



PAUL G. ALLEN SCHOOL
OF COMPUTER SCIENCE & ENGINEERING

CSE341: Programming Languages

Lecture 6

Tail Recursion

Exceptions

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Recursion

Should now be comfortable with recursion:

- No harder than using a loop (whatever that is 😊)
- Often much easier than a loop
 - When processing a tree (e.g., evaluate an arithmetic expression)
 - Examples like appending lists
 - Avoids mutation even for local variables
- Now:
 - How to reason about *efficiency* of recursion
 - The importance of *tail recursion*
 - Using an *accumulator* to achieve tail recursion
 - [No new language features here]

Call-stacks

While a program runs, there is a *call stack* of function calls that have started but not yet returned

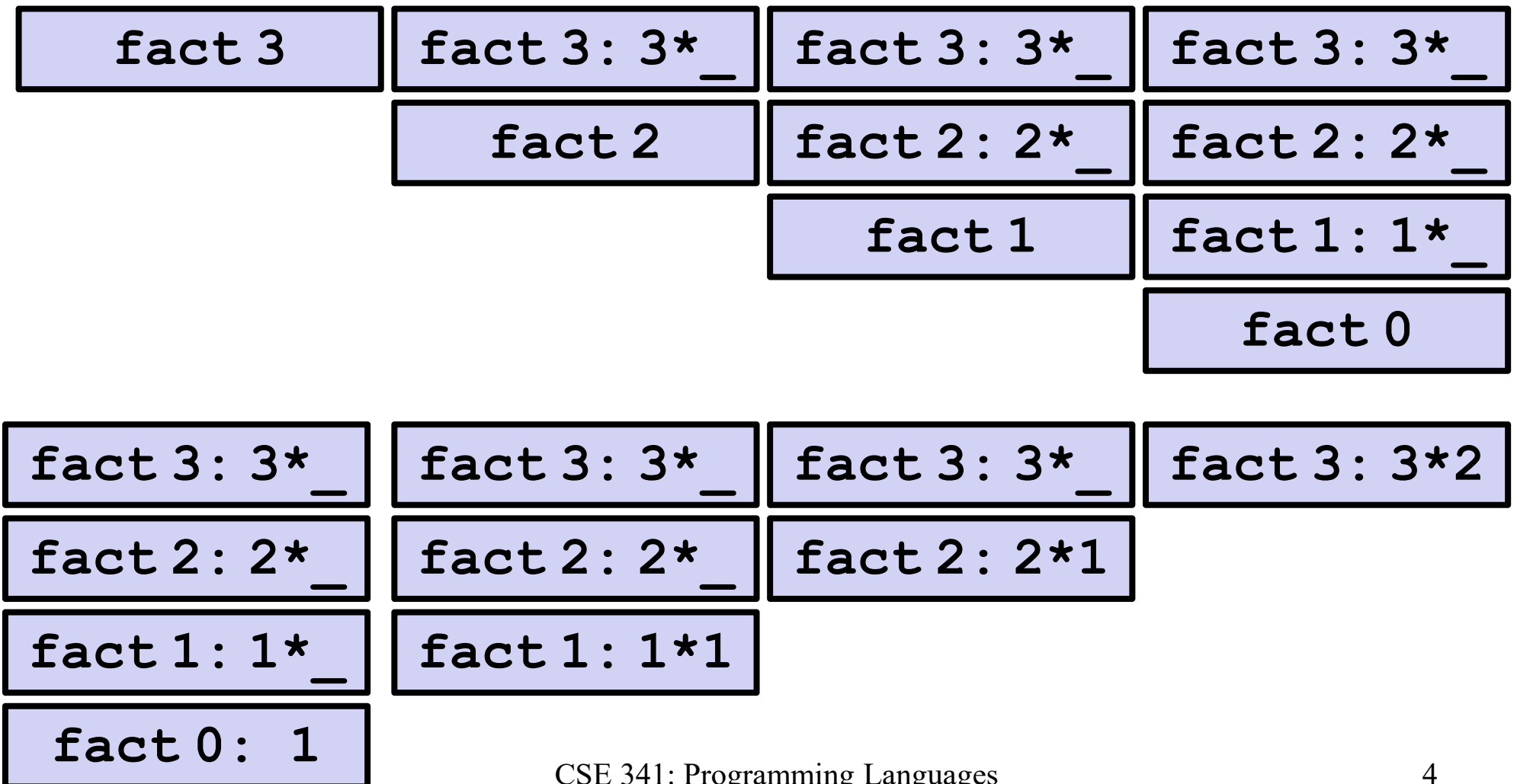
- Calling a function f pushes an instance of f on the stack
- When a call to f finishes, it is popped from the stack

