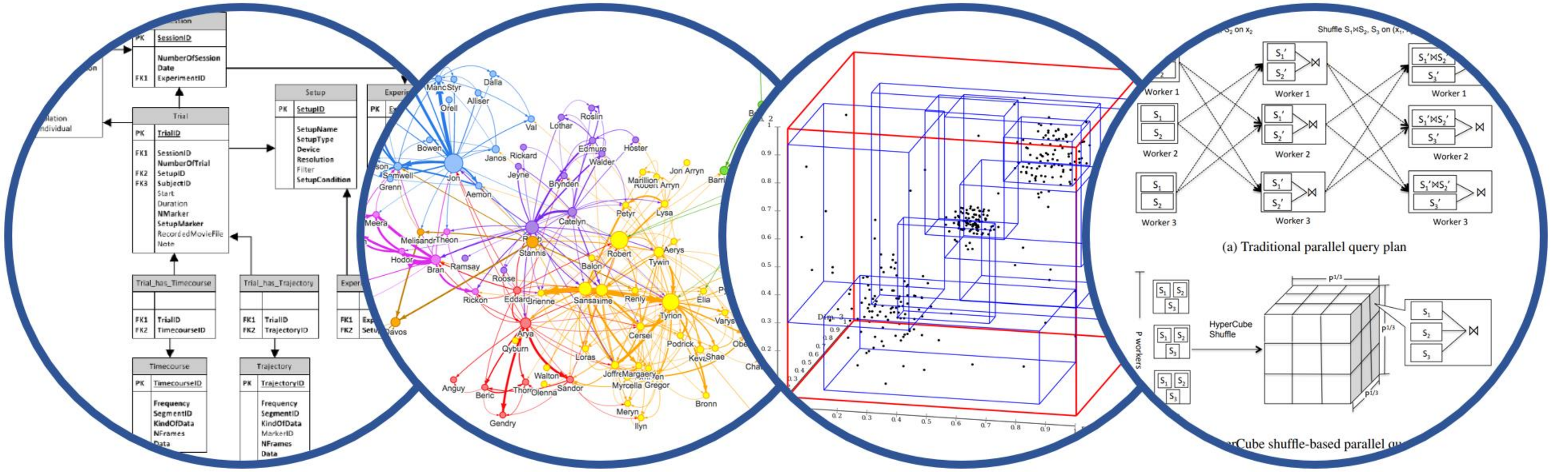

Introduction to Data Management

BCNF Decomposition

Paul G. Allen School of Computer Science and Engineering
University of Washington, Seattle

Announcements

- HW3 due tonight
- HW4 posted, due on Friday, Nov. 1st

Midterm

This coming Friday, 10/25, 9:30-10:20, in class

Topics:

- SQL
- Relational Algebra
- E/R Diagrams
- Functional Dependencies (FDs); no BCNF

longest

shorter

Closed books: cheat sheet included on the midterm

Short review session tomorrow in the sections

Recap

UID	Name	Phone	City
234	Fred	206-555-9999	Seattle
234	Fred	206-555-8888	Seattle
987	Joe	415-555-7777	SF

<u>UID</u>	Name	City
234	Fred	Seattle
987	Joe	SF

UID	Phone
234	206-555-9999
234	206-555-8888
987	415-555-7777

Anomalies:
- Redundancy
- Update
- Deletion

Functional dependencies
- **UID → Name, City**
- **UID ↛ Phone**

Inference

An Interesting Observation

If all these FDs are true:

Name → Color
Category → Dept
Color, Dept → Price

Then this FD is also true:

Name, Category → Price

Proof: (see last lecture)

Discussion

Two ways to infer new FDs:

- Armstrong axioms
- The closure operator

Armstrong's Axioms

Armstrong's Axioms

Reflexivity: if $Y \subseteq X$ then $X \to Y$

Augmentation: if $X \to Y$ then $XZ \to YZ$

Transitivity: if $X \to Y$ and $Y \to Z$ then $X \to Z$

Armstrong's Axioms

Reflexivity: if $Y \subseteq X$ then $X \rightarrow Y$

Called a trivial FD

Augmentation: if $X \rightarrow Y$ then $XZ \rightarrow YZ$

Transitivity: if $X \rightarrow Y$ and $Y \rightarrow Z$ then $X \rightarrow Z$

Using Armstrong's Axioms

Reflexivity:	if $Y \subseteq X$ then $X \to Y$
Augmentation:	if $X \to Y$ then $XZ \to YZ$
Transitivity:	if $X \to Y$ and $Y \to Z$ then $X \to Z$

1. Name → Color
2. Category → Dept
3. Color, Dept → Price

⇒

Name, Category → Price

Using Armstrong's Axioms

Reflexivity: if $Y \subseteq X$ then $X \to Y$
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1. Name → Color
2. Category → Dept
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⇒

Name, Category → Price

4. Name, Category → Color, Category (Augmentation of 1)

Using Armstrong's Axioms

Reflexivity: if $Y \subseteq X$ then $X \rightarrow Y$
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2. Category → Dept
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Name, Category → Price

4. Name, Category → Color, Category (Augmentation of 1)
5. Color, Category → Color, Dept (Augmentation of 2)

Using Armstrong's Axioms

Reflexivity: if $Y \subseteq X$ then $X \to Y$
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1. Name → Color
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⇒

