What is functional programming?

“Functional programming” can mean a few different things:

1. Avoiding mutation in most/all cases (done and ongoing)
2. Using functions as values (this unit)

... 
- Style encouraging recursion and recursive data structures
- Style closer to mathematical definitions
- Programming idioms using laziness (later topic, briefly)
- Anything not OOP or C? (not a good definition)

Not sure a definition of “functional language” exists beyond “makes functional programming easy / the default / required”
- No clear yes/no for a particular language
First-class functions

• First-class functions: Can use them wherever we use values
  – Functions are values too
  – Arguments, results, parts of tuples, bound to variables, carried by datatype constructors or exceptions, …

fun double x = 2*x
fun incr x = x+1
val a_tuple = (double, incr, double(incr 7))

• Most common use is as an argument / result of another function
  – Other function is called a higher-order function
  – Powerful way to factor out common functionality
Function Closures

• *Function closure*: Functions can use bindings from outside the function definition (in scope where function is defined)
  – Makes first-class functions *much* more powerful
  – Will get to this feature in a bit, after simpler examples

• Distinction between terms *first-class functions* and *function closures* is not universally understood
  – Important conceptual distinction even if terms get muddled
Onward

The next week:

- How to use first-class functions and closures
- The precise semantics
- Multiple powerful idioms
Functions as arguments

• We can pass one function as an argument to another function
  – Not a new feature, just never thought to do it before

  \[
  \text{fun } f \ (g, \ldots) = \ldots \ g \ (\ldots) \ \ldots \\
  \text{fun } h1 \ \ldots = \ \ldots \\
  \text{fun } h2 \ \ldots = \ \ldots \\
  \ldots \ f(h1, \ldots) \ \ldots \ f(h2, \ldots) \ \ldots 
  \]

• Elegant strategy for factoring out common code
  – Replace \( N \) similar functions with calls to 1 function where
  you pass in \( N \) different (short) functions as arguments

[See the code file for this lecture]
Example

Can reuse \texttt{n_times} rather than defining many similar functions

\begin{itemize}
  \item Computes $f(f(...f(x)))$ where number of calls is $n$
\end{itemize}

\begin{verbatim}
fun n_times (f,n,x) =
  if n=0
    then x
    else f (n_times(f,n-1,x))

fun double x = x + x
fun increment x = x + 1
val x1 = n_times(double,4,7)
val x2 = n_times(increment,4,7)
val x3 = n_times(tl,2,[4,8,12,16])

fun double_n_times (n,x) = n_times(double,n,x)
fun nth_tail (n,x) = n_times(tl,n,x)
\end{verbatim}
Relation to types

- Higher-order functions are often so “generic” and “reusable” that they have polymorphic types, i.e., types with type variables.

- But there are higher-order functions that are not polymorphic.

- And there are non-higher-order (first-order) functions that are polymorphic.

- Always a good idea to understand the type of a function, especially a higher-order function.
Types for example

fun n_times (f, n, x) = 
    if n=0 
      then x 
      else f (n_times(f, n-1, x))

- val n_times : ('a -> 'a) * int * 'a -> 'a
  - Simpler but less useful: (int -> int) * int * int -> int

- Two of our examples *instantiated* 'a with int
- One of our examples *instantiated* 'a with int list
- This *polymorphism* makes n_times more useful

- Type is *inferred* based on how arguments are used (later lecture)
  - Describes which types must be exactly something (e.g., int) and which can be anything but the same (e.g., 'a)
Polymorphism and higher-order functions

- Many higher-order functions are polymorphic because they are so reusable that some types, “can be anything”

- But some polymorphic functions are not higher-order
  - Example: `len : 'a list -> int`

- And some higher-order functions are not polymorphic
  - Example: `times_until_0 : (int->int)*int->int`

```haskell
fun times_until_zero (f,x) = 
   if x=0 then 0 else 1 + times_until_zero(f, f x)
```

Note: Would be better with tail-recursion
Toward anonymous functions

- Definitions unnecessarily at top-level are still poor style:

```ml
fun trip x = 3*x
fun triple_n_times (f,x) = n_times(trip,n,x)
```

