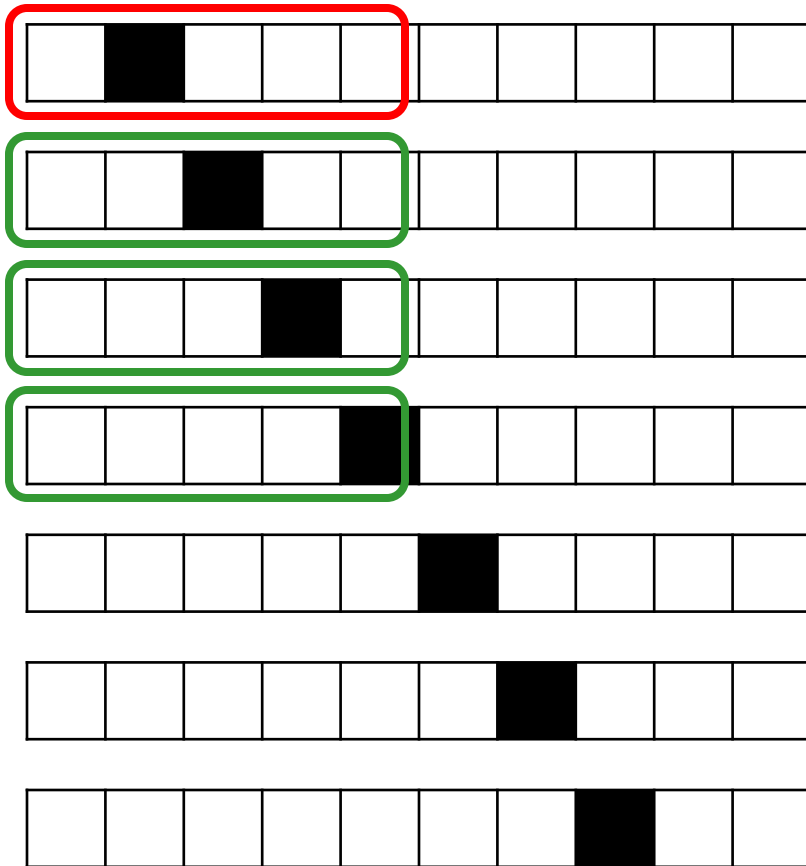
Read

Already
in buffer
pool



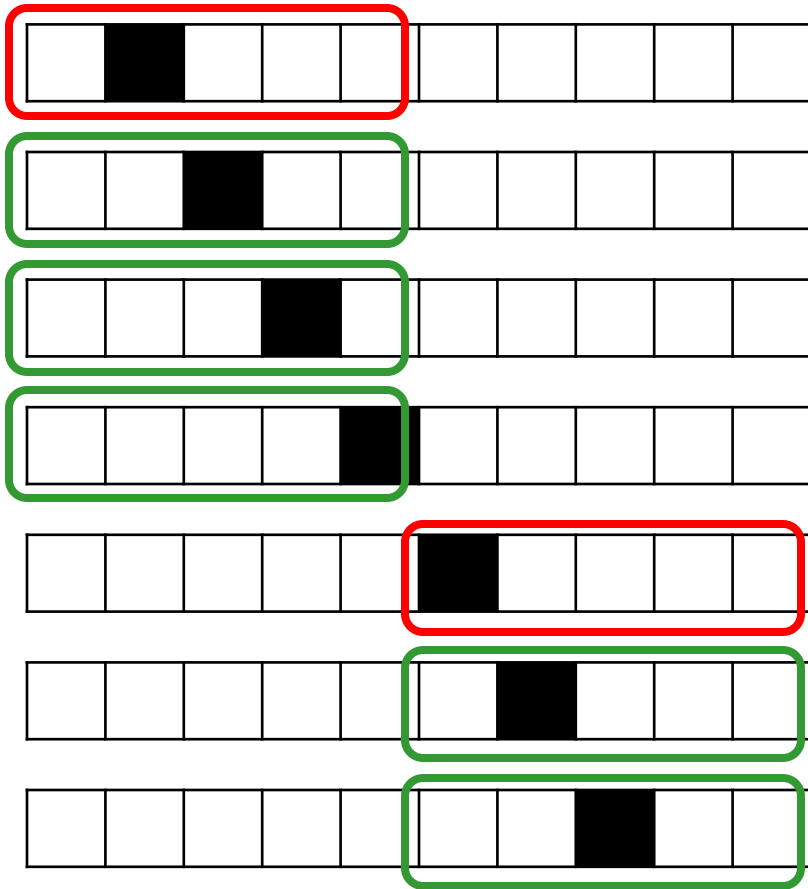
Sequential/Random Access

Sequential



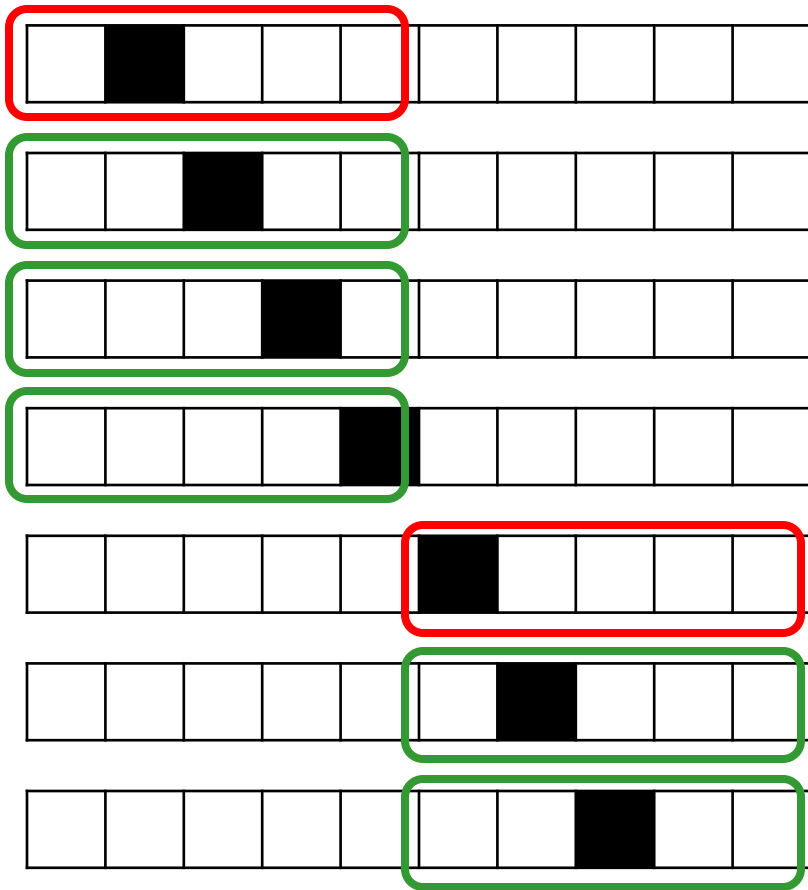