Name, Category → Price

4. Name, Category → Color, Category (Augmentation of 1)
5. Color, Category → Color, Dept (Augmentation of 2)
6. Color, Category → Price (Transitivity 5 and 3)

Using Armstrong's Axioms

Reflexivity: if $Y \subseteq X$ then $X \rightarrow Y$
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1. Name → Color
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Name, Category → Price

4. Name, Category → Color, Category (Augmentation of 1)
5. Color, Category → Color, Dept (Augmentation of 2)
6. Color, Category → Price (Transitivity 5 and 3)
7. Name, Category → Price (Transitivity 4 and 6)

Discussion

- Armstrong’s Axioms were introduced in the 70s, shortly after Codd’s relational model
- They are widely known today
- But they are cumbersome to use for inference
- Instead, the efficient inference method uses the **closure operator**: next.

The Closure Operator

The Closure of a set X

Fix a set of Functional Dependencies

The closure X^+ of a
set of attributes X
is the set of attributes A
such that $X \to A$.

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Closure(X):
   Repeat:
      find a FD Y → A
      such that Y ⊆ X and A ⊈ X
      X ≔ X ∪ A
   Until "no more change"
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Closure(X):
   Repeat:
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       X := X ∪ A
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Name → Color
Category → Dept
Color, Dept → Price

$\{\text{Name}, \text{Category}\}^+ =$

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$\{\mathrm{Color}\}^+ =$

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Name → Color
Category → Dept
Color, Dept → Price

$$\{\text{Name}, \text{Category}\}^+ = $$
$$= \{\text{Name}, \text{Category}, \text{Color}, \text{Dept}, \text{Price}\}$$

$$\{\text{Color}\}^+ = \{\text{Color}\}$$

Discussion so Far

- Goal is to detect/remove anomalies

- Anomalies are caused by unwanted FDs
  - **UID → Name, City** UID determines something
  - **UID ↛ Phone** UID is not a key

- Next : **Keys**

Keys

Keys and Superkeys

- Fix a relation $\mathrm{R}(\mathrm{A}_1, \ldots, \mathrm{A}_\mathrm{n})$ and a set of FDs

- A **super-key** is a set X such that $\mathrm{X} \rightarrow \mathrm{A}_\mathrm{i}$ for every attribute A_i

Keys and Superkeys

- Fix a relation $R(A_1, \ldots, A_n)$ and a set of FDs

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Equivalently:
$X^+ = A_1 \ldots A_n$

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- A **key** is a minimal super-key X

Keys and Superkeys

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- A **super-key** is a set X such that $X \rightarrow A_i$ for every attribute A_i

Equivalently:
$X^+ = A_1 \ldots A_n$

- A **key** is a minimal super-key X

In other words,
no super-key $Y \subsetneq X$
exists

Example: Find the Keys

UID	Name	Phone	City
234	Fred	206-555-9999	Seattle
234	Fred	206-555-8888	Seattle
987	Joe	415-555-7777	SF
…	…	…	…

UID → Name, City

UID^+ = UID, Name, City

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Not a key:
missing Phone

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UID$^+$ = UID, Name, City

Not a key: missing Phone

(UID, Phone)$^+$ = **??**

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UID^+ = UID, Name, City

Not a key: missing Phone

$(UID, Phone)^+$ = UID, Name, Phone, City

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Not a key: missing Phone

$(UID, Phone)^+$ = UID, Name, Phone, City

Key

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Super-Key

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UID → Name, City

UID^+ = UID, Name, City

Not a key: missing Phone

$(UID, Phone)^+$ = UID, Name, Phone, City

Key

$(UID, Name, Phone)^+$ = UID, Name, Phone, City

Super-Key

$Phone^+$ = Phone

Example: Find the Keys

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UID → Name, City

UID^+ = UID, Name, City

Not a key: missing Phone

$(UID, Phone)^+$ = UID, Name, Phone, City

Key

$(UID, Name, Phone)^+$ = UID, Name, Phone, City

Super-Key

$Phone^+$ = Phone

Not a (Super-)Key

Example: Find the Keys

Compute X^+, for larger and larger sets X, until X^+= [all-attributes]

Name → Color
Category → Dept
Color, Dept → Price

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Sets X of size 1

$Name^+$ = Name, Color;
$Color^+$ = Color;

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Name^+ = Name, Color;
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Sets X of size 1

$(\text{Name, Color})^+$ = Name, Color;

Sets X of size 2

Example: Find the Keys

Compute X^+, for larger and larger sets X, until X^+= [all-attributes]

Name → Color
Category → Dept
Color, Dept → Price

Name$^+$ = Name, Color;
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Category$^+$ = Category, Dept;
Dept$^+$ = Dept

Sets X of size 1

(Name, Color)$^+$ = Name, Color;
(Name, Category)$^+$ = Name, Color, Category, Dept, Price;

Sets X of size 2

Example: Find the Keys

Compute X^+, for larger and larger sets X, until X^+= [all-attributes]

Name → Color
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$Name^+$ = Name, Color;
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Sets X of size 1

$(Name, Color)^+$ = Name, Color;
$(Name, Category)^+$ = Name, Color, Category, Dept, Price;
// no need to try $(Name, Category, Dept)^+$ why?

