W PAUL G. ALLEN SCHOOL
OF COMPUTER SCIENCE & ENGINEERING

CSE341: Programming Languages

Lecture 6
Tail Recursion
Exceptions

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Recursion

Should now be comfortable with recursion:

- No harder than using a loop (whatever that is ☺)
- Often much easier than a loop
  - When processing a tree (e.g., evaluate an arithmetic expression)
  - Examples like appending lists
  - Avoids mutation even for local variables
- Now:
  - How to reason about *efficiency* of recursion
  - The importance of *tail recursion*
  - Using an *accumulator* to achieve tail recursion
  - [No new language features here]

Call-stacks

While a program runs, there is a *call stack* of function calls that have started but not yet returned

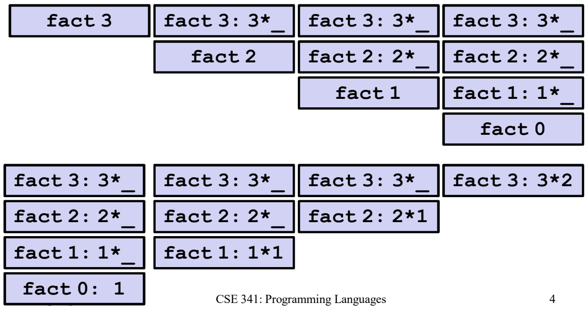
- Calling a function `f` pushes an instance of `f` on the stack
- When a call to `f` finishes, it is popped from the stack

These stack-frames store information like the value of local variables and "what is left to do" in the function

Due to recursion, multiple stack-frames may be calls to the same function

Example

```
fun fact n = if n=0 then 1 else n*fact(n-1)
val x = fact 3
```



fact 3

fact 3: 3*_
fact 2

fact 3: 3*_
fact 2: 2*_
fact 1

fact 3: 3*_
fact 2: 2*_
fact 1: 1*_
fact 0

fact 3: 3*_
fact 2: 2*_
fact 1: 1*_
fact 0: 1

fact 3: 3*_
fact 2: 2*_
fact 1: 1*1

fact 3: 3*_
fact 2: 2*1

fact 3: 3*2

Another Example

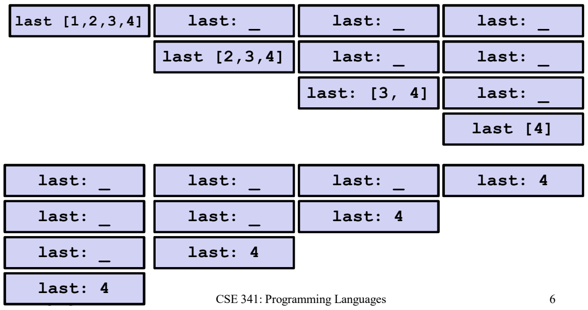
```
fun last lst =
   case lst of
       [x] => x
     | head::tail => last tail

val x = last [1,2,3,4]
```

Still recursive, but the result of recursive call *is* the result for the caller (no remaining work)

Example

```
fun last lst = ... (* see .sml *)
val x = last [1,2,3,4]
```



last [1,2,3,4]

last: _
last [2,3,4]

last: _
last: _
last: [3, 4]

last: _
last: _
last: _
last [4]

last: _
last: _
last: _
last: 4

last: _
last: _
last: 4

last: _
last: 4

last: 4

An optimization

It is unnecessary to keep around a stack-frame just so it can get a callee's result and return it without any further evaluation

ML recognizes these *tail calls* in the compiler and treats them differently:

- Pop the caller *before* the call, allowing callee to *reuse* the same stack space
- (Along with other optimizations,) as efficient as a loop

Reasonable to assume all functional-language implementations do tail-call optimization

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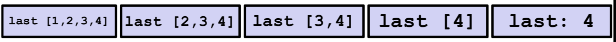
What really happens

```
fun last lst =
   case lst of
        [x] => x
      | head::tail => last tail

val x = last [1,2,3,4]
```


last [1,2,3,4]
last [2,3,4]
last [3,4]
last [4]
last: 4

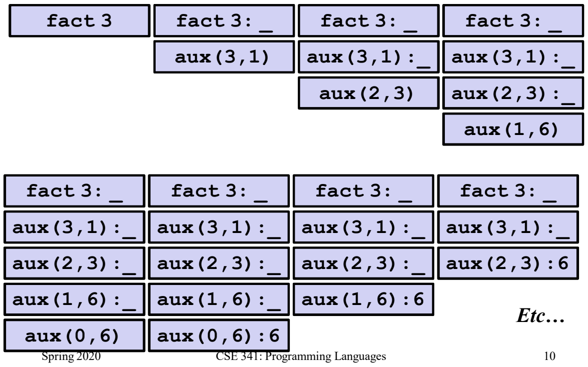
Factorial Revised

```
fun fact n =
    let fun aux(n,acc) =
            if n=0
            then acc
            else aux(n-1,acc*n)
    in
        aux(n,1)
    end

val x = fact 3
```