Sequential/Random Access

Sequential: 2 reads

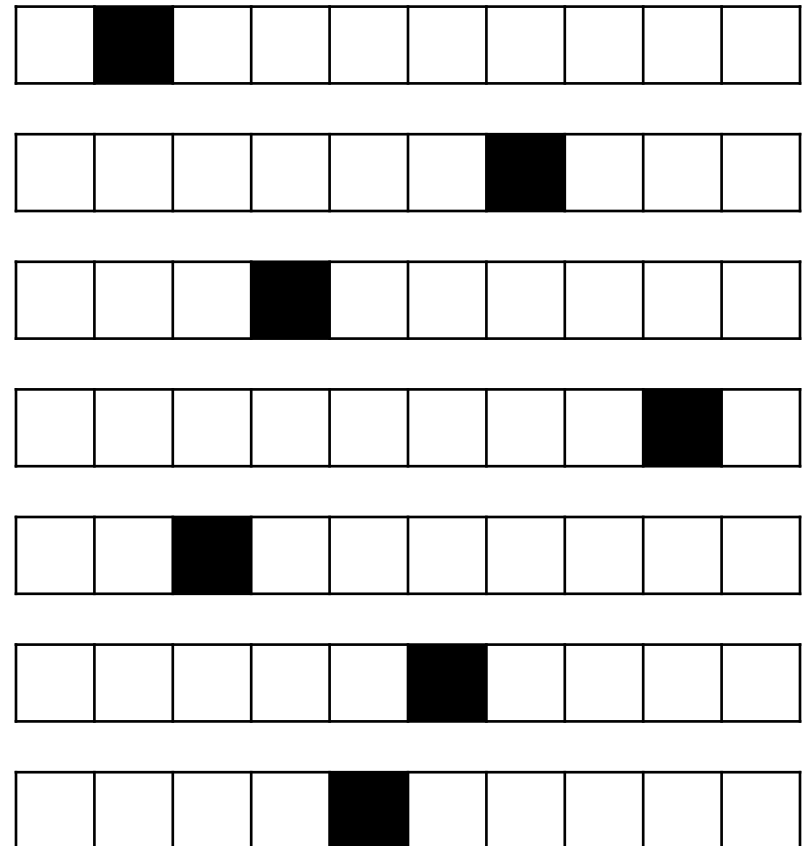


Sequential/Random Access

Sequential: 2 reads

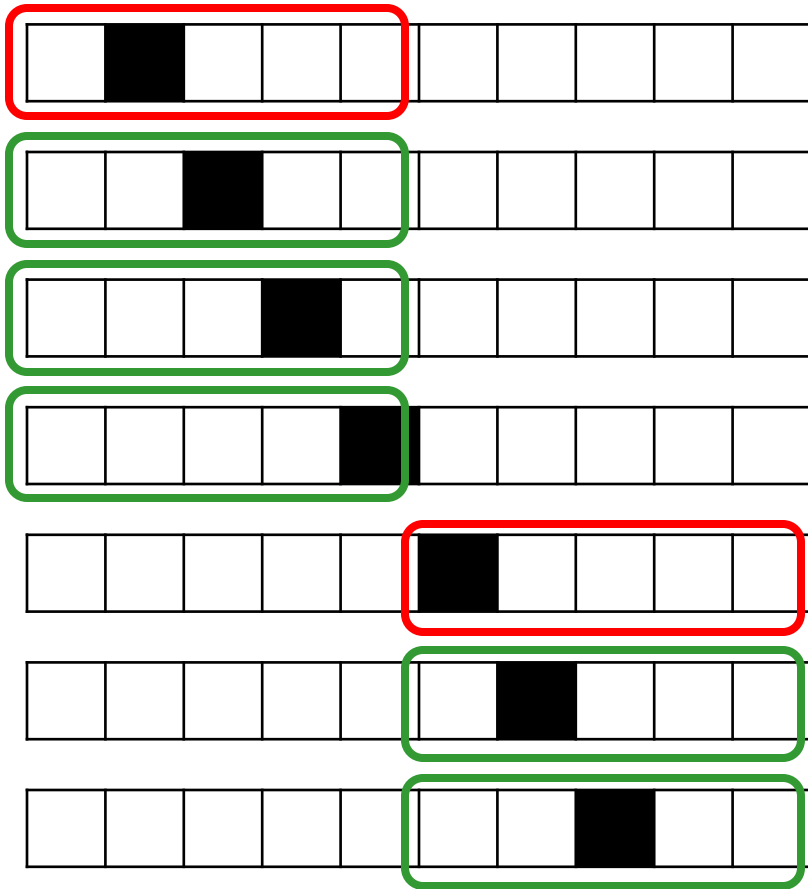