- So this is better (but not the best):

```ml
fun triple_n_times (f,x) =
  let fun trip y = 3*y
  in
    n_times(trip,n,x)
  end
```

- And this is even smaller scope
  - It makes sense but looks weird (poor style; see next slide)

```ml
fun triple_n_times (f,x) =
  n_times(let fun trip y = 3*y in trip end, n, x)
```
Anonymous functions

• This does not work: A function binding is not an expression

\[
\text{fun triple_n_times (f,x) =}
\ \ \ \ \ \ \ \ n\_\text{times}((\text{fun trip \ y = 3*y}), n, x)
\]

• This is the best way we were building up to: an expression form for anonymous functions

\[
\text{fun triple_n_times (f,x) =}
\ \ \ \ \ \ \ \ n\_\text{times}((\text{fn \ y => 3*y}), n, x)
\]

  - Like all expression forms, can appear anywhere
  - Syntax:
    • \text{fn not fun}
    • => not =
    • no function name, just an argument pattern
Using anonymous functions

• Most common use: Argument to a higher-order function
  – Don’t need a name just to pass a function

• But: Cannot use an anonymous function for a recursive function
  – Because there is no name for making recursive calls
  – If not for recursion, \texttt{fun} bindings would be syntactic sugar for \texttt{val} bindings and anonymous functions

\begin{verbatim}
fun triple x = 3*x
val triple = fn y => 3*y
\end{verbatim}
A style point

Compare:

\[
\text{if } x \text{ then true else false}
\]

With:

\[
(f n \ x \Rightarrow f \ x)
\]

So don’t do this:

\[
n\_\_\_\_\_\times((f n \ y \Rightarrow t l \ y),3,xs)
\]

When you can do this:

\[
n\_\_\_\_\times(t l,3,xs)
\]
Map

fun map (f,xs) =
  case xs of
    [] => []
  | x::xs' => (f x)::(map(f,xs'))

val map : ('a -> 'b) * 'a list -> 'b list

Map is, without doubt, in the “higher-order function hall-of-fame”
  – The name is standard (for any data structure)
  – You use it all the time once you know it: saves a little space, but more importantly, communicates what you are doing
  – Similar predefined function: List.map
    • But it uses currying (coming soon)
Filter

fun filter (f,xs) =
  case xs of
    [] => []
    | x::xs' => if f x then x::(filter(f,xs')) else filter(f,xs')

val filter : ('a -> bool) * 'a list -> 'a list

Filter is also in the hall-of-fame
  – So use it whenever your computation is a filter
  – Similar predefined function: List.filter
    • But it uses currying (coming soon)
Generalizing

Our examples of first-class functions so far have all:
– Taken one function as an argument to another function
– Processed a number or a list

But first-class functions are useful anywhere for any kind of data
– Can pass several functions as arguments
– Can put functions in data structures (tuples, lists, etc.)
– Can return functions as results
– Can write higher-order functions that traverse your own data structures

Useful whenever you want to abstract over “what to compute with”
– No new language features
Returning functions

- Remember: Functions are first-class values
  - For example, can return them from functions

- Silly example:

```haskell
fun double_or_triple f = 
  if f 7 
  then fn x => 2*x 
  else fn x => 3*x
```

Has type \((\text{int} \rightarrow \text{bool}) \rightarrow \text{int} \rightarrow \text{int}\)

But the REPL prints \((\text{int} \rightarrow \text{bool}) \rightarrow \text{int} \rightarrow \text{int}\) because it never prints unnecessary parentheses and 
\(t_1 \rightarrow t_2 \rightarrow t_3 \rightarrow t_4\) means \(t_1 \rightarrow (t_2 \rightarrow (t_3 \rightarrow t_4))\)
Other data structures

- Higher-order functions are not just for numbers and lists
- They work great for common recursive traversals over your own data structures (datatype bindings) too
- Example of a higher-order *predicate*:
  - Are all constants in an arithmetic expression even numbers?
  - Use a more general function of type
    \[(\text{int} \rightarrow \text{bool}) \times \text{exp} \rightarrow \text{bool}\]
  - And call it with \((\text{fn } x \Rightarrow x \mod 2 = 0)\)