CSE341: Programming Languages

Lecture 12
Equivalence

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Summer 2019

Slides originally created by Dan Grossman
Last Topic of Unit

More careful look at what “two pieces of code are equivalent” means

- Fundamental software-engineering idea

- Made easier with
  - Abstraction (hiding things)
  - Fewer side effects

Not about any “new ways to code something up”
Equivalence

Must reason about “are these equivalent” all the time
  – The more precisely you think about it the better

• Code maintenance: Can I simplify this code?

• Backward compatibility: Can I add new features without changing how any old features work?

• Optimization: Can I make this code faster?

• Abstraction: Can an external client tell I made this change?

To focus discussion: When can we say two functions are equivalent, even without looking at all calls to them?
  – May not know all the calls (e.g., we are editing a library)
A definition

Two functions are equivalent if they have the same “observable behavior” no matter how they are used anywhere in any program.

Given equivalent arguments, they:

- Produce equivalent results
- Have the same (non-)termination behavior
- Mutate (non-local) memory in the same way
- Do the same input/output
- Raise the same exceptions

Notice it is much easier to be equivalent if:

- There are fewer possible arguments, e.g., with a type system and abstraction
- We avoid side-effects: mutation, input/output, and exceptions
**Example**

Since looking up variables in ML has no side effects, these two functions are equivalent:

fun f x = x + x  
≡

val y = 2
fun f x = y * x

But these next two are not equivalent in general: it depends on what is passed for `f`

– Are equivalent *if* argument for `f` has no side-effects

fun g (f,x) = (f x) + (f x)  
≡

val y = 2
fun g (f,x) = y * (f x)

– Example: `g ((fn i => print "hi" ; i), 7)`
– Great reason for “pure” functional programming
Another example

These are equivalent only if functions bound to \( g \) and \( h \) do not raise exceptions or have side effects (printing, updating state, etc.)

- Again: pure functions make more things equivalent

\[
\begin{align*}
\text{fun } f \ x & = \\
& \quad \text{let} \\
& \quad \quad \text{val } y = g \ x \\
& \quad \quad \text{val } z = h \ x \\
& \quad \text{in} \\
& \quad \quad (y, z) \\
& \text{end}
\end{align*}
\]

\[
\begin{align*}
\text{fun } f \ x & = \\
& \quad \text{let} \\
& \quad \quad \text{val } z = h \ x \\
& \quad \quad \text{val } y = g \ x \\
& \quad \text{in} \\
& \quad \quad (y, z) \\
& \text{end}
\end{align*}
\]

- Example: \( g \) divides by 0 and \( h \) mutates a top-level reference
- Example: \( g \) writes to a reference that \( h \) reads from
One *that really matters*

Once again, turning the left into the right is great but only if the functions are pure:

\[
\text{map } f \ (\text{map } g \ x) \quad \text{map } (f \circ g) \ x
\]
Syntactic sugar

Using or not using syntactic sugar is always equivalent
– By definition, else not syntactic sugar

Example:

\[
\text{fun } f \ x = \begin{cases} 
\ x \ \land \ \text{also} \ g \ x & \text{if } x \\
\text{false} & \text{else}
\end{cases}
\]

But be careful about evaluation order
Standard equivalences

Three general equivalences that always work for functions
- In any (?) decent language

1. Consistently rename bound variables and uses

\[
\begin{align*}
\text{val } y &= 14 \\
\text{fun } f \ x &= x+y+x
\end{align*}
\]

\[
\begin{align*}
\text{val } y &= 14 \\
\text{fun } f \ z &= z+y+z
\end{align*}
\]

But notice you can’t use a variable name already used in the function body to refer to something else

\[
\begin{align*}
\text{val } y &= 14 \\
\text{fun } f \ x &= x+y+x
\end{align*}
\]

\[
\begin{align*}
\text{val } y &= 14 \\
\text{fun } f \ y &= y+y+y
\end{align*}
\]

\[
\begin{align*}
\text{fun } f \ x &= \ \\
&\text{let } \text{val } y = 3 \\
&\text{in } x+y \text{ end}
\end{align*}
\]

\[
\begin{align*}
\text{fun } f \ y &= \ \\
&\text{let } \text{val } y = 3 \\
&\text{in } y+y \text{ end}
\end{align*}
\]
Standard equivalences

Three general equivalences that always work for functions
- In (any?) decent language