Sets X of size 2

Example: Find the Keys

Compute X^+, for larger and larger sets X, until X^+= [all-attributes]

Name → Color
Category → Dept
Color, Dept → Price

Sets X of size 1

Name^+ = Name, Color;
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Sets X of size 2

$(\text{Name, Color})^+$ = Name, Color;
$(\text{Name, Category})^+$ = Name, Color, Category, Dept, Price;
// no need to try $(\text{Name, Category, Dept})^+$ why?
$(\text{Name, Dept})^+$ = … … and so on until we find all keys

Example: Find the Keys

Compute X^+, for larger and larger sets X, until X^+= [all-attributes]

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Sets X of size 1

Name^+ = Name, Color;
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Sets X of size 2

$(\text{Name, Color})^+$ = Name, Color;
$(\text{Name, Category})^+$ = Name, Color, Category, Dept, Price;
// no need to try $(\text{Name, Category, Dept})^+$ why?
$(\text{Name, Dept})^+$ = … … and so on until we find all keys

There is only one key: **Name, Category**

Example: Find the Keys

Compute X^+, for larger and larger sets X, until X^+= [all-attributes]

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Sets X of size 2

$(\text{Name, Color})^+$ = Name, Color;
$(\text{Name, Category})^+$ = Name, Color, Category, Dept, Price;
// no need to try $(\text{Name, Category, Dept})^+$ why?
$(\text{Name, Dept})^+$ = … … and so on until we find all keys

There is only one key: **Name, Category**

A quicker way: any key X must contain Name (why?) and Category (why?)

Keys are Not Unique

R(A,B,C)

A → B,C
B → A,C

Keys are Not Unique

R(A,B,C)

$\boxed{\begin{array}{l} A \rightarrow B,C \\ B \rightarrow A,C \end{array}}$

$A^+ = B^+ = ABC$

A is a key
B is a key

Keys are Not Unique

R(A,B,C)

$A \rightarrow B,C$
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$A^+ = B^+ = ABC$

A is a key
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In SQL
we must choose
either A or B
as primary key

Keys are Not Unique

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Don't confuse with

$$\boxed{A,B \rightarrow C}$$

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Don't confuse with

$A,B \rightarrow C$

$A^+ = A,\ B^+ = B$
$(AB)^+ = ABC$

AB is a key

Note the difference:
- AB is a key v.s.
- A and B are keys

Discussion

- Our redundancies come this FD:

  UID → Name, City

- The problem is that UID is not a key.

- Boyce-Codd Normal Form captures this intuition.

- Next: BCNF

BCNF

BCNF

- Fix a relation $R(A_1, \ldots, A_n)$ and a set of FDs

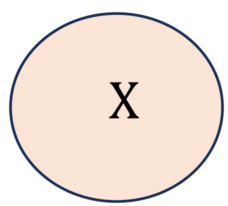
R is in Boyce-Codd Normal Form (BCNF), if every FD $X \rightarrow Y$ is either from a superkey X or is trivial: $Y \subseteq X$

- Equivalently: for every set X,
  either $X^+ = X$ or $X^+ = [\text{all}-\text{attributes}]$

Normalization

Algorithm BCNF $R(A_1, \dots, A_n)$

Find set X s.t. $X \subsetneq X^+ \subsetneq \{A_1, \dots, A_n\}$

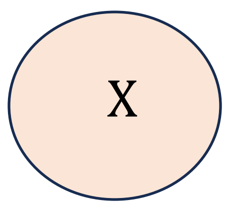


Normalization

Algorithm BCNF $\mathrm{R}(\mathrm{A}_1, \dots, \mathrm{A}_n)$

Find set X s.t. $\mathrm{X} \subsetneq \mathrm{X}^+ \subsetneq \{\mathrm{A}_1, \dots, \mathrm{A}_n\}$

If not found **then** return $\mathrm{R}(\mathrm{A}_1, \dots, \mathrm{A}_n)$ // already in BCNF



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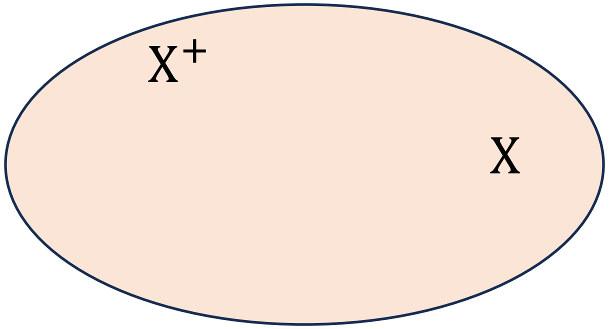

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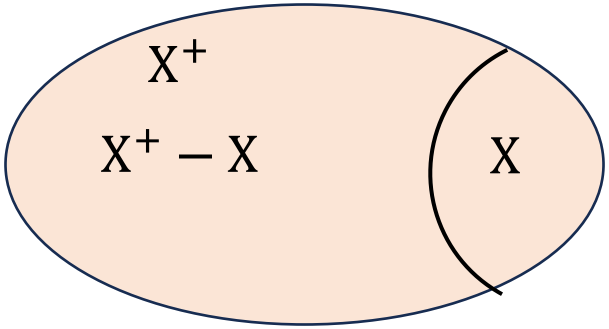

