What is functional programming?

“Functional programming” can mean a few different things:

1. Avoiding mutation in most/all cases (done and ongoing)
2. Using functions as values (this unit)

... 

• Style encouraging recursion and recursive data structures
• Style closer to mathematical definitions
• Programming idioms using laziness (later topic, briefly)
• Anything not OOP or C? (not a good definition)

Not sure a definition of “functional language” exists beyond “makes functional programming easy / the default / required”

– No clear yes/no for a particular language
First-class functions

- First-class functions: Can use them wherever we use values
  - Functions are values too
  - Arguments, results, parts of tuples, bound to variables, carried by datatype constructors or exceptions, ...

```plaintext
fun double x = 2*x
fun incr x = x+1
val a_tuple = (double, incr, double(incr 7))
```

- Most common use is as an argument / result of another function
  - Other function is called a higher-order function
  - Powerful way to factor out common functionality
Function Closures

• Function closure: Functions can use bindings from outside the function definition (in scope where function is defined)
  – Makes first-class functions much more powerful
  – Will get to this feature in a bit, after simpler examples

• Distinction between terms first-class functions and function closures is not universally understood
  – Important conceptual distinction even if terms get muddled
Onward

The next week:

- How to use first-class functions and closures
- The precise semantics
- Multiple powerful idioms
Functions as arguments

• We can pass one function as an argument to another function
  – Not a new feature, just never thought to do it before

```plaintext
fun f (g, ...) = ... g (...) ...  
fun h1 ... = ...  
fun h2 ... = ...  
... f(h1, ...) ... f(h2, ...) ...
```

• Elegant strategy for factoring out common code
  – Replace $N$ similar functions with calls to 1 function where you pass in $N$ different (short) functions as arguments

[See the code file for this lecture]
Example

Can reuse \texttt{n\_times} rather than defining many similar functions

– Computes $f(f(\ldots f(x)))$ where number of calls is $n$

\begin{verbatim}
fun n_times (f,n,x) = 
  if n=0
  then x
  else f (n_times(f,n-1,x))

fun double x = x + x
fun increment x = x + 1
val x1 = n_times(double,4,7)
val x2 = n_times(increment,4,7)
val x3 = n_times(tl,2,[4,8,12,16])

fun double_n_times (n,x) = n_times(double,n,x)
fun nth_tail (n,x) = n_times(tl,n,x)
\end{verbatim}
Map

fun map (f, xs) =
  case xs of
  [] => []
  | x::xs' => (f x)::(map(f,xs'))

val map : ('a -> 'b) * 'a list -> 'b list

Map is, without doubt, in the “higher-order function hall-of-fame”

- The name is standard (for any data structure)
- You use it all the time once you know it: saves a little space, but more importantly, communicates what you are doing
- Similar predefined function: List.map
  - But it uses currying (coming soon)
Filter

fun filter (f, xs) = 
  case xs of 
    [] => [] 
    | x :: xs' => if f x 
      then x :: (filter (f, xs')) 
      else filter (f, xs')

val filter : ('a -> bool) * 'a list -> 'a list

Filter is also in the hall-of-fame
  - So use it whenever your computation is a filter
  - Similar predefined function: List.filter
    • But it uses currying (coming soon)
Relation to types

• Higher-order functions are often so “generic” and “reusable” that they have polymorphic types, i.e., types with type variables

• But there are higher-order functions that are not polymorphic

• And there are non-higher-order (first-order) functions that are polymorphic

• Always a good idea to understand the type of a function, especially a higher-order function
Types for example

```fun n_times (f,n,x) =
    if n=0
    then x
    else f (n_times(f,n-1,x))
```

- `val n_times : ('a -> 'a) * int * 'a -> 'a`
  - Simpler but less useful: `(int -> int) * int * int -> int`

- Two of our examples instantiated 'a with `int`
- One of our examples instantiated 'a with `int list`
- This polymorphism makes `n_times` more useful