These stack-frames store information like the value of local variables and “what is left to do” in the function

Due to recursion, multiple stack-frames may be calls to the same function

Example

```
fun fact n = if n=0 then 1 else n*fact(n-1)
val x = fact 3
```



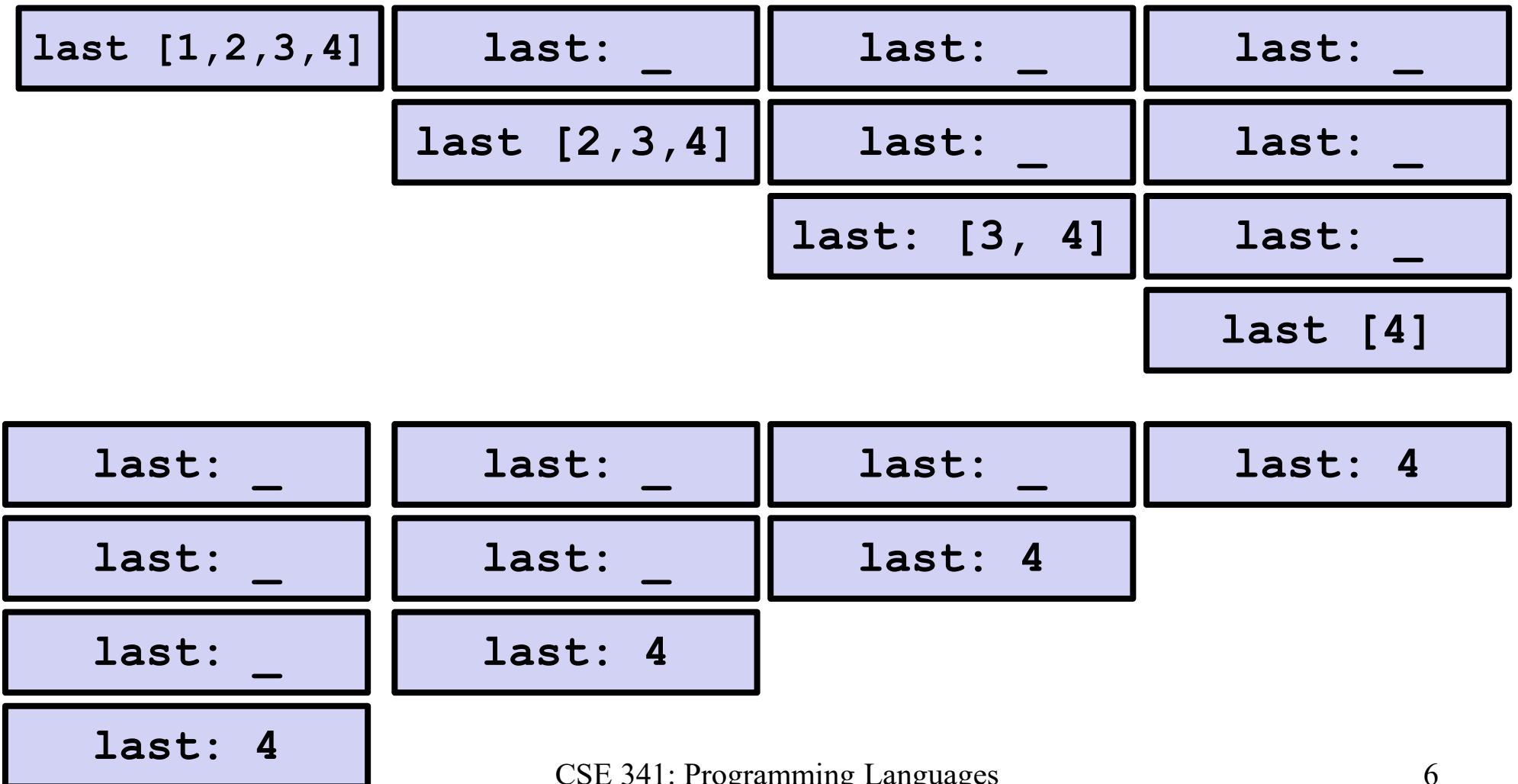
Another Example

```
fun last lst =  
  case lst of  
    [x] => x  
  | head::tail => last tail  
  
val x = last [1,2,3,4]
```

Still recursive, but the result of recursive call *is* the result for the caller (no remaining work)

Example

```
fun last lst = ... (* see .sml *)  
val x = last [1,2,3,4]
```



An optimization

It is unnecessary to keep around a stack-frame just so it can get a callee's result and return it without any further evaluation

ML recognizes these *tail calls* in the compiler and treats them differently:

- Pop the caller *before* the call, allowing callee to *reuse* the same stack space
- (Along with other optimizations,) as efficient as a loop

Reasonable to assume all functional-language implementations do tail-call optimization

What really happens

```
fun last lst =  
  case lst of  
    [x] => x  
  | head::tail => last tail  
  
val x = last [1,2,3,4]
```

last [1,2,3,4]

last [2,3,4]

last [3,4]

last [4]

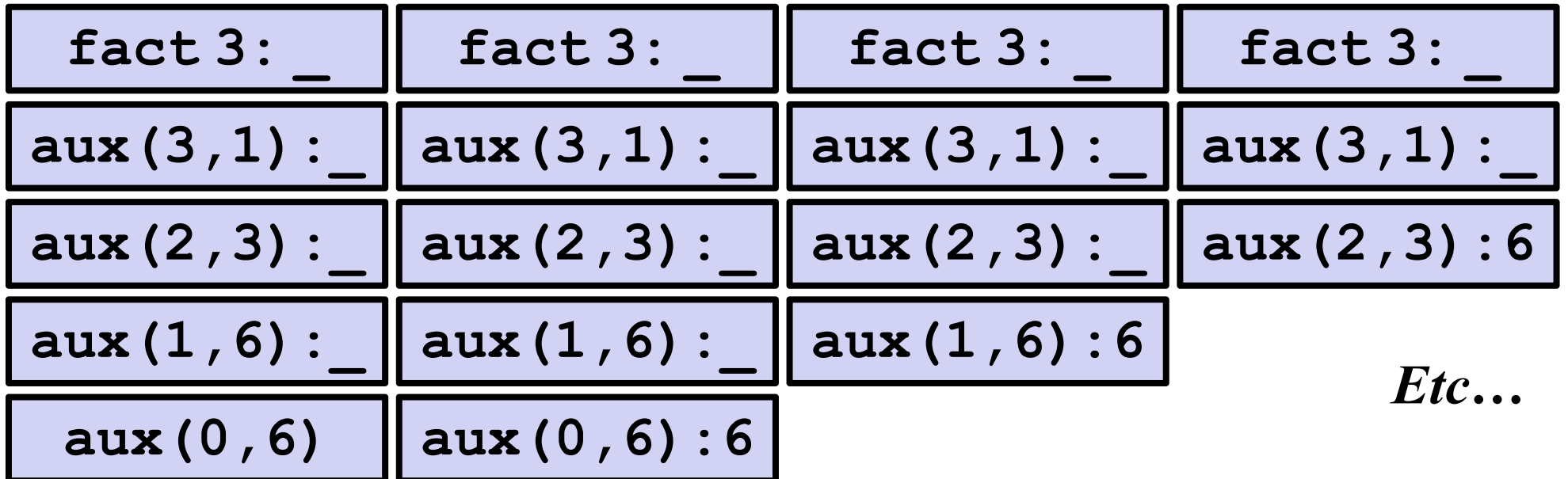
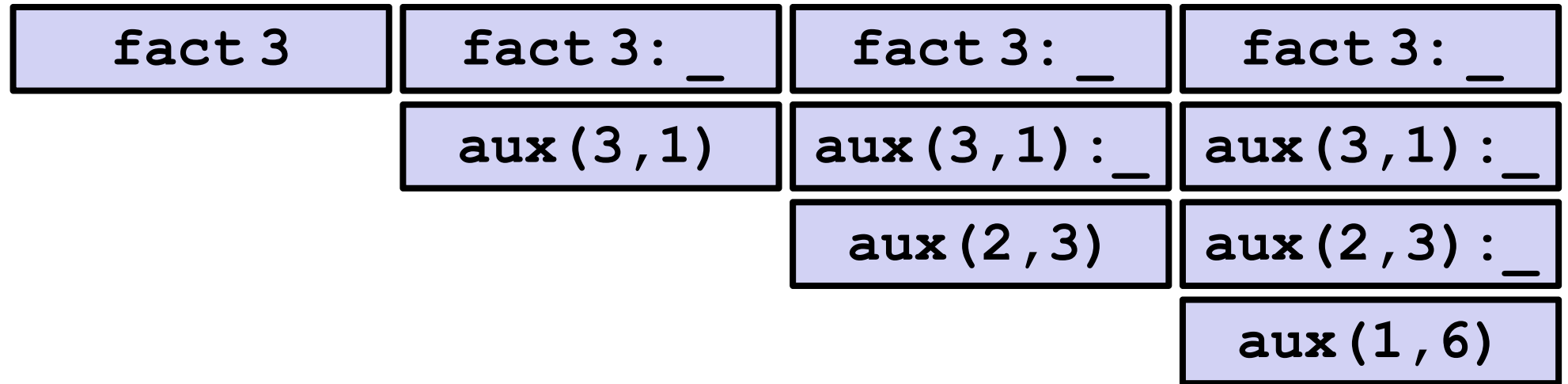
last: 4

Factorial Revised

```
fun fact n =  
  let fun aux(n, acc) =  
        if n=0  
        then acc  
        else aux(n-1, acc*n)  
      in  
        aux(n, 1)  
      end  
val x = fact 3
```

Still recursive, more complicated, but the result of recursive calls *is* the result for the caller (no remaining multiplication)

The call-stacks



Etc...