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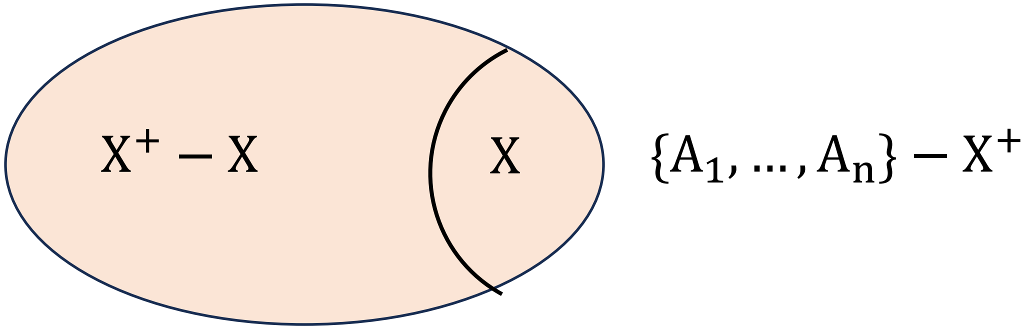

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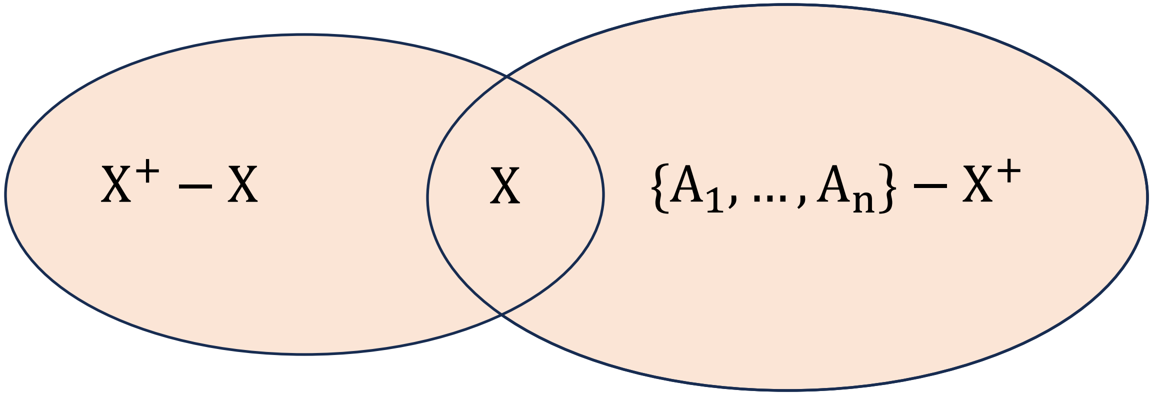

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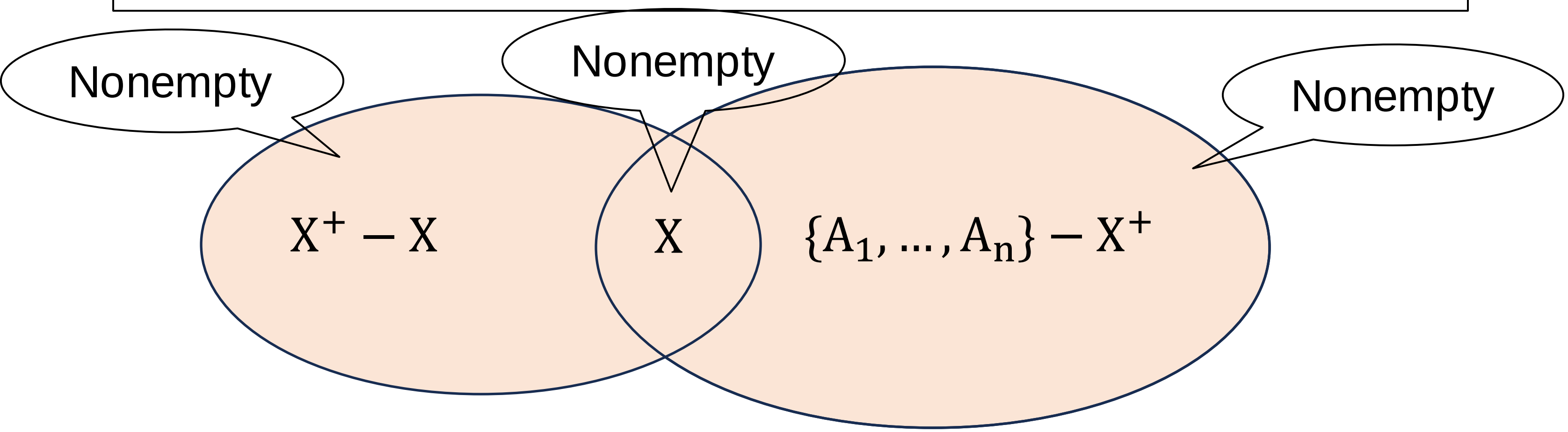

