Please do not turn the page until 8:30.

Rules:

• The exam is closed-book, closed-note, etc. except for both sides of one 8.5x11in piece of paper.

• **Please stop promptly at 10:20.**

• There are **130 points**, distributed **unevenly** among 9 questions (all with multiple parts).

• The exam is printed double-sided.

Advice:

• Read questions carefully. Understand a question before you start writing.

• Write down thoughts and intermediate steps so you can get partial credit. But clearly indicate what is your final answer.

• The questions are not necessarily in order of difficulty. Skip around. Make sure you get to all the questions.

• If you have questions, ask.

• Relax. You are here to learn.
1. (21 points)  (Racket programming)

(a) Write a Racket function `fold-basic` that behaves as follows:
- It is like SML's `List.foldl` function except it does not use currying and the order of arguments is as explained below.
- It takes three arguments: a function, an initial accumulator, and a list. (Do not check the arguments are what is expected.)
- It repeatedly applies the function to the accumulator (first argument to the function) and the next list element (second argument to the function) to produce the next accumulator.
- The result is the final accumulator.

(b) Consider this function

```racket
(define (mystery xs)
  (fold-basic (lambda (x y) (if (or (not x) (> y x)) y x)) #f xs))
```

For each of these calls, indicate “error” if the call fails with an error, else indicate the result of the call.

i. (mystery null)
ii. (mystery (list 4 5 9 3 7))
iii. (mystery (list 4 5 #f 3 7))

(c) Write a Racket function `fold-deep` that is like `fold-basic` except that if a list element is another list, it performs `fold-deep` with the same function and the current accumulator over that nested list. It should continue to “fold deeply” over any nesting of lists (lists inside lists inside lists...). For example, `(fold-deep + 0 (list 3 (list 4 (list 5) 6) (list 7)))` would evaluate to 25.

(d) What does this expression evaluate to?

```racket
(fold-deep
  (lambda (x y) (cons y x))
  null
  (list (list 3 4) null (list 3 5 (list "hi") 9) (cons 4 6) (cons 5 null)))
```
2. (12 points) (Scope and mutation) Suppose we evaluate all the code below as a single Racket program.

```racket
(define x 1)
(define y 2)

(define (g a)
  (+ a y))

(define (f1 z)
  (let ([x y]
         [y x])
    (+ x (g y) z)))

(define (f2 z)
  (let* ([x y]
         [y x])
    (+ x (g y) z)))

(define (f3 z)
  (let ([t x])
    (begin
      (set! x y)
      (set! y t)
      (let ([ans (+ x (g y) z)]
            [u x])
        (begin
          (set! x y)
          (set! y u)
          ans))))

(define part-a (f1 3))
(define part-b (f2 3))
(define part-c (f3 3))
(define part-d (f1 3))
(define part-e (f2 3))
(define part-f (f3 3))

(a) What is part-a bound to?
(b) What is part-b bound to?
(c) What is part-c bound to?
(d) What is part-d bound to?
(e) What is part-e bound to?
(f) What is part-f bound to?
```
3. (18 points) (Streams) As in class, we define a stream to be a thunk that when called returns a pair where the cdr of the pair is a stream.

(a) Write a function count-until-total-is-more that behaves as follows:
   - It takes two arguments: a stream s, which we assume contains only non-negative numbers, and a number limit.
   - It returns the minimum number of stream elements that need to be summed to make the sum strictly greater than limit.

   For example, if s represents the natural numbers 1,2,3,... and limit is 6, then the result should be 4. For full credit, do not define any helper functions. Hint: The base case is when limit is negative.

(b) Write a function every-third, that takes a stream s and returns a stream that contains every third element of s. For example if s represents 1,2,3,... then (every-third s) would represent 3,6,9,...

   For full credit, do not define any recursive helper functions (anonymous functions are fine).

