RECURSION

In pure Lisp there is no looping; recursion is used instead.

A recursive function is defined in terms of:

1. One or more base cases
2. Invocation of one or more simpler instances of itself.

Note that recursion is directly related to mathematical induction.

An inductive proof has:

1. A basis clause
2. A hypothesis that the theorem is true for some number K
3. An inductive clause that shows it is then true for K+1.

FACTORIAL

$$N! = N \cdot N-1 \cdot \ldots \cdot 1 = N(N-1)!$$

```
(defun factorial (N)
        (cond
                ((eql     N  0)   1)
                ((eql     N  1)   1)
                (T    (*  N (factorial   ( -   N  1))))
))
```

```
(factorial   3)
        (*   3   (factorial   2))
                (*   2   (factorial 1))
                       1
                2
        6
```

Can you prove by induction that (factorial N) produces N! for N ≥ O ?

1.36

PROOF

Basis: (factorial 0) produces 1
(factorial 1) produces 1

by definition of the first two cases in the cond.

Hypothesis: Suppose (factorial K)
correctly returns K!, K > 1

Inductive Part:

Then (factorial < K + 1 >) evaluates to
(* < K + 1 > (factorial (- < K + 1 > 1)))

which will be

(* < K + 1 > (factorial K))

or (K + 1) * K! = (K + 1) !

1.37

Doubling the Values of All Elements of a List

```
( defun   double   ( x )
      ( if    ( null   x )    NIL
              ( cons   ( *   ( first  x )     2 )
                       ( double   ( rest   x ) ) ) ) ) )
```

```
( double     ’( 2  0  6  ) )
  ( cons  4 ( double  ’( 0  6 ) ) )
           ( cons  0  ( double   ’( 6 ) ) )
                      ( cons  12  ( double NIL ) )
  ( 4  0  12 )
```

Summing a List

```
( defun   sum  ( L )
   ( if  ( null  L )  0
         ( +   ( first  L )  ( sum  ( rest  L ) ) ) ) ) )
```

1.38

FIBONACCI FUNCTION

$$f(n) = \begin{cases} f(n-1) + f(n-2) & n > 1 \\ 1 & n = 1 \\ 1 & n = 0 \end{cases}$$

```
( defun   fibonacci   ( N )
            ( if   ( or  (=  n  o)  (=  n  1) )
                1
                ( +  ( fibonacci  ( -  n  1 ) )
                     ( fibonacci  ( -  n  2 ) )  ) ) )
```

```
( fibonacci   4 )
 ( +   ( fibonacci   3 )   ( fibonacci   2 ) )
   (+  ( fibonacci  2)  (fibonacci   1))  (+  (fibonacci  1) (fibonacci 0) )
                              1                  1              1
                                                        2
(+  ( fibonacci 1) (fibonacci 0) )
         1              1
                  2
                        3
                                                             5
```

1.39

<u>FINDING THE INTEGERS FROM</u>
<u>I DOWN TO ZERO</u>

```
( defun   tozero  ( i )
   ( if   ( zerop   i )      '( 0 )
        ( cons    i  ( tozero   ( 1-    i ) ) ) ) ) )
```

```
( tozero    3 )
   ( cons   3 ( tozero   2 ) )
             ( cons   2   ( tozero   1 ) )
                          ( cons  1  (tozero  0 ) )
                                        ( 0 )
                      (   1   0   )
              (   2   1   0   )
     (   3   2   1   0   )
```

1.40

SEARCHING FOR A VALUE IN A LIST

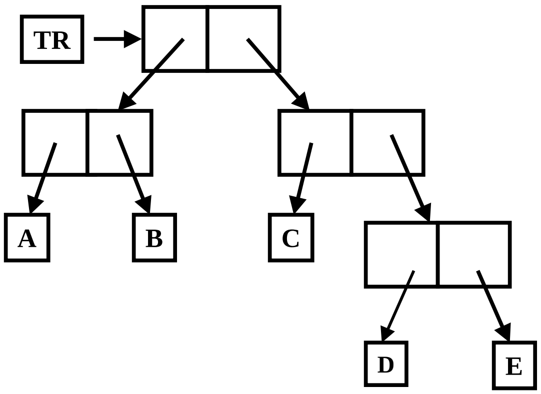
```
( defun  isin   ( val   L )
    ( cond
        ( ( null   L )    NIL )
        ( ( eql    val    ( car   L ) )     T )
        ( T       ( isin    val   ( cdr   L ) ) )
) )
```

```
( isin  8    '( 6    2    3    4    1    31 ) )
 ( isin   8   '( 2  3   4   8   1   31 ) )
  ( isin   8   '( 3   4   8   1   31 ) )
   ( isin   8   '( 4   8   1   31 ) )
    ( isin  8    '( 8   1   31 ) )
        T
```