Random

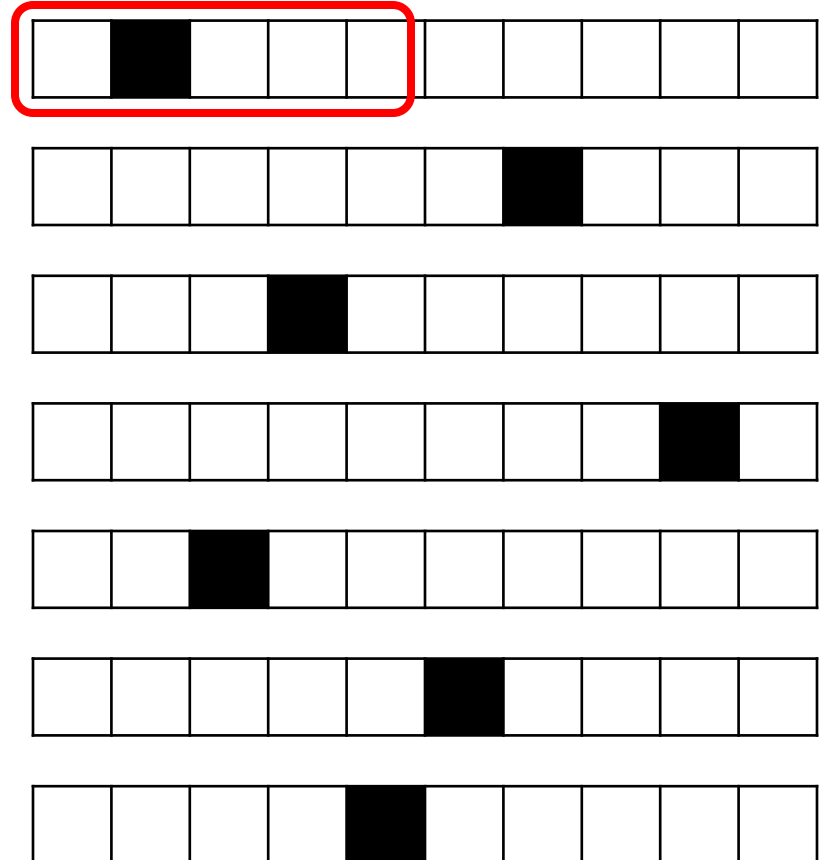


Sequential/Random Access

Sequential: 2 reads

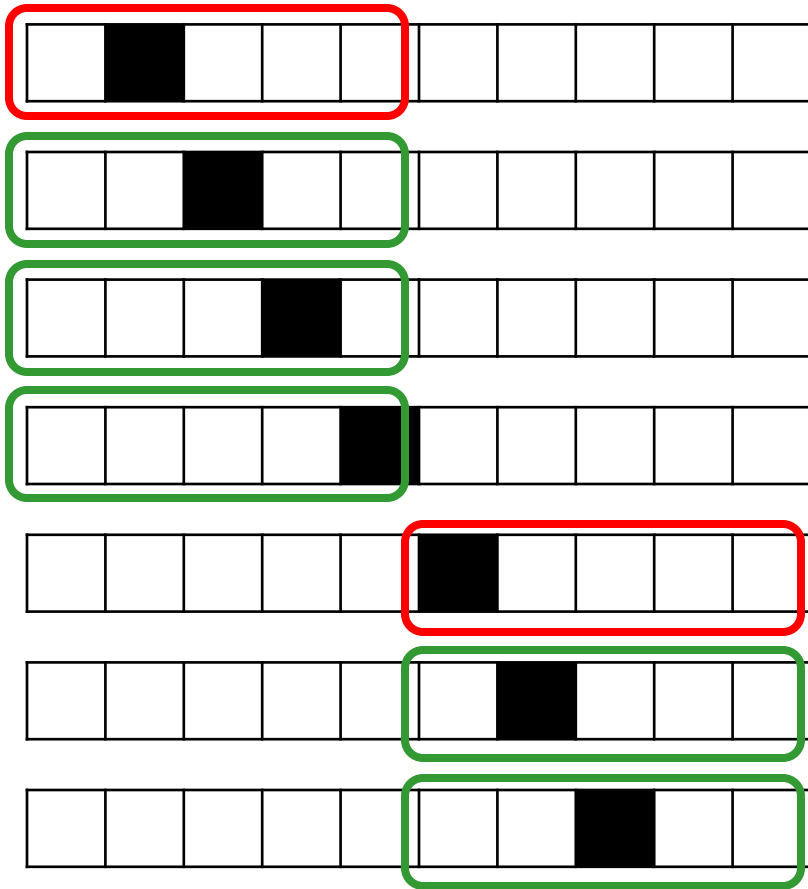


Random

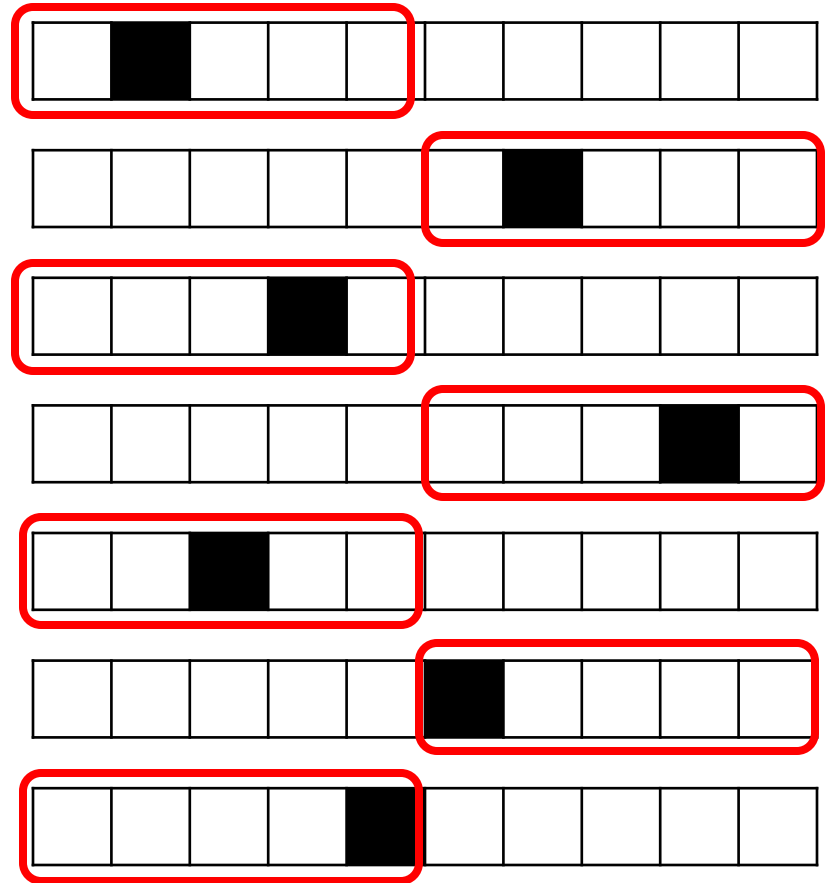


Sequential/Random Access

Sequential: 2 reads



Random: many reads



Discussion

Sequential scan:

- Needs to read all blocks
- But uses sequential access to disk

Index lookup:

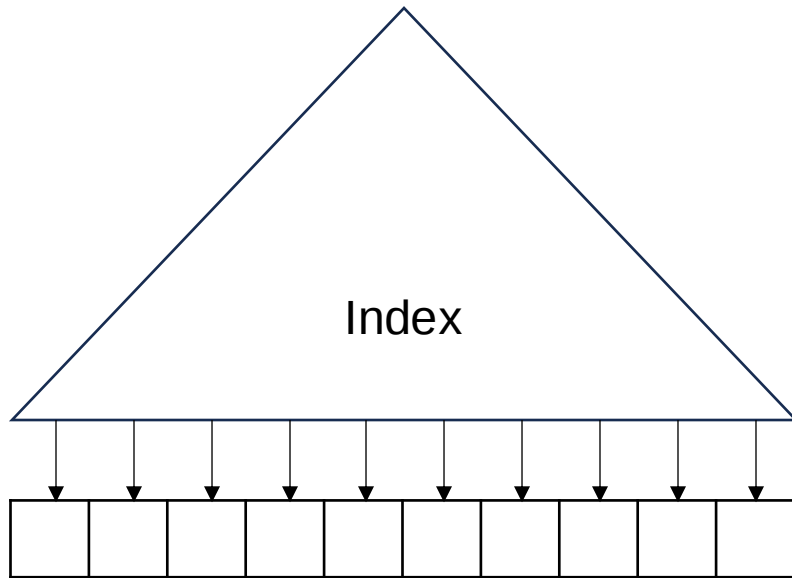
- Reads far fewer blocks (sometimes just 1)
- But it uses random access

Clustered v.s. Unclustered Indexes

- An index is **clustered** if the data file is sorted in by the index key
- Otherwise, it is called unclustered

Clustered v.s. Unclustered Indexes

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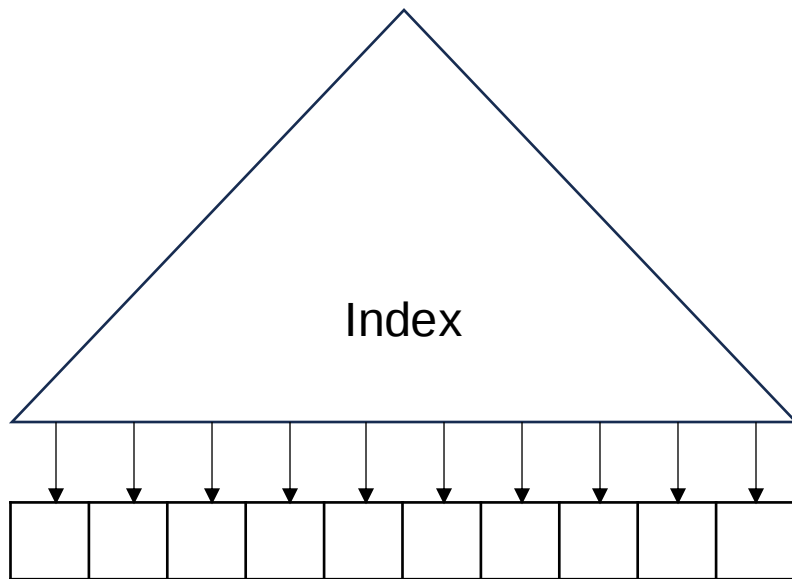


Data file

Clustered

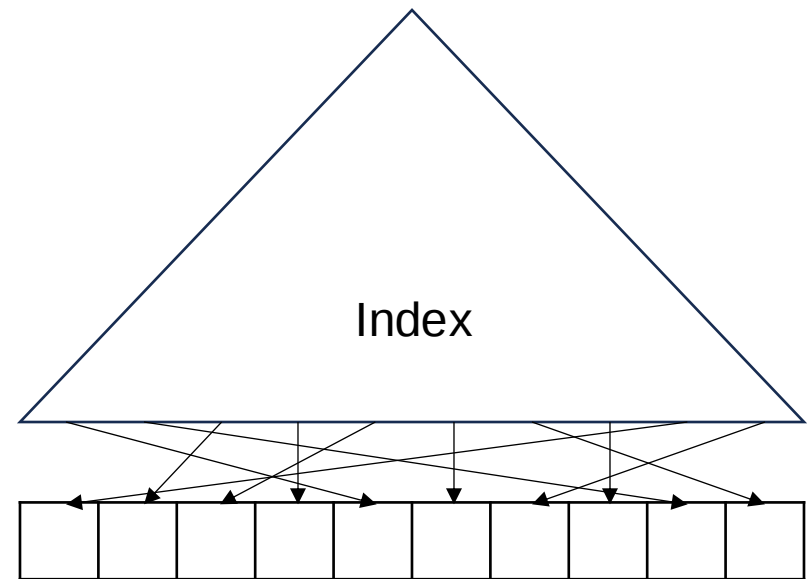
Clustered v.s. Unclustered Indexes

- An index is **clustered** if the data file is sorted in by the index key
- Otherwise, it is called unclustered