2. Use a helper function or do not

\[
\begin{align*}
\text{val } y &= 14 \\
\text{fun } g \ z &= (z+y+z)+z
\end{align*}
\]

\[
\begin{align*}
\text{val } y &= 14 \\
\text{fun } f \ x &= x+y+x \\
\text{fun } g \ z &= (f \ z)+z
\end{align*}
\]

But notice you need to be careful about environments

\[
\begin{align*}
\text{val } y &= 14 \\
\text{val } y &= 7 \\
\text{fun } g \ z &= (z+y+z)+z
\end{align*}
\]

\[
\begin{align*}
\text{val } y &= 14 \\
\text{fun } f \ x &= x+y+x \\
\text{val } y &= 7 \\
\text{fun } g \ z &= (f \ z)+z
\end{align*}
\]
Standard equivalences

Three general equivalences that always work for functions
- In (any?) decent language

3. Unnecessary function wrapping

\[
\begin{align*}
\text{fun} \ f \ x &= x+x \\
\text{fun} \ g \ y &= f \ y \\
\text{val} \ g &= f
\end{align*}
\]

But notice that if you compute the function to call and \textit{that computation} has side-effects, you have to be careful

\[
\begin{align*}
\text{fun} \ f \ x &= x+x \\
\text{fun} \ h () &= (\text{print} \ "hi"; \ f) \\
\text{fun} \ g \ y &= (h()) \ y \\
\text{fun} \ h () &= (\text{print} \ "hi"; \ f) \\
\text{val} \ g &= (h())
\end{align*}
\]
One more

If we ignore types, then ML let-bindings can be syntactic sugar for calling an anonymous function:

```ml
let val x = e1 in e2 end
(fn x => e2) e1
```

- These both evaluate $e_1$ to $v_1$, then evaluate $e_2$ in an environment extended to map $x$ to $v_1$
- So *exactly* the same evaluation of expressions and result

But in ML, there is a type-system difference:
- $x$ on the left can have a polymorphic type, but not on the right
- Can always go from right to left
- If $x$ need not be polymorphic, can go from left to right
What about performance?

According to our definition of equivalence, these two functions are equivalent, but we learned one is awful

– (Actually we studied this before pattern-matching)

\[
\begin{align*}
\text{fun} & \quad \text{max} \ \text{xs} = \\
& \quad \text{case} \ \text{xs} \ \text{of} \\
& \quad \quad \text{[]} \Rightarrow \text{raise Empty} \\
& \quad \quad \text{x::[]} \Rightarrow \ x \\
& \quad \quad \text{x::xs'} \Rightarrow \\
& \quad \quad \quad \text{if} \ x > \text{max} \ \text{xs'} \\
& \quad \quad \quad \text{then} \ x \\
& \quad \quad \quad \text{else} \ \text{max} \ \text{xs'}
\end{align*}
\]

\[
\begin{align*}
\text{fun} & \quad \text{max} \ \text{xs} = \\
& \quad \text{case} \ \text{xs} \ \text{of} \\
& \quad \quad \text{[]} \Rightarrow \text{raise Empty} \\
& \quad \quad \text{x::[]} \Rightarrow \ x \\
& \quad \quad \text{x::xs'} \Rightarrow \\
& \quad \quad \quad \text{let} \\
& \quad \quad \quad \quad \text{val} \ y = \text{max} \ \text{xs'} \\
& \quad \quad \quad \quad \text{in} \\
& \quad \quad \quad \quad \quad \text{if} \ x > y \\
& \quad \quad \quad \quad \quad \text{then} \ x \\
& \quad \quad \quad \quad \quad \text{else} \ y \\
& \quad \quad \quad \quad \text{end}
\end{align*}
\]
Different definitions for different jobs

• PL Equivalence (341): given same inputs, same outputs and effects
  – Good: Lets us replace bad $\max$ with good $\max$
  – Bad: Ignores performance in the extreme

• Asymptotic equivalence (332): Ignore constant factors
  – Good: Focus on the algorithm and efficiency for large inputs
  – Bad: Ignores “four times faster”

• Systems equivalence (333): Account for constant overheads, performance tune
  – Good: Faster means different and better
  – Bad: Beware overtuning on “wrong” (e.g., small) inputs; definition does not let you “swap in a different algorithm”

Claim: Computer scientists implicitly (?) use all three every (?) day