Equivalence

Must reason about "are these equivalent" all the time
- The more precisely you think about it the better
  • Code maintenance: Can I simplify this code?
  • Backward compatibility: Can I add new features without changing how any old features work?
  • Optimization: Can I make this code faster?
  • Abstraction: Can an external client tell I made this change?

To focus discussion: When can we say two functions are equivalent, even without looking at all calls to them?
- May not know all the calls (e.g., we are editing a library)

A definition

Two functions are equivalent if they have the same "observable behavior" no matter how they are used anywhere in any program

Given equivalent arguments, they:
- Produce equivalent results
- Have the same (non-)termination behavior
- Mutate (non-local) memory in the same way
- Do the same input/output
- Raise the same exceptions

Notice it is much easier to be equivalent if:
- There are fewer possible arguments, e.g., with a type system and abstraction
- We avoid side-effects: mutation, input/output, and exceptions

Example

Since looking up variables in ML has no side effects, these two functions are equivalent:

\[
\begin{align*}
\text{fun } f \ x &= x + x \\
\text{fun } g \ (f, x) &= (f \ x) + (f \ x)
\end{align*}
\]

But these next two are not equivalent in general: it depends on what is passed for \( f \)
- Are equivalent if argument for \( f \) has no side-effects

\[
\begin{align*}
\text{val } y &= 2 \\
\text{fun } g \ (f, x) &= (f \ x) + (f \ x)
\end{align*}
\]

Example: \( g ((\text{fn } i \rightarrow \text{print } "hi" ; i), 7) \)
- Great reason for "pure" functional programming

Another example

These are equivalent only if functions bound to \( g \) and \( h \) do not raise exceptions or have side effects (printing, updating state, etc.)
- Again: pure functions make more things equivalent
**One that really matters**

Once again, turning the left into the right is great but only if the functions are pure:

```ml
map f (map g xs)  # map (f o g) xs
```

**Syntactic sugar**

Using or not using syntactic sugar is always equivalent

– By definition, else not syntactic sugar

Example:

```ml
fun f x =
  x andalso g x

fun f x =
  if x
  then g x
  else false
```

But be careful about evaluation order

```ml
fun f x =
  if g x
  then x
  else false
```

**Standard equivalences**

Three general equivalences that always work for functions

– In (any?) decent language

1. Consistently rename bound variables and uses

```ml
val y = 14
fun f x = x+y+x
```

But notice you can’t use a variable name already used in the

```ml
fun f x = x+y+x
```

function body to refer to something else

```ml
fun f x =
  let val y = 3
  in x+y end
```

```ml
fun f x =
  let val y = 3
  in y+y end
```

**Standard equivalences**

Three general equivalences that always work for functions

– In (any?) decent language

2. Use a helper function or do not

```ml
val y = 14
fun g z = (f z)+z
```

But notice you need to be careful about environments

```ml
val y = 14
val y = 7
fun g z = (f z)+z
```

**Standard equivalences**

Three general equivalences that always work for functions

– In (any?) decent language

3. Unnecessary function wrapping

```ml
fun f x = x+x
fun g y = f y
```

But notice that if you compute the function to call and that computation has side-effects, you have to be careful

```ml
fun f x = x+x
fun h () = (print "hi";
  f)
fun g y = (h()) y
```

But in ML, there is a type-system difference:

– x on the left can have a polymorphic type, but not on the right
– Can always go from right to left
– If x need not be polymorphic, can go from left to right
What about performance?

According to our definition of equivalence, these two functions are equivalent, but we learned one is awful

   (Actually we studied this before pattern-matching)

Different definitions for different jobs

- **PL Equivalence (341):** given same inputs, same outputs and effects
  - Good: Lets us replace bad max with good max
  - Bad: Ignores performance in the extreme

- **Asymptotic equivalence (332):** Ignore constant factors
  - Good: Focus on the algorithm and efficiency for large inputs
  - Bad: Ignores “four times faster”

- **Systems equivalence (333):** Account for constant overheads, performance tune
  - Good: Faster means different and better
  - Bad: Beware overtuning on “wrong” (e.g., small) inputs; definition does not let you “swap in a different algorithm”

Claim: Computer scientists implicitly (?) use all three every (?) day