Data file

Clustered

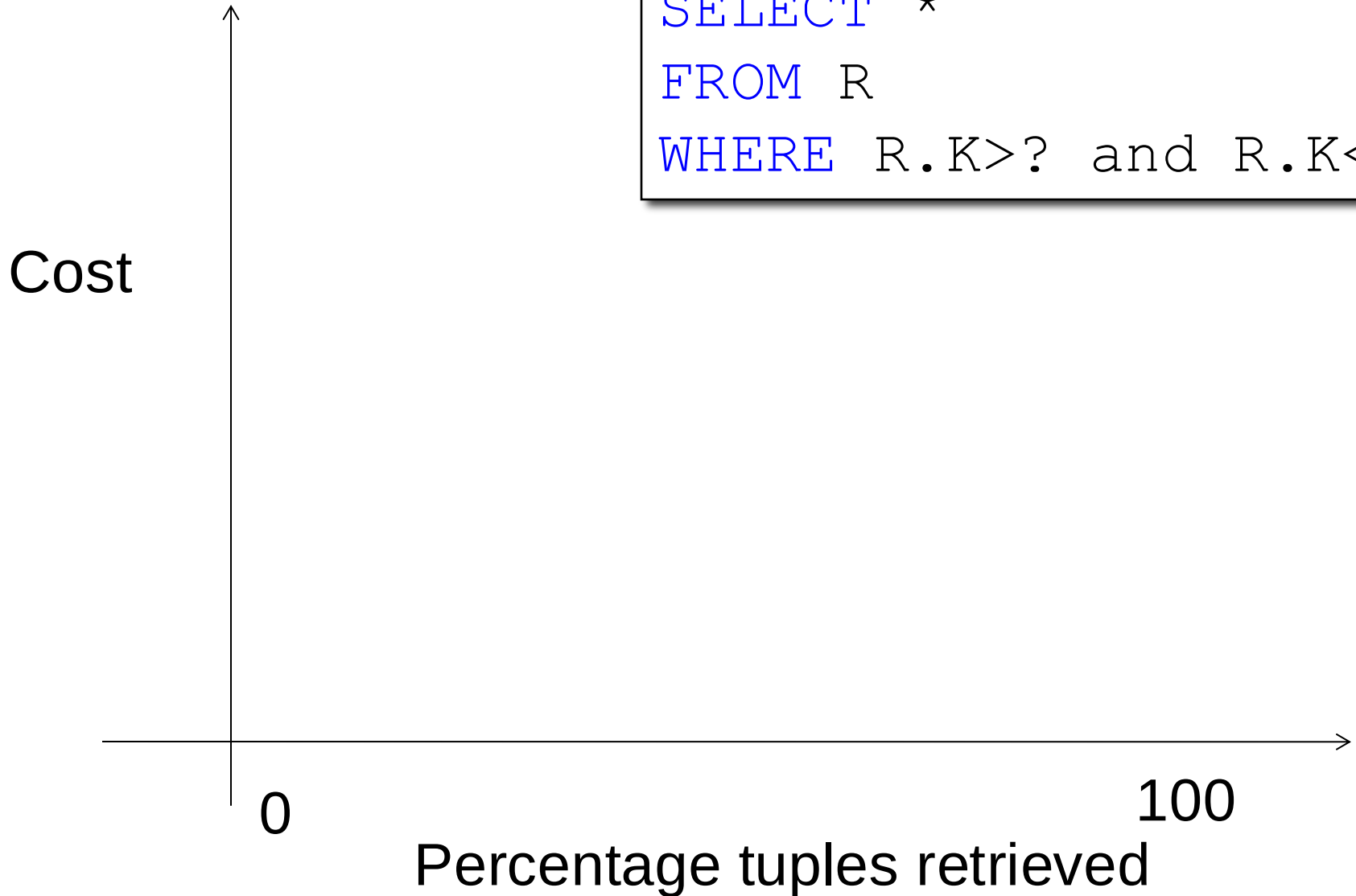


Data file

Unclustered

Clustered v.s. Unclustered Indexes

```
SELECT *  
FROM R  
WHERE R.K > ? and R.K < ?
```



Clustered v.s. Unclustered Indexes

```
SELECT *  
FROM R  
WHERE R.K > ? and R.K < ?
```

