



PAUL G. ALLEN SCHOOL  
OF COMPUTER SCIENCE & ENGINEERING

# CSE341: Programming Languages

## Lecture 11 *Type Inference*

Brett Wortzman

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# *Type-checking*

- (Static) **type-checking** can reject a program before it runs to prevent the possibility of some errors
  - A feature of **statically typed languages**
- **Dynamically typed languages** do little (none?) such checking
  - So might try to treat a number as a function at run-time
- Will study relative advantages after some Racket
  - Racket, Ruby (and Python, Javascript, ...) dynamically typed
- ML (and Java, C#, Scala, C, C++) is statically typed
  - Every binding has one type, determined “at compile-time”

# *Implicitly typed*

- ML is statically typed
- ML is **implicitly typed**: rarely need to write down types

```
fun f x = (* infer val f : int -> int *)
  if x > 3
  then 42
  else x * 2

fun g x = (* report type error *)
  if x > 3
  then true
  else x * 2
```

- Statically typed: Much more like Java than Javascript!

# *Type inference*

- **Type inference** problem: Give every binding/expression a type such that type-checking succeeds
  - Fail if and only if no solution exists
- In principle, could be a pass before the type-checker
  - But often implemented together
- Type inference can be easy, difficult, or *impossible*
  - Easy: Accept all programs
  - Easy: Reject all programs
  - Subtle, elegant, and *not magic*: ML

# Overview

- Will describe ML type inference via several examples
  - General algorithm is a slightly more advanced topic
  - Supporting nested functions also a bit more advanced
- Enough to help you “do type inference in your head”
  - And appreciate it is not magic

# Key steps

- Determine types of bindings in order
  - (Except for mutual recursion)
  - So you cannot use later bindings: will not type-check
- For each **val** or **fun** binding:
  - Analyze definition for all necessary facts (constraints)
  - Example: If see  $x > 0$ , then  $x$  must have type **int**
  - Type error if no way for all facts to hold (over-constrained)
- Afterward, use type variables (e.g., ' **a** ) for any unconstrained types
  - Example: An unused argument can have any type
- (Finally, enforce the *value restriction*, discussed later)

## *Very simple example*

After this example, will go much more step-by-step

- Like the automated algorithm does

```
val x = 42 (* val x : int *)

fun f (y, z, w) =
  if y (* y must be bool *)
  then z + x (* z must be int *)
  else 0 (* both branches have same type *)
(* f must return an int
   f must take a bool * int * ANYTHING
   so val f : bool * int * 'a -> int
   *)
```

# *Relation to Polymorphism*

- Central feature of ML type inference: it can infer types with type variables
  - Great for code reuse and understanding functions
- But remember there are two orthogonal concepts
  - Languages can have type inference without type variables
  - Languages can have type variables without type inference



# Key Idea

- Collect all the facts needed for type-checking
- These facts constrain the type of the function
- See code and/or reading notes for:
  - Two examples without type variables
  - And one example that does not type-check
  - Then examples for polymorphic functions
    - Nothing changes, just under-constrained: some types can “be anything” but may still need to be the same as other types

Material after here is optional,  
but is an important part of the full story

## *Two more topics*

- ML type-inference story so far is too lenient
  - Value restriction limits where polymorphic types can occur
  - See why and then what
- ML is in a “sweet spot”
  - Type inference more difficult without polymorphism
  - Type inference more difficult with subtyping

Important to “finish the story” but these topics are:

- A bit more advanced
- A bit less elegant
- Will not be on the exam

# The Problem

As presented so far, the ML type system is *unsound!*

- Allows putting a value of type `t1` (e.g., `int`) where we expect a value of type `t2` (e.g., `string`)

A combination of polymorphism and mutation is to blame:

```
val r = ref NONE (* val r : 'a option ref *)
val _ = r := SOME "hi"
val i = 1 + valOf (!r)
```

- Assignment type-checks because (infix) `:=` has type `'a ref * 'a -> unit`, so instantiate with `string`
- Dereference type-checks because `!` has type `'a ref -> 'a`, so instantiate with `int`

## What to do

To restore soundness, need a stricter type system that rejects at least one of these three lines

```
val r = ref NONE (* val r : 'a option ref *)  
val _ = r := SOME "hi"  
val i = 1 + valOf (!r)
```

- And cannot make special rules for reference types because type-checker cannot know the definition of all type synonyms
  - Due to module system

```
type 'a foo = 'a ref  
val f = ref (* val f : 'a -> 'a foo *)  
val r = f NONE
```

## *The fix*

```
val r = ref NONE (* val r : ?.X1 option ref *)  
val _ = r := SOME "hi"  
val i = 1 + valOf (!r)
```

- Value restriction: a variable-binding can have a polymorphic type only if the expression is a variable or value
  - Function calls like `ref NONE` are neither
- Else get a warning and unconstrained types are filled in with dummy types (basically unusable)
- Not obvious this suffices to make type system sound, but it does

## *The downside*

As we saw previously, the value restriction can cause problems when it is unnecessary because we are not using mutation

```
val pairWithOne = List.map (fn x => (x,1))  
(* does not get type 'a list -> ('a*int) list *)
```

The type-checker does not know `List.map` is not making a mutable reference

Saw workarounds in previous segment on partial application

- Common one: wrap in a function binding

```
fun pairWithOne xs = List.map (fn x => (x,1)) xs  
(* 'a list -> ('a*int) list *)
```

## *A local optimum*

- Despite the value restriction, ML type inference is elegant and fairly easy to understand
- More difficult *without* polymorphism
  - What type should length-of-list have?
- More difficult *with* subtyping
  - Suppose pairs are supertypes of wider tuples
  - Then `val (y, z) = x` constrains `x` to have at least two fields, not exactly two fields
  - Depending on details, languages can support this, but types often more difficult to infer and understand
  - Will study subtyping later, but not with type inference