PAUL G. ALLEN SCHOOL OF COMPUTER SCIENCE & ENGINEERING

CSE341: Programming Languages

Lecture 24
Subtyping

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Last major topic: Subtyping

Build up key ideas from first principles

- In pseudocode because:
  - No time for another language
  - Simpler to first show subtyping without objects

Then:

- How does subtyping relate to types for OOP?
  - Brief sketch only
- What are the relative strengths of subtyping and generics?
- How can subtyping and generics combine synergistically?

A tiny language

- Can cover most core subtyping ideas by just considering *records with mutable fields*

- Will make up our own syntax
  - ML has records, but no subtyping or field-mutation
  - Racket and Ruby have no type system
  - Java uses class/interface names and rarely fits on a slide

Records (half like ML, half like Java)

Record creation (field names and contents):

`{f1=e1, f2=e2, …, fn=en}` Evaluate **`ei`**, make a record

Record field access:

`e.f` Evaluate **`e`** to record **`v`** with an **`f`** field, get contents of **`f`** field

Record field update

`e1.f = e2` Evaluate **`e1`** to a record **`v1`** and **`e2`** to a value **`v2`**; Change **`v1`**'s **`f`** field (which must exist) to **`v2`**; Return **`v2`**

A Basic Type System

Record types: What fields a record has and type for each field

```
{f1:t1, f2:t2, …, fn:tn}
```

Type-checking expressions:

- If **`e1`** has type **`t1`**, …, **`en`** has type **`tn`**, then **`{f1=e1, …, fn=en}`** has type **`{f1:t1, …, fn:tn}`**
- If **`e`** has a record type containing **`f : t`**, then **`e.f`** has type **`t`**
- If **`e1`** has a record type containing **`f : t`** and **`e2`** has type **`t`**, then **`e1.f = e2`** has type **`t`**

This is safe

These evaluation rules and typing rules prevent ever trying to access a field of a record that does not exist

Example program that type-checks (in a made-up language):

```
fun distToOrigin (p:{x:real,y:real}) =
  Math.sqrt(p.x*p.x + p.y*p.y)

val pythag : {x:real,y:real} = {x=3.0, y=4.0}
val five : real = distToOrigin(pythag)
```

Motivating subtyping

But according to our typing rules, this program does not type-check

- It does nothing wrong and seems worth supporting

```
fun distToOrigin (p:{x:real,y:real}) =
  Math.sqrt(p.x*p.x + p.y*p.y)

val c : {x:real,y:real,color:string} =
   {x=3.0, y=4.0, color="green"}

val five : real = distToOrigin(c)
```

A good idea: allow extra fields

Natural idea: If an expression has type

```
{f1:t1, f2:t2, …, fn:tn}
```

Then it can *also* have a type with some fields removed

This is what we need to type-check these function calls:

```
fun distToOrigin (p:{x:real,y:real}) = …
fun makePurple (p:{color:string}) =
    p.color = "purple"

val c :{x:real,y:real,color:string} =
   {x=3.0, y=4.0, color="green"}

val _ = distToOrigin(c)
val _ = makePurple(c)
```

Keeping subtyping separate

A programming language already has a lot of typing rules and we do not want to change them

– Example: The type of an actual function argument must ***equal*** the type of the function parameter

We can do this by adding "just two things to our language"

– *Subtyping*: Write **`t1 <: t2`** for **`t1`** is a subtype of **`t2`**

– One new typing rule that uses subtyping:

If **`e`** has type **`t1`** and **`t1 <: t2`**,

then **`e`** (also) has type **`t2`**

Now all we need to do is define **`t1 <: t2`**

Subtyping is not a matter of opinion

- Misconception: If we are making a new language, we can have whatever typing and subtyping rules we want

- Not if you want to prevent what you claim to prevent [soundness]
  - Here: No accessing record fields that do not exist

- Our typing rules were *sound* before we added subtyping
  - We should keep it that way

- Principle of *substitutability*: If **`t1 <: t2`**, then any value of type **`t1`** must be usable in every way a **`t2`** is
  - Here: Any value of subtype needs all fields any value of supertype has

Four good rules

For our record types, these rules all meet the substitutability test:

1. "Width" subtyping: A supertype can have a subset of fields with the same types
2. "Permutation" subtyping: A supertype can have the same set of fields with the same types in a different order
3. Transitivity: If **`t1 <: t2`** and **`t2 <: t3`**, then **`t1 <: t3`**
4. Reflexivity: Every type is a subtype of itself

(4) may seem unnecessary, but it composes well with other rules in a full language and "does no harm"

More record subtyping?