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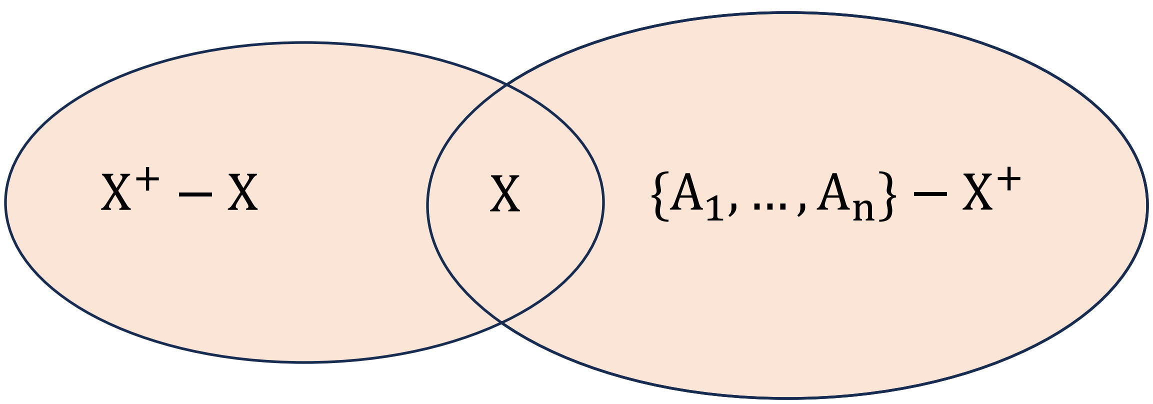
If not found **then** return $R(A_1, \ldots, A_n)$ // already in BCNF



Normalization

Algorithm BCNF $R(A_1, \ldots, A_n)$

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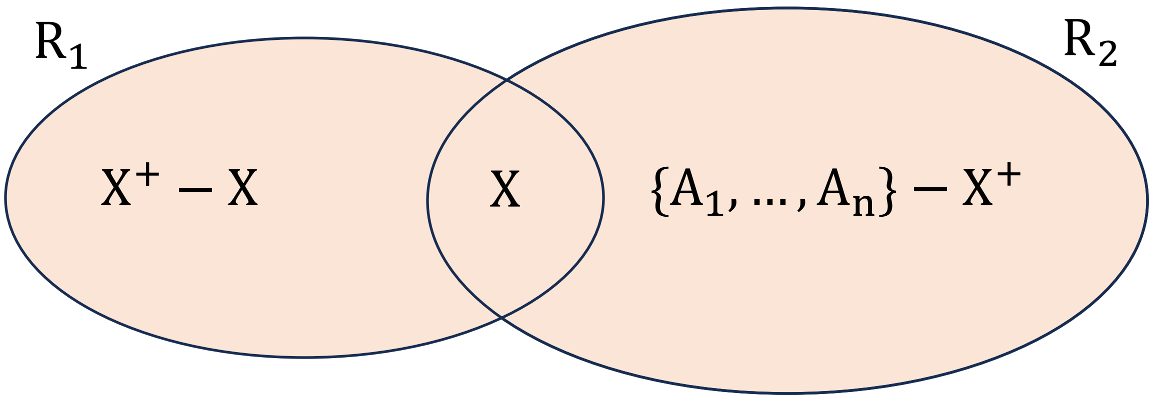
Decompose: $R(A_1, \ldots, A_n) = R_1(X^+) \bowtie R_2(\{A_1, \ldots, A_n\} - X^+)$



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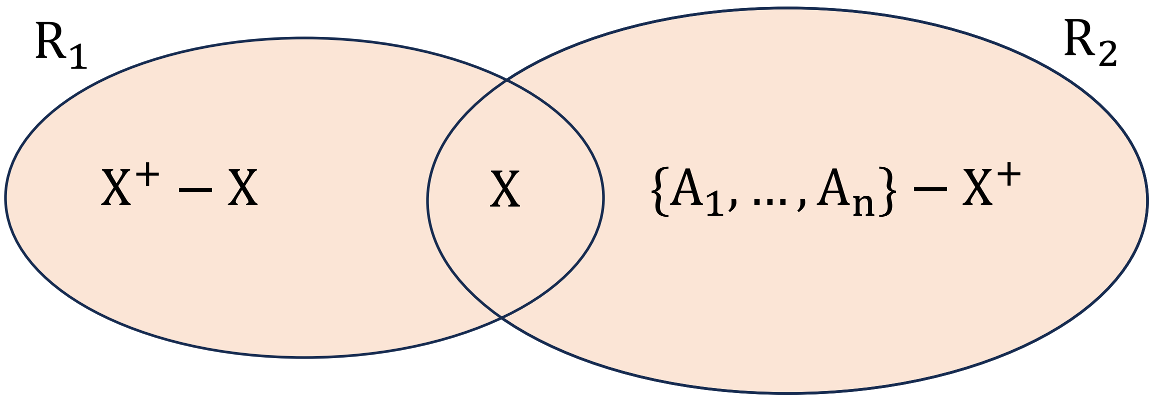
Decompose: $R(A_1, \dots, A_n) = R_1(X^+) \bowtie R_2(\{A_1, \dots, A_n\} - X^+)$