(c) Write a function every-nth that takes a stream s and a positive integer n and returns a stream that contains every n\textsuperscript{th} element of s. For example, (every-nth s 3) would be equivalent to (every-third s).

   For full credit, use one locally defined recursive helper function.
4. (16 points) (Interpreter implementation) Here is some of the code we provided you for Homework 5 (MUPL).

(struct var (string) #:transparent) ;; a variable, e.g., (var "foo")
(struct int (num) #:transparent) ;; a constant number, e.g., (int 17)
(struct add (e1 e2) #:transparent) ;; add two expressions
(struct isgreater (e1 e2) #:transparent) ;; if e1 > e2 then 1 else 0
(struct ifnz (e1 e2 e3) #:transparent) ;; if not zero e1 then e2 else e3
(struct fun (nameopt formal body) #:transparent) ;; a recursive(?) 1-argument function
(struct call (funexp actual) #:transparent) ;; function call
(struct mlet (var e body) #:transparent) ;; a local binding (let var = e in body)
(struct apair (e1 e2) #:transparent) ;; make a new pair
(struct first (e) #:transparent) ;; get first part of a pair
(struct second (e) #:transparent) ;; get second part of a pair
(struct munit () #:transparent) ;; unit value -- good for ending a list
(struct ismunit (e) #:transparent) ;; if e is unit then 1 else 0

(define (envlookup env str)
  (cond [(null? env) (error "unbound variable during evaluation" str)]
        [(equal? (car (car env)) str) (cdr (car env))]
        [#t (envlookup (cdr env) str)]))

(define (eval-under-env e env)
  (cond [(var? e) (envlookup env (var-string e))]
        [(add? e) (let ([v1 (eval-under-env (add-e1 e) env)]
                        [v2 (eval-under-env (add-e2 e) env)])
                        (if (and (int? v1)
                                  (int? v2))
                            (int (+ (int-num v1)
                                    (int-num v2)))
                            (error "MUPL addition applied to non-number")))])

Assume all the not-shown pieces are implemented correctly, including raising an appropriate error message if a subexpression evaluates to the wrong type of MUPL value.

In this question, we extend the MUPL language with this expression form:

(struct callpair (e) #:transparent)

It has this semantics:

- The subexpression should evaluate to a pair where the first component of the pair is a closure, else it is an error.
- The result is the result of calling the first component of the pair with the second component of the pair as the argument.

See the next page for the questions.
(a) Consider this implementation of the callpair case in “the big cond” of eval-under-env:

\[
\text{(callpair? e) (eval-under-env (mlet "x" (callpair-e e)
\quad \text{(call (first (var "x")) (second (var "x")))
\quad \text{env})]
\]

Answer (i) and then either (ii) or (iii) as explained below:

i. Yes or No: Is this implementation correct in cases where the subexpression evaluates to a pair whose first component is a closure?

ii. If your answer to part (i) was yes, answer this: If a callpair’s subexpression evaluates to something that is not a pair, what will happen when the callpair is evaluated?

iii. If your answer to part (i) was no, answer this: If a callpair’s subexpression evaluates to something that is a pair, what will happen when the callpair is evaluated?

(b) Repeat part (a) for this implementation of callpair:

\[
\text{(callpair? e) (eval-under-env (mlet "x" (callpair-e e)
\quad \text{(call (apair-e1 (var "x")) (apair-e2 (var "x")))
\quad \text{env})]
\]

(c) Repeat part (a) for this implementation of callpair:

\[
\text{(callpair? e) (let ([pr (eval-under-env e env)])
\quad \text{(if (not (apair? pr))
\quad \text{(error "argument to callpair did not evaluate to a pair")
\quad \text{(eval-under-env (call (apair-e1 pr) (apair-e2 pr))
\quad \text{env})])]
\]