[Warning: I am misleading you ☺]

Subtyping rules so far let us drop fields but not change their types

Example: A circle has a center field holding another record

```
fun circleY (c:{center:{x:real,y:real}, r:real}) =
   c.center.y

val sphere:{center:{x:real,y:real,z:real}, r:real} =
  {center={x=3.0,y=4.0,z=0.0}, r=1.0}

val _ = circleY(sphere)
```

For this to type-check, we need:

```
{center:{x:real,y:real,z:real}, r:real}
                  <:
   {center:{x:real,y:real}, r:real}
```

Do not have this subtyping – could we?

```
{center:{x:real,y:real,z:real}, r:real}
                 <:
   {center:{x:real,y:real}, r:real}
```

- No way to get this yet: we can drop **`center`**, drop **`r`**, or permute order, but cannot "reach into a field type" to do subtyping

- So why not add another subtyping rule… "Depth" subtyping:
  If **`ta <: tb`**, then **`{f1:t1, …, f:ta, …, fn:tn} <:`**
  `{f1:t1, …, f:tb, …, fn:tn}`

- Depth subtyping (along with width on the field's type) lets our example type-check

Stop!

- It is nice and all that our new subtyping rule lets our example type-check

- But it is not worth it if it breaks soundness
  - Also allows programs that can access missing record fields

- Unfortunately, it breaks soundness ☹

Mutation strikes again

If **`ta <: tb`**,
then **`{f1:t1, …, f:ta, …, fn:tn} <:`**
`{f1:t1, …, f:tb, …, fn:tn}`

```
fun setToOrigin (c:{center:{x:real,y:real}, r:real})=
   c.center = {x=0.0, y=0.0}

val sphere:{center:{x:real,y:real,z:real}, r:real} =
  {center={x=3.0, y=4.0, z=0.0}, r=1.0}

val _ = setToOrigin(sphere)
val _ = sphere.center.z (* kaboom! (no z field) *)
```

Moral of the story

- In a language with records/objects with getters and setters, depth subtyping is unsound
  - Subtyping cannot change the type of fields

- If fields are immutable, then depth subtyping is sound!
  - Yet another benefit of outlawing mutation!
  - Choose two of three: setters, depth subtyping, soundness

- Remember: subtyping is not a matter of opinion

Picking on Java (and C#)

Arrays should work just like records in terms of depth subtyping

- But in Java, if **`t1 <: t2`**, then **`t1[] <: t2[]`**
- So this code type-checks, surprisingly

```
class Point { … }
class ColorPoint extends Point { … }
…
void m1(Point[] pt_arr) {
  pt_arr[0] = new Point(3,4);
}
String m2(int x) {
  ColorPoint[] cpt_arr = new ColorPoint[x];
  for(int i=0; i < x; i++)
     cpt_arr[i] = new ColorPoint(0,0,"green");
  m1(cpt_arr); // !
  return cpt_arr[0].color; // !
}
```

Why did they do this?

- More flexible type system allows more programs but prevents fewer errors
  - Seemed especially important before Java/C# had generics
- Good news: despite this "inappropriate" depth subtyping
  - **`e.color`** will never fail due to there being no **`color`** field
  - Array *reads* **`e1[e2]`** always return a (subtype of) **`t`** if **`e1`** is a **`t[]`**
- Bad news: to get the good news
  - **`e1[e2]=e3`** can fail even if **`e1`** has type **`t[]`** and **`e3`** has type **`t`**
  - Array *stores* check the *run-time class* of **`e1`**'s elements and do not allow storing a supertype
  - No type-system help to avoid such bugs / performance cost

So what happens

```
void m1(Point[] pt_arr) {
  pt_arr[0] = new Point(3,4); // can throw
}
String m2(int x) {
  ColorPoint[] cpt_arr = new ColorPoint[x];
  …
  m1(cpt_arr); // "inappropriate" depth subtyping
  ColorPoint c = cpt_arr[0]; // fine, cpt_arr
    // will always hold (subtypes of) ColorPoints
  return c.color; // fine, a ColorPoint has a color
}
```

- Causes code in **`m1`** to throw an **`ArrayStoreException`**
  – Even though logical error is in **`m2`**
  – At least run-time checks occur only on array stores, not on field accesses like **`c.color`**

null

- Array stores probably the most *surprising* choice for flexibility over static checking
- But **`null`** is the most *common* one in practice
  - **`null`** is not an object; it has *no* fields or methods
  - But Java and C# let it have *any* object type (backwards, huh?!)
  - So, in fact, we do *not* have the static guarantee that evaluating **`e`** in **`e.f`** or **`e.m(…)`** produces an object that has an **`f`** or **`m`**
  - The "or **`null`**" caveat leads to run-time checks and errors, as you have surely noticed
- Sometimes **`null`** is convenient (like ML's option types)
  - But also having "cannot be **`null`**" types would be nice

Now functions

- Already know a caller can use subtyping for arguments passed
  - Or on the result

- More interesting: When is one function type a subtype of another?