- Type is inferred based on how arguments are used (later lecture)
  - Describes which types must be exactly something (e.g., `int`) and which can be anything but the same (e.g., 'a)
Polymorphism and higher-order functions

• Many higher-order functions are polymorphic because they are so reusable that some types, “can be anything”

• But some polymorphic functions are not higher-order
  – Example: `len : 'a list -> int`

• And some higher-order functions are not polymorphic
  – Example: `times_until_0 : (int->int)*int->int`

```ocaml
fun times_until_zero (f,x) = 
  if x=0 then 0 else 1 + times_until_zero(f, f x)
```

Note: Would be better with tail-recursion
Toward anonymous functions

• Definitions unnecessarily at top-level are still poor style:

```plaintext
fun trip x = 3*x
fun triple_n_times (f,x) = n_times(trip,n,x)
```

• So this is better (but not the best):

```plaintext
fun triple_n_times (f,x) =
  let fun trip y = 3*y
  in
    n_times(trip,n,x)
  end
end
```

• And this is even smaller scope
  – It makes sense but looks weird (poor style; see next slide)

```plaintext
fun triple_n_times (f,x) =
  n_times(let fun trip y = 3*y in trip end, n, x)
```
Anonymous functions

• This does not work: A function binding is not an expression

```plaintext
fun triple_n_times (f, x) = 
n_times((fun trip y = 3*y), n, x)
```

• This is the best way we were building up to: an expression form for anonymous functions

```plaintext
fun triple_n_times (f, x) = 
n_times((fn y => 3*y), n, x)
```

  – Like all expression forms, can appear anywhere
  – Syntax:
    • `fn` not `fun`
    • `=>` not `=
    • no function name, just an argument pattern
Using anonymous functions

- Most common use: Argument to a higher-order function
  - Don’t need a name just to pass a function

- But: Cannot use an anonymous function for a recursive function
  - Because there is no name for making recursive calls
  - If not for recursion, `fun` bindings would be syntactic sugar for `val` bindings and anonymous functions

```plaintext
fun triple x = 3*x
val triple = fn y => 3*y
```
A style point

Compare:

```
if x then true else false
```

With:

```
(fn x => f x)
```

So don’t do this:

```
n_times((fn y => tl y),3,xs)
```

When you can do this:

```
n_times(tl,3,xs)
```
Generalizing

Our examples of first-class functions so far have all:
  – Taken one function as an argument to another function
  – Processed a number or a list

But first-class functions are useful anywhere for any kind of data
  – Can pass several functions as arguments
  – Can put functions in data structures (tuples, lists, etc.)
  – Can return functions as results
  – Can write higher-order functions that traverse your own data structures

Useful whenever you want to abstract over “what to compute with”
  – No new language features
Returning functions

• Remember: Functions are first-class values
  – For example, can return them from functions

• Silly example:

```
fun double_or_triple f =
  if f 7
  then fn x => 2*x
  else fn x => 3*x
```

Has type \((\text{int} \rightarrow \text{bool}) \rightarrow (\text{int} \rightarrow \text{int})\)

But the REPL prints \((\text{int} \rightarrow \text{bool}) \rightarrow \text{int} \rightarrow \text{int}\)
because it never prints unnecessary parentheses and
\( \text{t1} \rightarrow \text{t2} \rightarrow \text{t3} \rightarrow \text{t4} \) means \(\text{t1} \rightarrow (\text{t2} \rightarrow (\text{t3} \rightarrow \text{t4}))\)
Other data structures

• Higher-order functions are not just for numbers and lists

• They work great for common recursive traversals over your own data structures (datatype bindings) too

• Example of a higher-order predicate:
  – Are all constants in an arithmetic expression even numbers?
  – Use a more general function of type
    \[(\text{int} \to \text{bool}) \ast \text{exp} \to \text{bool}\]
  – And call it with \((\text{fn } x \Rightarrow x \mod 2 = 0)\)