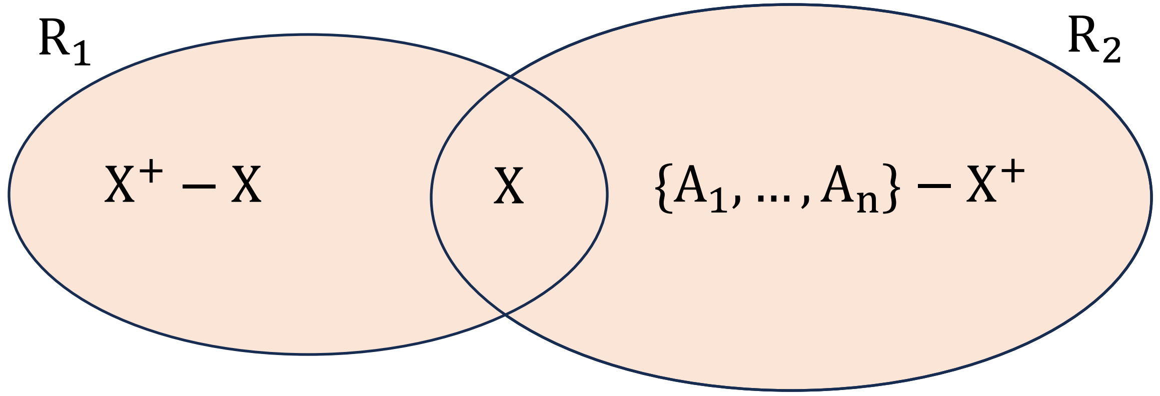
Call recursively BCNF on $R_1(X^+)$
Call recursively BCNF on $R_2(\{A_1, \dots, A_n\} - X^+)$



Example: Decompose in BCNF

UID	Name	Phone	City
234	Fred	206-555-9999	Seattle
234	Fred	206-555-8888	Seattle
987	Joe	415-555-7777	SF
…	…	…	…

UID → Name, City



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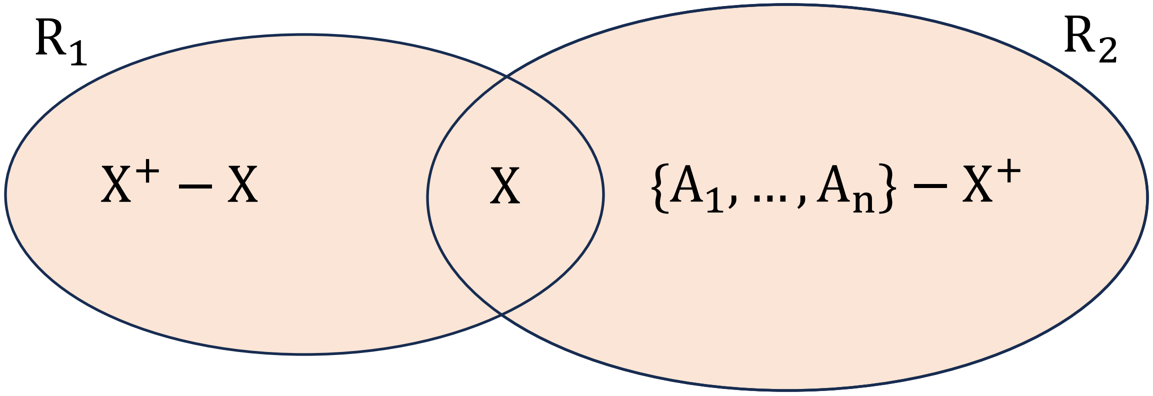
234	Fred	206-555-9999	Seattle
234	Fred	206-555-8888	Seattle
987	Joe	415-555-7777	SF
…	…	…	…

UID → Name, City

Find set X s.t. $X \subsetneq X^+ \subsetneq \{\text{UID}, \text{Name}, \text{Phone}, \text{City}\}$



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234	Fred	206-555-9999	Seattle
234	Fred	206-555-8888	Seattle
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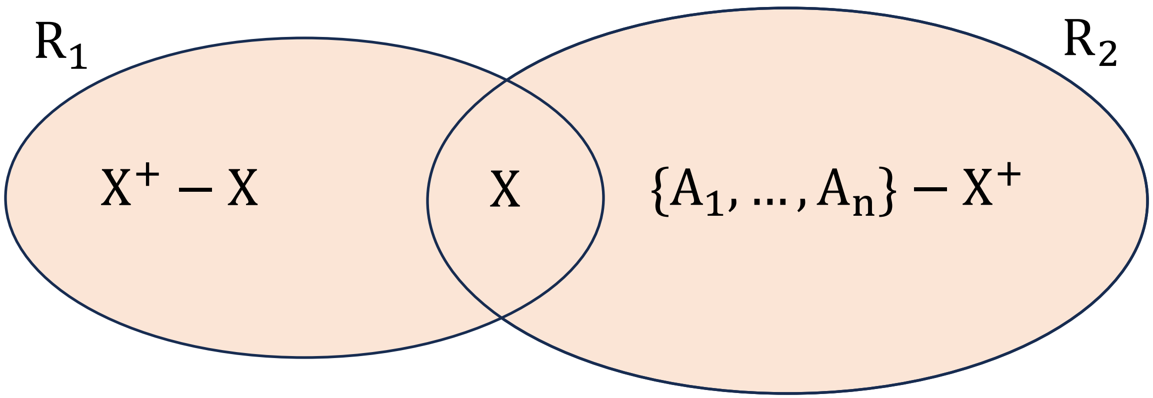
…	…	…	…

UID → Name, City

Find set X s.t. $X \subsetneq X^+ \subsetneq \{\mathrm{UID}, \mathrm{Name}, \mathrm{Phone}, \mathrm{City}\}$