  - Important for higher-order functions: If a function expects an argument of type **`t1 -> t2`**, can you pass a **`t3 -> t4`** instead?

  - Coming next: Important for understanding methods
    - (An object type is a lot like a record type where "method positions" are immutable and have function types)

Example

```
fun distMoved (f : {x:real,y:real}->{x:real,y:real},
               p : {x:real,y:real}) =
   let val p2 : {x:real,y:real} = f p
       val dx : real = p2.x - p.x
       val dy : real = p2.y - p.y
   in Math.sqrt(dx*dx + dy*dy) end

fun flip p = {x = ~p.x, y=~p.y}
val d = distMoved(flip, {x=3.0, y=4.0})
```

No subtyping here yet:

- **`flip`** has exactly the type **`distMoved`** expects for **`f`**
- Can pass **`distMoved`** a record with extra fields for **`p`**, but that's old news

Return-type subtyping

```
fun distMoved (f : {x:real,y:real}->{x:real,y:real},
               p : {x:real,y:real}) =
   let val p2 : {x:real,y:real} = f p
       val dx : real = p2.x - p.x
       val dy : real = p2.y - p.y
   in Math.sqrt(dx*dx + dy*dy) end

fun flipGreen p = {x = ~p.x, y=~p.y, color="green"}
val d = distMoved(flipGreen, {x=3.0, y=4.0})
```

- Return type of **`flipGreen`** is **`{x:real,y:real,color:string}`**, but **`distMoved`** expects a return type of **`{x:real,y:real}`**
- Nothing goes wrong: If **`ta <: tb`**, then **`t -> ta <: t -> tb`**
  - A function can return "*more* than it needs to"
  - Jargon: "Return types are *covariant*"

This is wrong

```
fun distMoved (f : {x:real,y:real}->{x:real,y:real},
               p : {x:real,y:real}) =
   let val p2 : {x:real,y:real} = f p
       val dx : real = p2.x - p.x
       val dy : real = p2.y - p.y
   in Math.sqrt(dx*dx + dy*dy) end

fun flipIfGreen p = if p.color = "green" (*kaboom!*)
                    then {x = ~p.x, y=~p.y}
                    else {x = p.x, y=p.y}
val d = distMoved(flipIfGreen, {x=3.0, y=4.0})
```

- Argument type of **`flipIfGreen`** is **`{x:real,y:real,color:string}`**, but it is called with a **`{x:real,y:real}`**
- Unsound! **`ta <: tb`** does **NOT** allow **`ta -> t <: tb -> t`**

The other way works!

```
fun distMoved (f : {x:real,y:real}->{x:real,y:real},
               p : {x:real,y:real}) =
   let val p2 : {x:real,y:real} = f p
       val dx : real = p2.x - p.x
       val dy : real = p2.y - p.y
   in Math.sqrt(dx*dx + dy*dy) end

fun flipX_Y0 p = {x = ~p.x, y=0.0}
val d = distMoved(flipX_Y0, {x=3.0, y=4.0})
```

- Argument type of **`flipX_Y0`** is **`{x:real}`**, but it is called with a **`{x:real,y:real}`**, which is fine
- If **`tb <: ta`**, then **`ta -> t <: tb -> t`**
  - A function can assume "*less* than it needs to" about arguments
  - Jargon: "Argument types are *contravariant*"

Can do both

```
fun distMoved (f : {x:real,y:real}->{x:real,y:real},
               p : {x:real,y:real}) =
   let val p2 : {x:real,y:real} = f p
       val dx : real = p2.x - p.x
       val dy : real = p2.y - p.y
   in Math.sqrt(dx*dx + dy*dy) end

fun flipXMakeGreen p = {x = ~p.x, y=0.0, color="green"}
val d = distMoved(flipXMakeGreen, {x=3.0, y=4.0})
```

- **`flipXMakeGreen`** has type

  `{x:real} -> {x:real,y:real,color:string}`

- Fine to pass a function of such a type as function of type

  `{x:real,y:real} -> {x:real,y:real}`

- If **`t3 <: t1`** and **`t2 <: t4`**, then **`t1 -> t2 <: t3 -> t4`**

Conclusion

- If `t3 <: t1` and `t2 <: t4`, then `t1 -> t2 <: t3 -> t4`
  - Function subtyping contravariant in argument(s) and covariant in results
- Also essential for understanding subtyping and methods in OOP
- Most unintuitive concept in the course
  - Smart people often forget and convince themselves covariant arguments are okay
  - These people are always mistaken
  - At times, you or your boss or your friend may do this
  - Remember: A guy with a PhD in PL ***jumped up and down*** insisting that function/method subtyping is always contravariant in its argument -- covariant is unsound