CSE 341 AB: Section 3

Josh Pollock
joshpoll@cs.uw.edu
OH: Thursdays 4:30pm - 5:30pm

Intros

Introduce yourself to someone new!

- What's your name?
- How's your quarter?
- [insert question here]

- Share questions you have about course content.
  - If the other person can answer it, great!
  - If you both don't know, hold on to it.

Questions?

Agenda (Lots of Cool Stuff!)

- SML Standard Library
- Datatype Polymorphism
- Tracing Functions (For Real!!)
- **Higher-Order Functions**
  - Returning Functions
  - map, filter, join, bind/flat_map
  - **foldr**
  - foldl
- Revisiting HW1

  (see the section code)

SML Standard Library

Online Documentation

http://www.standardml.org/Basis/index.html

http://www.smlnj.org/doc/smlnj-lib/Manual/toc.html

Helpful Subset

Top-Level	http://www.standardml.org/Basis/top-level-chapter.html
List	http://www.standardml.org/Basis/list.html
ListPair	http://www.standardml.org/Basis/list-pair.html
Real	http://www.standardml.org/Basis/real.html
String	http://www.standardml.org/Basis/string.html

Datatype Polymorphism

- Last week we saw polymorphic *functions* that use parametric polymorphism.
- This week we'll look at polymorphic *datatypes*.
- We've already seen them, but you can make your own, too!
- As with polymorphic functions, type variables in polymorphic datatypes must be substituted *consistently*.
- Demo!

Four Kinds of Functions

		Output	
		Term	Type
Input	Term	"Normal" Functions `f (x, y) = x + y`	???
	Type	Parametric Polymorphism (fake syntax) `f ('a) (x) = x : 'a`	Datatype Polymorphism `datatype 'a list = ...`

Four Kinds of Functions

		Output	
		Term	Type
Input	Term	“Normal” Functions `f (x, y) = x + y`	Dependent Types outside course scope :(
	Type	Parametric Polymorphism (fake syntax) `f ('a) (x) = x : 'a`	Datatype Polymorphism `datatype 'a list = ...`

Function Closures

Functions **ARE NOT** Values

Closures **ARE** Values

Function Closures

Function closures are the most unique value we'll see.

- The only value that's not an expression.
- Store *code* and *bindings*.

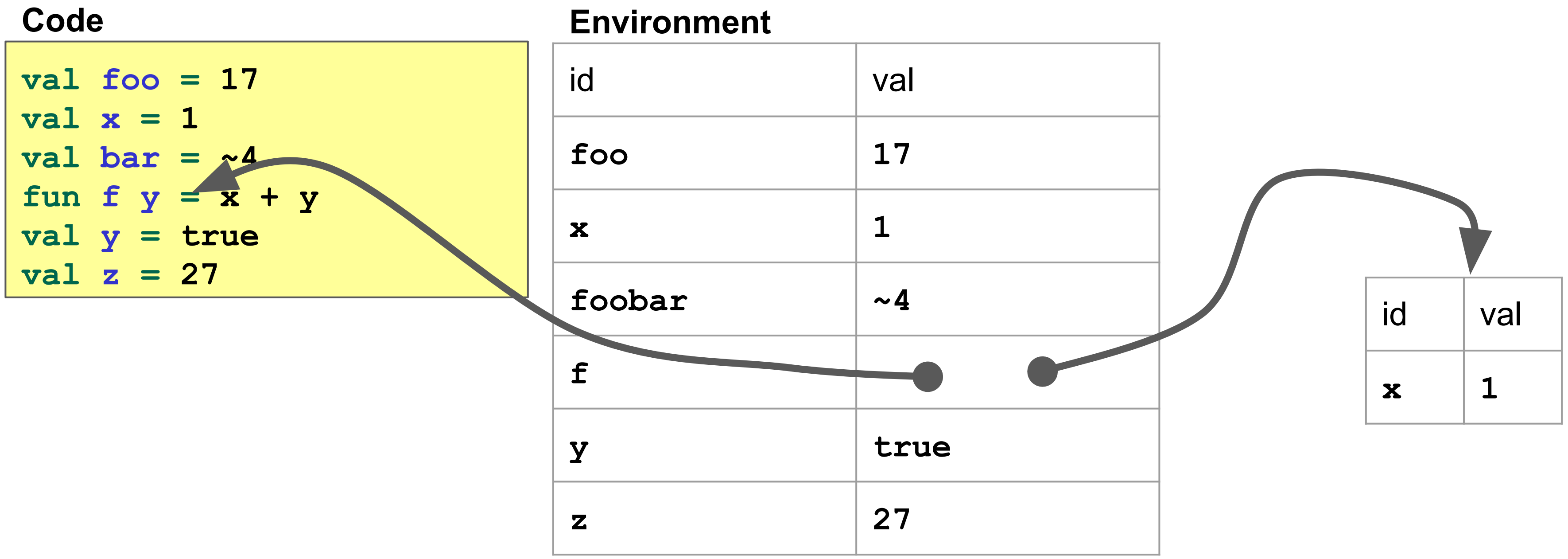
- Keep pointers to the code and to an environment.
- The environment stores the bindings that weren't bound by the function.
    - These are called **free variables** or **open bindings**.
    - The environment *closes* the function.