$X = \mathrm{UID}$, $X^+ = \{\mathrm{UID}, \mathrm{Name}, \mathrm{City}\}$



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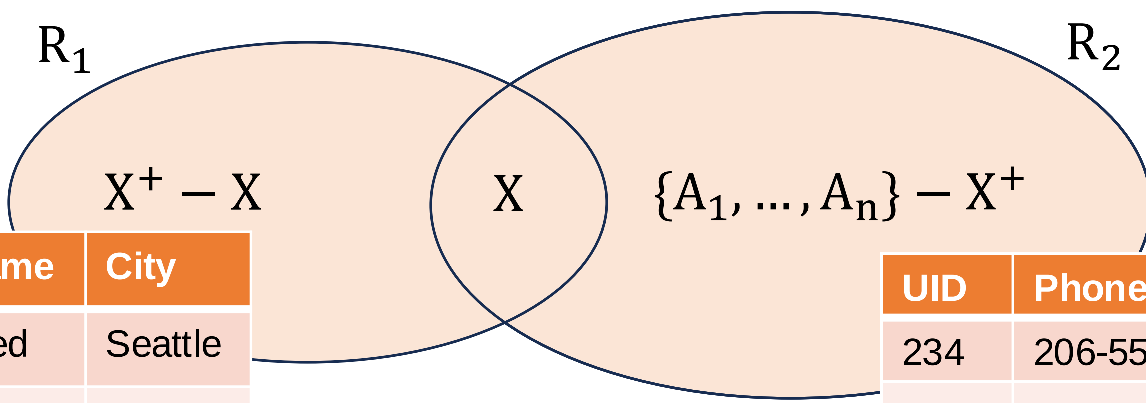
234	Fred	206-555-8888	Seattle
987	Joe	415-555-7777	SF
…	…	…	…

$\boxed{\text{UID} \rightarrow \text{Name, City}}$

Find set X s.t. $X \subsetneq X^+ \subsetneq \{\text{UID}, \text{Name}, \text{Phone}, \text{City}\}$

$X = \text{UID}, \qquad X^+ = \{\text{UID}, \text{Name}, \text{City}\}$



UID	Name	City
234	Fred	Seattle
…		

UID	Phone
234	206-555-9999
…	…

Example: Decompose in BCNF

R(Name, Color, Category, Dept, Price)

Name → Color
Category → Dept
Color, Dept → Price

Example: Decompose in BCNF

Name → Color
Category → Dept
Color, Dept → Price

R(Name, Color, Category, Dept, Price)

$$\mathrm{X} = \mathrm{Name}$$
$$\mathrm{X}^{+} = \{\mathrm{Name}, \mathrm{Color}\}$$

Example: Decompose in BCNF

Name → Color
Category → Dept
Color, Dept → Price

R(Name, Color, Category, Dept, Price)

$X = \text{Name}$
$X^+ = \{\text{Name}, \text{Color}\}$

R_1(<u>Name</u>, Color)

R_2(Name,Category,Dept,Price)

Example: Decompose in BCNF

Name → Color
Category → Dept
Color, Dept → Price

R(Name, Color, Category, Dept, Price)

$X = \text{Name}$
$X^{+} = \{\text{Name}, \text{Color}\}$

R_1(<u>Name</u>, Color)

R_2(Name,Category,Dept,Price)

BCNF because:
Name+ = Name, Color
Color^{+} = Color

Example: Decompose in BCNF

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R_2(Name,Category,Dept,Price)

BCNF because:
Name+ = Name, Color
Color^{+} = Color

$X = \text{Category}$
$X^{+} = \{\text{Category}, \text{Dept}\}$

Example: Decompose in BCNF

Name → Color
Category → Dept
Color, Dept → Price

R(Name, Color, Category, Dept, Price)

$X = \text{Name}$
$X^+ = \{\text{Name}, \text{Color}\}$

R_1(Name, Color)

R_2(Name,Category,Dept,Price)

BCNF because:
Name+ = Name, Color
Color^+ = Color

$X = \text{Category}$
$X^+ = \{\text{Category}, \text{Dept}\}$

R_3(Category, Dept)

R_4(Name, Category, Price)

BCNF

Example: Decompose in BCNF

Name → Color
Category → Dept
Color, Dept → Price

R(Name, Color, Category, Dept, Price)

$X = \text{Name}$
$X^{+} = \{\text{Name}, \text{Color}\}$

R_1(Name, Color)

R_2(Name,Category,Dept,Price)

BCNF because:
Name+ = Name, Color
Color^{+} = Color

$X = \text{Category}$
$X^{+} = \{\text{Category}, \text{Dept}\}$

R_3(Category, Dept)

R_4(Name, Category, Price)

BCNF

BCNF
Name,Category is a key because:
$(\text{Name,Category})^{+}$=…,Price

Example: Decompose in BCNF

Name → Color
Category → Dept
Color, Dept → Price

R(Name, Color, Category, Dept, Price)

$X = \text{Name}$
$X^+ = \{\text{Name}, \text{Color}\}$

R_1(Name, Color)

R_2(Name,Category,Dept,Price)

BCNF because:
Name+ = Name, Color
Color$^+$ = Color

$X = \text{Category}$
$X^+ = \{\text{Category}, \text{Dept}\}$

R_3(Category, Dept)