Still recursive, more complicated, but the result of recursive calls *is* the result for the caller (no remaining multiplication)

The call-stacks


fact 3

fact 3: _
aux(3,1)

fact 3: _
aux(3,1):_
aux(2,3)

fact 3: _
aux(3,1):_
aux(2,3):_
aux(1,6)

fact 3: _
aux(3,1):_
aux(2,3):_
aux(1,6):_
aux(0,6)

fact 3: _
aux(3,1):_
aux(2,3):_
aux(1,6):_
aux(0,6):6

fact 3: _
aux(3,1):_
aux(2,3):_
aux(1,6):6

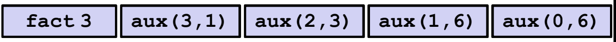
fact 3: _
aux(3,1):_
aux(2,3):6

Etc…

What really happens

```
fun fact n =
    let fun aux(n,acc) =
            if n=0
            then acc
            else aux(n-1,acc*n)
    in
        aux(n,1)
    end

val x = fact 3
```


fact 3
aux(3,1)
aux(2,3)
aux(1,6)
aux(0,6)

Moral of tail recursion

- Where reasonably elegant, feasible, and important, rewriting functions to be *tail-recursive* can be much more efficient
  - Tail-recursive: recursive calls are tail-calls
- There is a methodology that can often guide this transformation:
  - Create a helper function that takes an *accumulator*
  - Old base case becomes initial accumulator
  - New base case becomes final accumulator

Methodology already seen

```
fun fact n =
    let fun aux(n,acc) =
            if n=0
            then acc
            else aux(n-1,acc*n)
    in
        aux(n,1)
    end

val x = fact 3
```

`fact 3` | `aux(3,1)` | `aux(2,3)` | `aux(1,6)` | `aux(0,6)`

Another example

```
fun sum xs =
   case xs of
       [] => 0
     | x::xs’ => x + sum xs’
```

```
fun sum xs =
    let fun aux(xs,acc) =
            case xs of
               [] => acc
             | x::xs’ => aux(xs’,x+acc)
    in
        aux(xs,0)
    end
```

And another

```
fun rev xs =
   case xs of
      [] => []
    | x::xs’ => (rev xs’) @ [x]
```

```
fun rev xs =
    let fun aux(xs,acc) =
           case xs of
              [] => acc
            | x::xs’ => aux(xs’,x::acc)
    in
       aux(xs,[])
    end
```

Actually much better

```
fun rev xs =
   case xs of
      [] => []
    | x::xs’ => (rev xs’) @ [x]
```

- For `fact` and `sum`, tail-recursion is faster but both ways linear time
- Non-tail recursive `rev` is quadratic because each recursive call uses append, which must traverse the first list
  - And 1+2+…+(length-1) is almost length*length/2
  - Moral: beware list-append, especially within outer recursion
- Cons constant-time (and fast), so accumulator version much better

Always tail-recursive?

There are certainly cases where recursive functions cannot be evaluated in a constant amount of space

Most obvious examples are functions that process trees

In these cases, the natural recursive approach is the way to go
- You could get one recursive call to be a tail call, but rarely worth the complication

Also beware the wrath of premature optimization
- Favor clear, concise code
- But do use less space if inputs may be large

What is a tail-call?

The “nothing left for caller to do” intuition usually suffices
- If the result of `f x` is the “immediate result” for the enclosing function body, then `f x` is a tail call

But we can define “tail position” recursively
- Then a “tail call” is a function call in “tail position”

…

Precise definition

A *tail call* is a function call in *tail position*

- If an expression is not in tail position, then no subexpressions are

- In `fun f p = e`, the body `e` is in tail position
- If `if e1 then e2 else e3` is in tail position, then `e2` and `e3` are in tail position (but `e1` is not). (Similar for case-expressions)
- If `let b1 … bn in e end` is in tail position, then `e` is in tail position (but no binding expressions are)
- Function-call *arguments* `e1 e2` are not in tail position
- …

Exceptions

An exception binding introduces a new kind of exception

```
exception MyUndesirableCondition
exception MyOtherException of int * int
```

The `raise` primitive raises (a.k.a. throws) an exception

```
raise MyUndesirableException
raise (MyOtherException (7,9))
```

A handle expression can handle (a.k.a. catch) an exception
- If doesn't match, exception continues to propagate

```
e1 handle MyUndesirableException => e2
e1 handle MyOtherException(x,y) => e2
```

Actually…

Exceptions are a lot like datatype constructors…

- Declaring an exception adds a constructor for type `exn`

- Can pass values of `exn` anywhere (e.g., function arguments)
  - Not too common to do this but can be useful

- `handle` can have multiple branches with patterns for type `exn`