1.41

Two-Sided Recursion

Searching for a Value in a Tree. Assume this kind of structure

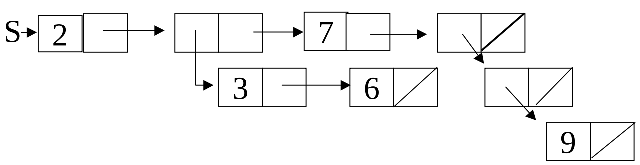



```
( defun     search        ( val  TR )
   ( cond
      ( ( null    TR )      NIL )
      ( ( atom  TR )      (eql    TR    val ) )
      (   T
            (or
                 ( search   val    ( car  TR ) )
                 ( search   val    ( cdr  TR ) ) )
) )
```

Another One with Similar Form

Determine if there are any odd numbers in an arbitrary list structure.

```
( defun    any-odd  ( x )
   ( cond
      ( ( null x )       nil)
      ( ( numberp   x )  ( oddp   x ) )
      (   T       ( or
                          ( any-odd    ( first   x ) )
                          ( any-odd    ( rest    x ) ) ) ) )
) )
```



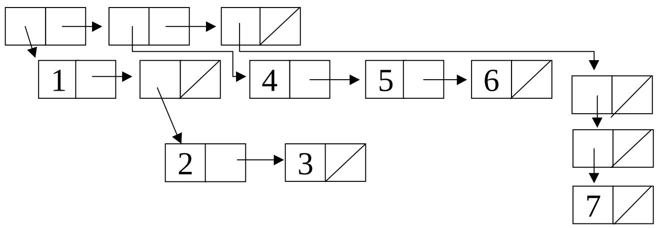
(any-odd S)

Is this efficient?

ONE MORE

```
( defun   flatten      ( x )
   ( cond
      ( ( null  x )    NIL )
      ( ( symbolp   x )   ( list   x )
      (   T
            ( append
                  ( flatten   ( first   x ) )
                  ( flatten   ( rest    x ) ) ) )
) )
```

(flatten ’((1 (2 3)) (4 5 6)(((7))))



(1 2 3 4 5 6 7)

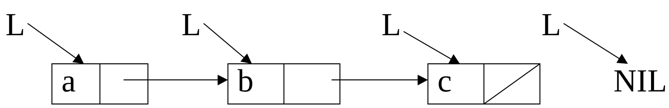
LOOP

1.44

(loop < sequence of forms >)

where one form is (return < val >)

```
( loop
        ( when  ( null  L )  ( return  NIL ) )
        ( print  ( car   L ) )
        ( setf   L   ( cdr   L ) )
)
```



-- loops can be more efficient

-- people like loops

-- but they involve assignment and thus side effects

ITERATION

1.45

Common Lisp provides looping functions.

dotimes

(dotimes (< count parm > < upper bound > < result >)
< sequence of forms >)

```
( dotimes   ( i    10)    ( print  i ) )
```

0 1 2 3 4 5 6 7 8 9 NIL returned

dolist

(dolist (< element parm > < list form > < result >)
< sequence of forms >)

```
( dolist    ( q   ’( a b c ) )   ( print  q ) )
```

A B C NIL returned

FILE I/O WITH ITERATION

```
( with--open--file
    ( < stream  name >
      < file  specs >
      :direction    < :input  or   :output > )
      < sequence  of  forms > )


( setf   L   NIL )
( with--open--file    ( fi   ''myinput.lsp''
          :direction        :input )
    ( dotimes         ( i    10 )
                      ( setf   val   ( read  fi ) )
                      ( setf   L     ( cons   val   L ) ) ) )
( with--open--file     ( fo   ''myoutput.lsp''
          :direction        :output )
    ( dolist       ( q     L )   ( print   q   fo ) ) )
```