Cost

Sequential scan

0

100

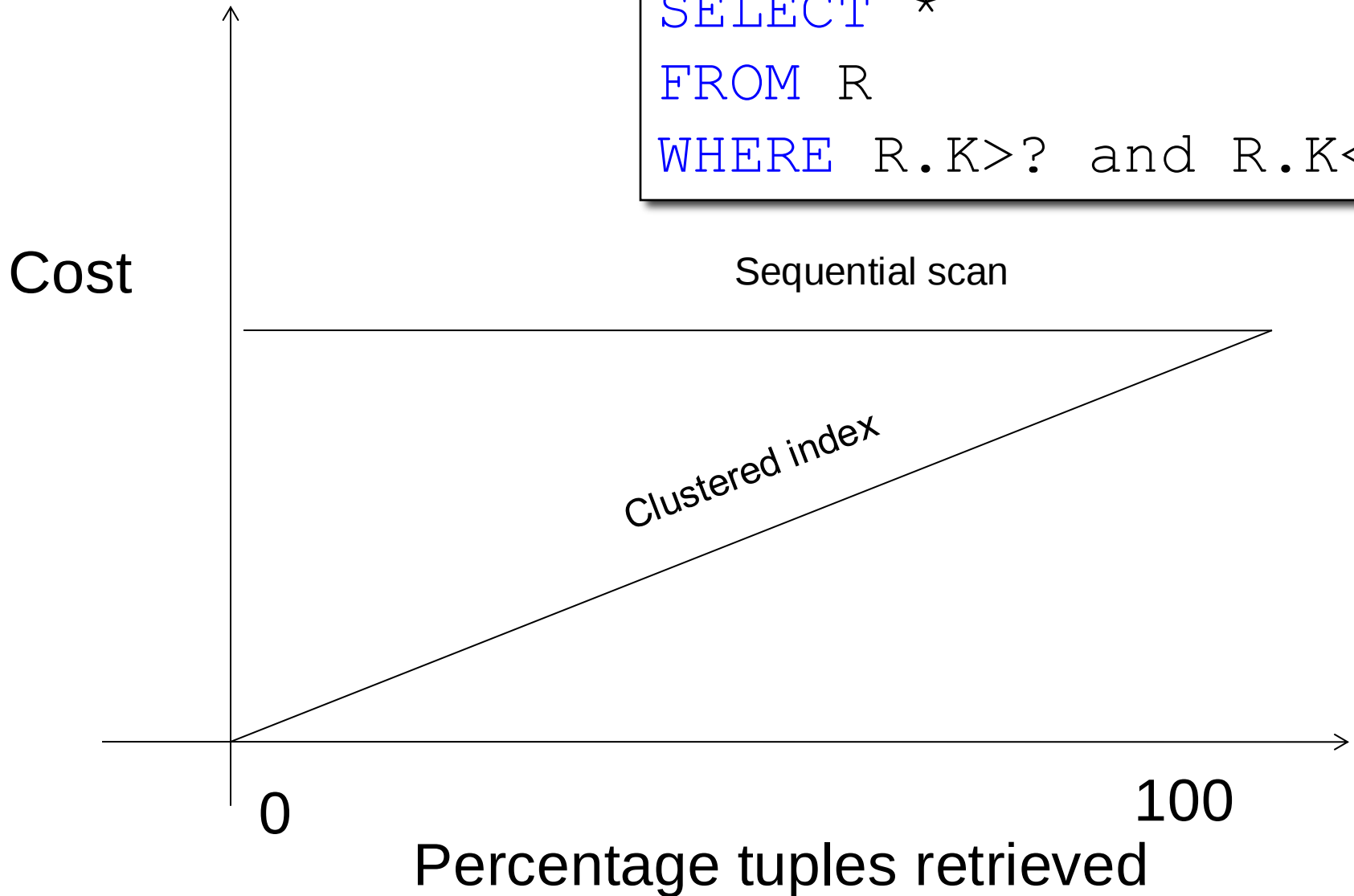
Percentage tuples retrieved

Clustered v.s. Unclustered Indexes

```
SELECT *
```

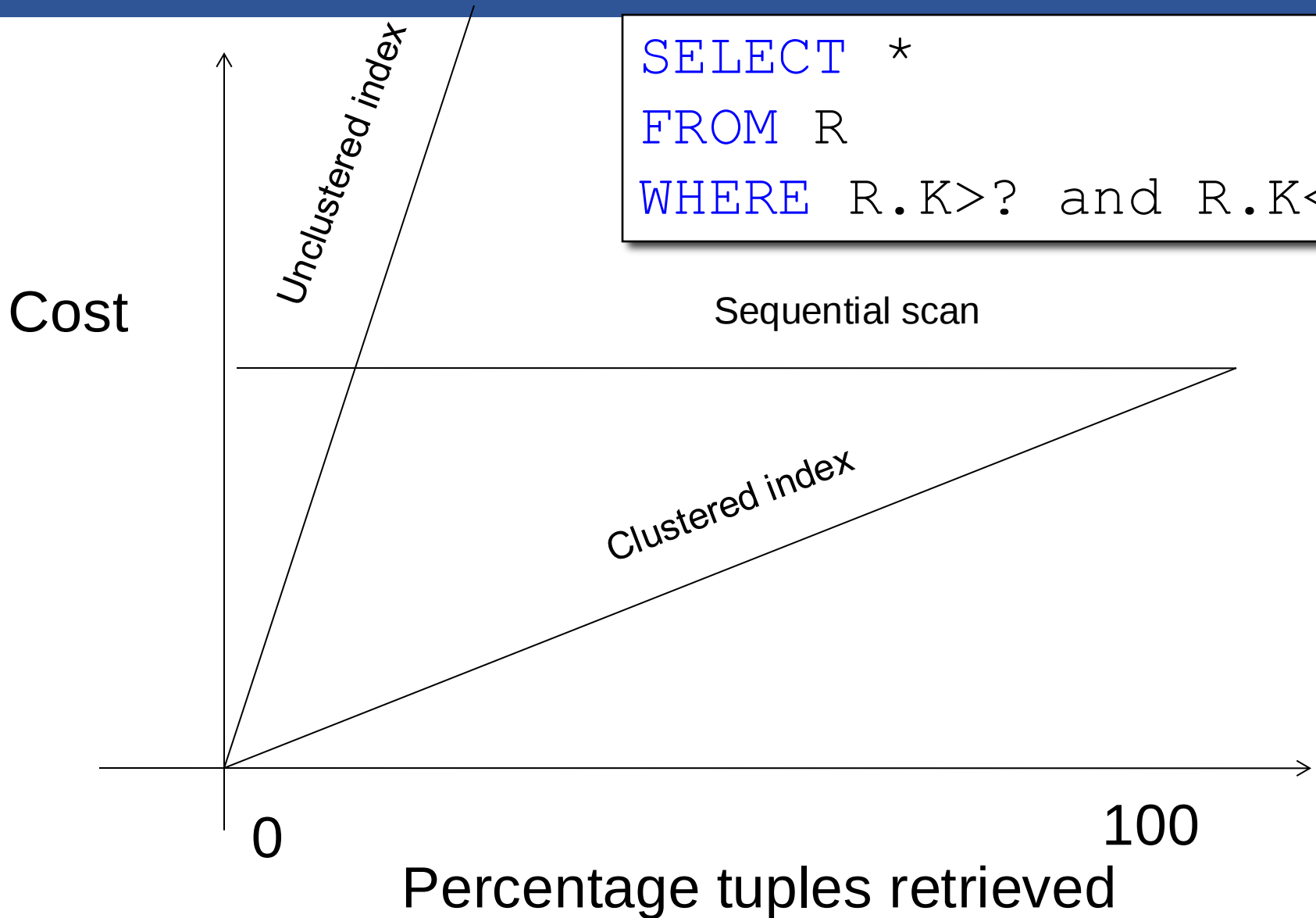
```
FROM R
```

```
WHERE R.K>? and R.K<?
```



Clustered v.s. Unclustered Indexes

```
SELECT *  
FROM R  
WHERE R.K > ? and R.K < ?
```



To Cluster or Not

- Rule of thumb:

Random reading 1-2% of file $\approx$ full sequential scan

- Range queries benefit mostly from clustering because they may read more than 1-2%

Indexes in SQL

Getting Practical: Creating Indexes in SQL

```
CREATE TABLE V(M int, N varchar(20), P int);
```

```
CREATE INDEX V1 ON V(N);
```

Getting Practical: Creating Indexes in SQL

```
CREATE TABLE V(M int, N varchar(20), P int);
```

```
CREATE INDEX V1 ON V(N);
```

```
CREATE INDEX V2 ON V(P, M);
```

Getting Practical: Creating Indexes in SQL

```
CREATE TABLE V(M int, N varchar(20), P int);
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```
CREATE INDEX V1 ON V(N);
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CREATE INDEX V2 ON V(P, M);
```

What does this mean?

Getting Practical: Creating Indexes in SQL

```
CREATE TABLE V(M int, N varchar(20), P int);
```

```
CREATE INDEX V1 ON V(N);
```

```
CREATE INDEX V2 ON V(P, M);
```

```
select *  
from V  
where P=55 and M=77
```

Yes

What does this mean?

Getting Practical: Creating Indexes in SQL

```
CREATE TABLE V(M int, N varchar(20), P int);
```

```
CREATE INDEX V1 ON V(N);
```

```
CREATE INDEX V2 ON V(P, M);
```

```
select *  
from V  
where P=55 and M=77
```

Yes

What does this mean?

```
select *  
from V  
where P=55
```

Yes

Getting Practical: Creating Indexes in SQL

```
CREATE TABLE V(M int, N varchar(20), P int);
```

```
CREATE INDEX V1 ON V(N);
```

```
CREATE INDEX V2 ON V(P, M);
```

```
select *  
from V  
where P=55 and M=77
```

Yes

What does this mean?

```
select *  
from V  
where P=55
```

Yes

```
select *  
from V  
where M=77
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No

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CREATE INDEX V3 ON V(M, N);
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CREATE INDEX V2 ON V(P, M);
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CREATE INDEX V3 ON V(M, N);
```

```
CREATE UNIQUE INDEX V4 ON V(N);
```

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from V  
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```
select *  
from V  
where P=55
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Yes

```
CREATE CLUSTERED INDEX V5 ON V(N);
```

```
select *  
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Not supported
in SQLite