Nested patterns

- We can nest patterns as deep as we want
  - Just like we can nest expressions as deep as we want
  - Often avoids hard-to-read, wordy nested case expressions
- So the full meaning of pattern-matching is to compare a pattern against a value for the “same shape” and bind variables to the “right parts”
  - More precise recursive definition coming after examples

Useful example: zip/unzip 3 lists

```sml
fun zip3 lists =
  case lists of
    ([],[],[]) => []
  | (hd1::tl1,hd2::tl2,hd3::tl3) =>
    (hd1,hd2,hd3)::zip3(tl1,tl2,tl3)
  | _ => raise ListLengthMismatch

fun unzip3 triples =
  case triples of
    [] => ([],[],[])
  | (a,b,c)::tl =>
    let val
      (l1,l2,l3) = unzip3 tl
    in
      (a::l1,b::l2,c::l3)
    end

More examples in .sml files
```

Style

- Nested patterns can lead to very elegant, concise code
  - Avoid nested case expressions if nested patterns are simpler
  - Example: unzip3 and nondecreasing
  - A common idiom is matching against a tuple of datatypes to compare them
    - Examples: zip3 and multsign
- Wildcards are good style: use them instead of variables when you do not need the data
  - Examples: len and multsign

Examples

- Pattern a::b::c::d matches all lists with >=3 elements
- Pattern a::b::c::[] matches all lists with 3 elements
- Pattern (a,b),(c,d)::e matches all non-empty lists of pairs of pairs
Exceptions

An exception binding introduces a new kind of exception

```
exception MyUndesirableCondition
exception MyOtherException of int * int
```

The `raise` primitive raises (a.k.a. throws) an exception

```
raise MyUndesirableException
raise (MyOtherException (7,9))
```

A handle expression can handle (a.k.a. catch) an exception

- If doesn't match, exception continues to propagate

```
e1 handle MyUndesirableException => e2
e1 handle MyOtherException(x,y) => e2
```

Actually...

Exceptions are a lot like datatype constructors...

- Declaring an exception adds a constructor for type `exn`
- Can pass values of `exn` anywhere (e.g., function arguments)
  - Not too common to do this but can be useful
- `handle` can have multiple branches with patterns for type `exn`

Recursion

Should now be comfortable with recursion:

- No harder than using a loop (whatever that is 😊)
- Often much easier than a loop
  - When processing a tree (e.g., evaluate an arithmetic expression)
  - Examples like appending lists
  - Avoids mutation even for local variables
- Now:
  - How to reason about efficiency of recursion
  - The importance of tail recursion
  - Using an accumulator to achieve tail recursion
  - [No new language features here]

Call-stacks

While a program runs, there is a call stack of function calls that have started but not yet returned

- Calling a function `f` pushes an instance of `f` on the stack
- When a call to `f` finishes, it is popped from the stack

These stack-frames store information like the value of local variables and "what is left to do" in the function

Due to recursion, multiple stack-frames may be calls to the same function

Example

```
fun fact n = if n=0 then 1 else n*fact(n-1)
val x = fact 3
```

```
fact 3
  fact 3:3*  fact 3:3*  fact 3:3*
  fact 2  fact 2:2*  fact 2:2*
  fact 1  fact 1:1*  fact 0
```

Still recursive, more complicated, but the result of recursive calls is the result for the caller (no remaining multiplication)

Example Revised

```
fun fact n = let fun aux(n,acc) =
  if n=0 then acc
  else aux(n-1,acc*n)
in
  aux(n,1)
end
val x = fact 3
```

```
fact 3
  fact 3:3*  fact 3:3*  fact 3:3*
  fact 2  fact 2:2*  fact 2:2*
  fact 1  fact 1:1*  fact 0
```
The call-stacks

```
fun fact n = let fun aux(n,acc) = if n=0 then acc else aux(n-1,acc*n) in aux(n,1) end
val x = fact 3
```

An optimization

It is unnecessary to keep around a stack-frame just so it can get a callee’s result and return it without any further evaluation.

ML recognizes these **tail calls** in the compiler and treats them differently:
- Pop the caller before the call, allowing callee to reuse the same stack space
- (Along with other optimizations,) as efficient as a loop

Reasonable to assume all functional-language implementations do tail-call optimization

What really happens

```
fun fact n = let fun aux(n,acc) = if n=0 then acc else aux(n-1,acc*n) in aux(n,1) end
val x = fact 3
```

Moral of tail recursion

- Where reasonably elegant, feasible, and important, rewriting functions to be tail-recursive can be much more efficient
  - Tail-recursive: recursive calls are tail-calls
- There is a **methodology** that can often guide this transformation:
  - Create a helper function that takes an **accumulator**
  - Old base case becomes initial accumulator
  - New base case becomes final accumulator

Methodology already seen

```
fun fact n = let fun aux(n,acc) = if n=0 then acc else aux(n-1,acc*n) in aux(n,1) end
val x = fact 3
```

Another example

```
fun sum xs = case xs of [] => 0 | x::xs' => x + sum xs'

fun sum xs = let fun aux(xs,acc) = case xs of [] => acc | x::xs' => aux(xs',x+acc) in aux(xs,0) end
```
And another

```haskell
fun rev xs = 
  case xs of 
      [] => [] 
    | x:xs' => (rev xs') @ [x]
```

```haskell
let fun aux(xs,acc) = 
      case xs of 
          [] => acc 
        | x:xs' => aux(xs',x::acc) 
      in 
      aux(xs,[]) 
end
```

Actually much better

```haskell
fun rev xs = 
  case xs of 
      [] => [] 
    | x:xs' => (rev xs') @ [x]
```

- For `fact` and `sum`, tail-recursion is faster but both ways linear time
- Non-tail recursive `rev` is quadratic because each recursive call uses append, which must traverse the first list
  - And 1+2+…+(length-1) is almost length*length/2
- Moral: beware list-append, especially within outer recursion
- Cons constant-time (and fast), so accumulator version much better

Always tail-recursive?

There are certainly cases where recursive functions cannot be evaluated in a constant amount of space

Most obvious examples are functions that process trees

In these cases, the natural recursive approach is the way to go
- You could get one recursive call to be a tail call, but rarely worth the complication

Also beware the wrath of premature optimization
- Favor clear, concise code
- But do use less space if inputs may be large

What is a tail-call?

The “nothing left for caller to do” intuition usually suffices
- If the result of `f x` is the “immediate result” for the enclosing function body, then `f x` is a tail call

But we can define “tail position” recursively
- Then a “tail call” is a function call in “tail position”

Precise definition

A tail call is a function call in tail position

- If an expression is not in tail position, then no subexpressions are
- In `fun f p = e`, the body `e` is in tail position
- If `if e1 then e2 else e3` is in tail position, then `e2` and `e3` are in tail position (but `e1` is not). (Similar for case-expressions)
- If `let b1 ... be in e end` is in tail position, then `e` is in tail position (but no binding expressions are)
- Function-call arguments `e1 e2` are not in tail position