Function Closures Visualized!

Code

```
val foo = 17
val x = 1
val bar = ~4
fun f y = x + y
val y = true
val z = 27
```

Function Closures Visualized!



Code

```
val foo = 17
val x = 1
val bar = ~4
fun f y = x + y
val y = true
val z = 27
```

Environment

id	val
foo	17
x	1
foobar	~4
f	● ●
y	true
z	27

id	val
x	1

Function Closures Visualized!

Pointers are hard to draw.

Code

i0

```
val foo = 17
val x = 1
val bar = ~4
fun f y = x + y
val y = true
val z = 27
```

Environment

id	val
foo	17
x	1
foobar	~4
f	i0 e0
y	true
z	27

e0

id	val
x	1

Function Closures Visualized!

We need to support recursion!

Code

i0

```
val foo = 17
val x = 1
val bar = ~4
fun f y = x + y
val y = true
val z = 27
```

Environment

id	val
foo	17
x	1
foobar	~4
f	i0 e0
y	true
z	27

e0

id	val
x	1
f	i0 e0

Tracing Function Closures

Higher-Order Functions

Returning Functions

Demo!

Higher-Order Functions

Higher-order functions really give functional programming its “flavor.”

Today we’ll look at higher-order functions for data manipulation.

They separate data manipulation into two parts:

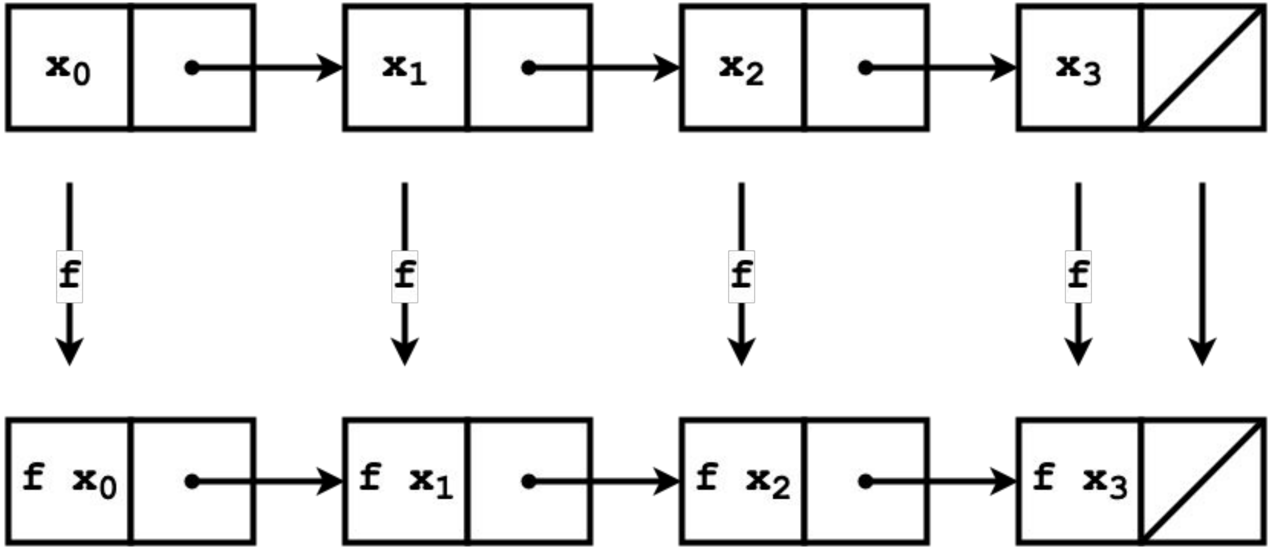
- Structure traversal
- Computation

It’s also easy to write our own structure traversals.

We’re not stuck with a small set like `if`, `while`, and `for`.

`map`

Solution: **map**



map

$$\begin{tikzcd}
\texttt{[\ \ x0,} \arrow[d] & \texttt{x1,} \arrow[d] & \texttt{x2,} \arrow[d] & \texttt{x3]} \arrow[d] \\
\texttt{[f x0,} & \texttt{f x1,} & \texttt{f x2,} & \texttt{f x3]}
\end{tikzcd}$$

map

```
(* types annotated for clarity *)
fun map  (f  : 'a -> 'b,
          xs : 'a list) : 'b list =
  case xs of
    [] => []
  | x::xs' => (f x)::(map (f, xs'))
```

`filter`

`join`

`bind/flat_map`

`foldr`

The One to Rule Them All: **`foldr`**

Remember we can think of constructors as abstract functions and values.

```
Cons : 'a * 'a my_list -> 'a my_list

Nil : 'a my_list
```

`foldr` replaces the constructors with functions you choose.

The One to Rule Them All: **`foldr`**

$$\begin{tikzcd}
& {::} \arrow[dl] \arrow[dr] & & & & \\
x_0 & & {::} \arrow[dl] \arrow[dr] & & & \\
& x_1 & & {::} \arrow[dl] \arrow[dr] & & \\
& & x_2 & & {::} \arrow[dl] \arrow[dr] & \\
& & & x_3 & & {[]}
\end{tikzcd}$$

$$\begin{tikzcd}
& f \arrow[dl] \arrow[dr] & & & & \\
x_0 & & f \arrow[dl] \arrow[dr] & & & \\
& x_1 & & f \arrow[dl] \arrow[dr] & & \\
& & x_2 & & f \arrow[dl] \arrow[dr] & \\
& & & x_3 & & z
\end{tikzcd}$$

The One to Rule Them All: `foldr`

$$\begin{tikzcd}
& {::} \arrow[dl] \arrow[dr] & & & & \\
x_0 & & {::} \arrow[dl] \arrow[dr] & & & \\
& x_1 & & {::} \arrow[dl] \arrow[dr] & & \\
& & x_2 & & {::} \arrow[dl] \arrow[dr] & \\
& & & x_3 & & {[]}
\end{tikzcd}$$

$$\begin{tikzcd}
& f \arrow[dl] \arrow[dr] & & & & \\
x_0 & & f \arrow[dl] \arrow[dr] & & & \\
& x_1 & & f \arrow[dl] \arrow[dr] & & \\
& & x_2 & & f \arrow[dl] \arrow[dr] & \\
& & & x_3 & & z
\end{tikzcd}$$

```
(op ::) : 'a * 'a list -> 'a list
[] : 'a list
```

```
f : 'a * 'b -> 'b
z : 'b
```