BCNF

R_4(Name, Category, Price)

BCNF
Name,Category is a key because:
(Name,Category)$^+$=…,Price

Final Decomposition

Example: Decompose in BCNF

Name → Color
Category → Dept
Color, Dept → Price

We lost the FD
Color, Dept → Price

R(Name, Color, Category, Dept, Price)

$X = \text{Name}$
$X^{+} = \{\text{Name}, \text{Color}\}$

R_1(Name, Color)

R_2(Name,Category,Dept,Price)

BCNF because:
Name+ = Name, Color
Color^{+} = Color

$X = \text{Category}$
$X^{+} = \{\text{Category}, \text{Dept}\}$

R_3(Category, Dept)

R_4(Name, Category, Price)

BCNF

BCNF
Name,Category is a key because:
$(\text{Name,Category})^{+}$=…,Price

Final Decomposition

Example: Decompose in BCNF

R(Name, Color, Category, Dept, Price)

Name → Color
Category → Dept
Color, Dept → Price

Decomposition is not unique

Example: Decompose in BCNF

Name → Color
Category → Dept
Color, Dept → Price

R(Name, Color, Category, Dept, Price)

$$X = \text{Color}, \text{Dept}$$
$$X^+ = \{\text{Color}, \text{Dept}, \text{Price}\}$$

Decomposition is not unique

Example: Decompose in BCNF

Name → Color
Category → Dept
Color, Dept → Price

R(Name, Color, Category, Dept, Price)

$X = \text{Color}, \text{Dept}$
$X^{+} = \{\text{Color}, \text{Dept}, \text{Price}\}$

$R_1(\underline{\text{Color, Dept}}, \text{Price})$

$R_2(\text{Name, Category, Color, Dept})$

BCNF

Decomposition is not unique

Example: Decompose in BCNF

Name → Color
Category → Dept
Color, Dept → Price

R(Name, Color, Category, Dept, Price)

$$X = \text{Color, Dept}$$
$$X^+ = \{\text{Color, Dept, Price}\}$$

$R_1(\underline{\text{Color, Dept}}, \text{Price})$

BCNF

$R_2(\text{Name, Category, Color, Dept})$

$$X = \text{Name}$$
$$X^+ = \{\text{Name, Color}\}$$

Decomposition is not unique

Example: Decompose in BCNF

Name → Color
Category → Dept
Color, Dept → Price

R(Name, Color, Category, Dept, Price)

$X = \text{Color}, \text{Dept}$
$X^+ = \{\text{Color}, \text{Dept}, \text{Price}\}$

R_1(<u>Color, Dept</u>, Price)

BCNF

R_2(Name, Category, Color, Dept)

$X = \text{Name}$
$X^+ = \{\text{Name}, \text{Color}\}$

R_3(<u>Name</u>, Color)

BCNF

R_4(Name, Category, Dept)

Decomposition is not unique

Example: Decompose in BCNF

Name → Color
Category → Dept
Color, Dept → Price

R(Name, Color, Category, Dept, Price)

$X = \text{Color}, \text{Dept}$
$X^{+} = \{\text{Color}, \text{Dept}, \text{Price}\}$

R_1(Color, Dept, Price)

BCNF

R_2(Name, Category, Color, Dept)

$X = \text{Name}$
$X^{+} = \{\text{Name}, \text{Color}\}$

R_3(Name, Color)

BCNF

R_4(Name, Category, Dept)

$X = \text{Category}$
$X^{+} = \{\text{Category}, \text{Dept}\}$

Decomposition is not unique

Example: Decompose in BCNF

Name → Color
Category → Dept
Color, Dept → Price

R(Name, Color, Category, Dept, Price)

$X = \text{Color, Dept}$
$X^+ = \{\text{Color, Dept, Price}\}$

R_1(Color, Dept, Price)

BCNF

R_2(Name, Category, Color, Dept)

$X = \text{Name}$
$X^+ = \{\text{Name, Color}\}$

R_3(Name, Color)

BCNF

R_4(Name, Category, Dept)

$X = \text{Category}$
$X^+ = \{\text{Category, Dept}\}$

R_5(Category, Dept)

BCNF

R_6(Name, Category)

BCNF

Decomposition is not unique

Example: Decompose in BCNF

Name → Color
Category → Dept
Color, Dept → Price

All FDs are recovered

R(Name, Color, Category, Dept, Price)

$X = \text{Color, Dept}$
$X^+ = \{\text{Color, Dept, Price}\}$

R_1(<u>Color, Dept</u>, Price)

BCNF

R_2(Name, Category, Color, Dept)

$X = \text{Name}$
$X^+ = \{\text{Name, Color}\}$

R_3(<u>Name</u>, Color)

BCNF

R_4(Name, Category, Dept)

$X = \text{Category}$
$X^+ = \{\text{Category, Dept}\}$

R_5(<u>Category</u>, Dept)

BCNF

R_6(Name, Category)

BCNF

Final decomposition

Decomposition is not unique

Discussion

- The BCNF decomposition eliminates all anomalies
- In general, we may not be able to recover all FDs
- The 3rd Normal Form is another kind of decomposition, which recovers all FDs, but does not eliminate all anomalies
- We won’t discuss 3NF: it is very similar to BCNF but a lot more complicated