What really happens

```
fun fact n =  
  let fun aux(n, acc) =  
        if n=0  
        then acc  
        else aux(n-1, acc*n)  
    in  
      aux(n, 1)  
    end  
val x = fact 3
```

fact 3

aux(3, 1)

aux(2, 3)

aux(1, 6)

aux(0, 6)

Moral of tail recursion

- Where reasonably elegant, feasible, and important, rewriting functions to be *tail-recursive* can be much more efficient
 - Tail-recursive: recursive calls are tail-calls
- There is a *methodology* that can often guide this transformation:
 - Create a helper function that takes an *accumulator*
 - Old base case becomes initial accumulator
 - New base case becomes final accumulator

Methodology already seen

```
fun fact n =  
  let fun aux(n, acc) =  
        if n=0  
        then acc  
        else aux(n-1, acc*n)  
  in  
    aux(n, 1)  
  end  
val x = fact 3
```

fact 3

aux(3, 1)

aux(2, 3)

aux(1, 6)

aux(0, 6)

Another example

```
fun sum xs =  
  case xs of  
    [] => 0  
  | x::xs' => x + sum xs'
```

```
fun sum xs =  
  let fun aux(xs, acc) =  
        case xs of  
          [] => acc  
        | x::xs' => aux(xs', x+acc)  
      in  
        aux(xs, 0)  
      end
```

And another

```
fun rev xs =  
  case xs of  
    [] => []  
  | x::xs' => (rev xs') @ [x]
```

```
fun rev xs =  
  let fun aux(xs, acc) =  
        case xs of  
          [] => acc  
        | x::xs' => aux(xs', x::acc)  
      in  
        aux(xs, [])  
      end
```

Actually much better

```
fun rev xs =  
  case xs of  
    [] => []  
  | x::xs' => (rev xs') @ [x]
```

- For **fact** and **sum**, tail-recursion is faster but both ways linear time
- Non-tail recursive **rev** is quadratic because each recursive call uses append, which must traverse the first list
 - And $1+2+\dots+(\text{length}-1)$ is almost $\text{length}*\text{length}/2$
 - Moral: beware list-append, especially within outer recursion
- Cons constant-time (and fast), so accumulator version much better

Always tail-recursive?

There are certainly cases where recursive functions cannot be evaluated in a constant amount of space

Most obvious examples are functions that process trees

In these cases, the natural recursive approach is the way to go

- You could get one recursive call to be a tail call, but rarely worth the complication

Also beware the wrath of premature optimization

- Favor clear, concise code
- But do use less space if inputs may be large

What is a tail-call?

The “nothing left for caller to do” intuition usually suffices

- If the result of $f\ x$ is the “immediate result” for the enclosing function body, then $f\ x$ is a tail call

But we can define “tail position” recursively

- Then a “tail call” is a function call in “tail position”

...

Precise definition

A tail call is a function call in *tail position*

- If an expression is not in tail position, then no subexpressions are
- In **fun f p = e**, the body **e** is in tail position
- If **if e1 then e2 else e3** is in tail position, then **e2** and **e3** are in tail position (but **e1** is not). (Similar for case-expressions)
- If **let b1 ... bn in e end** is in tail position, then **e** is in tail position (but no binding expressions are)
- Function-call *arguments* **e1 e2** are not in tail position
- ...

Exceptions

An exception binding introduces a new kind of exception

```
exception MyUndesirableCondition  
exception MyOtherException of int * int
```

The `raise` primitive raises (a.k.a. throws) an exception

```
raise MyUndesirableException  
raise (MyOtherException (7,9))
```

A handle expression can handle (a.k.a. catch) an exception

- If doesn't match, exception continues to propagate

```
e1 handle MyUndesirableException => e2  
e1 handle MyOtherException(x,y) => e2
```

Actually...

Exceptions are a lot like datatype constructors...

- Declaring an exception adds a constructor for type **exn**
- Can pass values of **exn** anywhere (e.g., function arguments)
 - Not too common to do this but can be useful
- **handle** can have multiple branches with patterns for type **exn**