(`op ::` is the prefix version of `::`)

foldr

```
(* types annotated for clarity *)
fun foldr (f  : 'a * 'b -> 'b,
           z  : 'b,
           xs : 'a list) : 'b =
  case xs of
    [] => z
  | x::xs' => f (x, foldr (f, z, xs'))
```

```
foldr (fn (x, acc) => x::acc, [3, 4], [1, 2])
foldr (fn (x, acc) => x::acc, [3, 4], [1, 2])

1::(foldr (fn (x, acc) => x::acc, [3, 4], [2])
foldr (fn (x, acc) => x::acc, [3, 4], [1, 2])

1::(foldr (fn (x, acc) => x::acc, [3, 4], [2])

1::(2::(foldr (fn (x, acc) => x::acc, [3, 4], []))
foldr (fn (x, acc) => x::acc, [3, 4], [1, 2])

1::(foldr (fn (x, acc) => x::acc, [3, 4], [2])

1::(2::(foldr (fn (x, acc) => x::acc, [3, 4], []))

1::(2::[3, 4])
```

```
foldr (fn (x, acc) => x::acc, [3, 4], [1, 2])

1::(foldr (fn (x, acc) => x::acc, [3, 4], [2])

1::(2::(foldr (fn (x, acc) => x::acc, [3, 4], []))

1::(2::[3, 4])

1::[2, 3, 4]
```

```
foldr (fn (x, acc) => x::acc, [3, 4], [1, 2])

1::(foldr (fn (x, acc) => x::acc, [3, 4], [2])

1::(2::(foldr (fn (x, acc) => x::acc, [3, 4], []))

1::(2::[3, 4])

1::[2, 3, 4]

[1, 2, 3, 4]
```

`foldl`

What about tail recursion?

Reversing and summing are needlessly slow with **`foldr`**.

Solution: `foldl`

```
(* types annotated for clarity *)
fun foldl (f   : 'b * 'a -> 'b,
           acc : 'b,
           xs  : 'a list) : 'b =
  case xs of
    [] => acc
  | x::xs' => foldl (f, f (acc, x), xs')
```

`foldl` generalizes the accumulator pattern

foldr vs foldl

$$\begin{tikzcd}
& f \arrow[dl] \arrow[dr] & & & & \\
x_0 & & f \arrow[dl] \arrow[dr] & & & \\
& x_1 & & f \arrow[dl] \arrow[dr] & & \\
& & x_2 & & f \arrow[dl] \arrow[dr] & \\
& & & x_3 & & z
\end{tikzcd}$$

$$\begin{tikzcd}
& & & & f \arrow[dl] \arrow[dr] & \\
& & & f \arrow[dl] \arrow[dr] & & x_3 \\
& & f \arrow[dl] \arrow[dr] & & x_2 & \\
& f \arrow[dl] \arrow[dr] & & x_1 & & \\
z & & x_0 & & &
\end{tikzcd}$$

foldr goes down then up
foldl only goes up

```
foldl (fn (acc, x) => x::acc, [], [1, 2, 3])
foldl (fn (acc, x) => x::acc, [], [1, 2, 3])

foldl (fn (acc, x) => x::acc, [1], [2, 3])
foldl (fn (acc, x) => x::acc, [], [1, 2, 3])

foldl (fn (acc, x) => x::acc, [1], [2, 3])

foldl (fn (acc, x) => x::acc, [2, 1], [3])
foldl (fn (acc, x) => x::acc, [], [1, 2, 3])

foldl (fn (acc, x) => x::acc, [1], [2, 3])

foldl (fn (acc, x) => x::acc, [2, 1], [3])

foldl (fn (acc, x) => x::acc, [3, 2, 1], [])
foldl (fn (acc, x) => x::acc, [], [1, 2, 3])

foldl (fn (acc, x) => x::acc, [1], [2, 3])

foldl (fn (acc, x) => x::acc, [2, 1], [3])

foldl (fn (acc, x) => x::acc, [3, 2, 1], [])

[3, 2, 1]
```

But `foldr` Is Still “Better”

You should use `foldl` when you need tail recursion.

BUT…

- You can write `foldl` in terms of `foldr`.
  - We may get to this next week.
- `foldr` generalizes naturally to other datatypes, and `foldl` does not.

Generalizing `foldr`.

Most common datatypes have a natural version of `foldr`.*

Generalize the types of the datatype constructors.

Match clause → function (or constant if it has no arguments).

See section file for examples.

*It's called a *catamorphism.*

Higher-Order Functions Are Difficult But Useful

HO functions allow for (among other things) better separation of concerns.

Today we saw how you to separate traversal strategies and computation.

It will probably require some time for these functions to sink in.

But once they do, they make your code easier to read and write!