(d) Repeat part (a) for this implementation of callpair:

\[
\text{(callpair? e) (let ([pr (eval-under-env (callpair-e e) env)])
\quad \text{(if (not (apair? pr))
\quad \text{(error "argument to callpair did not evaluate to a pair")
\quad \text{(eval-under-env (call (apair-e1 pr) (apair-e2 pr))
\quad \text{env})])]
\]
5. (16 points) (Soundness/Completeness) Suppose a committee is designing a new language BestEver and wishes to statically prevent bad thing OhNoes from happening in the language. Two subcommittees have presented competing type-system proposals called LionTypes and TigerTypes. The committee chair proposes two combination type systems to resolve the competition:

- WhyNotBoth, a type system that accepts a program if and only if LionTypes and TigerTypes accept the program.
- EitherWay, a type system that accepts a program if and only if at least one of LionTypes or TigerTypes accepts the program.

Answer yes/no/maybe — no explanations needed.

(a) If LionTypes is sound and TigerTypes is sound, is WhyNotBoth sound?

(b) If LionTypes is sound and TigerTypes is sound, is EitherWay sound?

(c) If LionTypes is sound and TigerTypes is unsound, is WhyNotBoth sound?

(d) If LionTypes is sound and TigerTypes is unsound, is EitherWay sound?

(e) If LionTypes is unsound and TigerTypes is unsound, is WhyNotBoth sound?

(f) If LionTypes is unsound and TigerTypes is unsound, is EitherWay sound?

(g) If LionTypes is sound and TigerTypes is complete, is WhyNotBoth complete?

(h) If LionTypes is sound and TigerTypes is complete, is EitherWay complete?
6. (8 points) (Ruby programming)
Consider the code below, which adds two different methods to the `Array` class that are intended to compute whether an array is sorted, assuming its elements are comparable. (Reminder: The `shift` method returns the 0th element after removing it from the array and moving all other elements “over” one position toward the beginning of the array.)

```ruby
class Array
  def is_sorted1
    n = shift
    all? { |i| n2 = n; n = i; n2 <= i }
  end
  def is_sorted2
    a = Array.new(self)
    n = a.shift
    a.all? { |i| n2 = n; n = i; n2 <= i }
  end
end
```

(a) Does a call to `is_sorted1` always return the correct true/false result?

(b) Does a call to `is_sorted2` always return the correct true/false result?

(c) Explain in 1–2 English sentences why `is_sorted1` and `is_sorted2` are not equivalent.

(d) Provide two tests that show `is_sorted1` and `is_sorted2` are not equivalent. The tests should be identical except one calls `is_sorted1` wherever the other calls `is_sorted2`. Tests can just be top-level Ruby code.
7. (8 points) (Ruby mixins) Recall Ruby’s Comparable mixin defines >, >=, etc.

(a) Add a method `between` to Comparable that takes two arguments and evaluates to `true` if and only if the receiver is strictly greater than the first argument and strictly less than the second argument.

(b) Add a method `less_than_all?` to Comparable that takes one argument and returns `true` if and only if the receiver is (strictly) less than all elements in the argument. The method should `assume` the argument:
   - is `enumerable` (in the sense of the Enumerable mixin)
   - contains only items that can be `compared` (in the sense of the Comparable mixin) to the receiver.
8. (18 points) (Function vs. object-oriented decomposition) Consider this SML code that implements its own lists with a few standard operations:

```sml
datatype 'a mylist = Empty | NonEmpty of 'a * ('a mylist)

exception Bad of string

fun head xs =
  case xs of
    Empty => raise (Bad "head of empty list")
  | NonEmpty(x,_) => x

fun tail xs =
  case xs of
    Empty => raise (Bad "tail of empty list")
  | NonEmpty(_,ys) => ys

fun length xs =
  case xs of
    Empty => 0
  | NonEmpty(_,ys) => 1 + length ys

fun is_longer (xs,ys) = length xs > length ys

fun append (xs,ys) =
  case xs of
    Empty => ys
  | NonEmpty(x,zs) => NonEmpty(x, append(zs,ys))
```

(a) Write an SML function `sum` of type `int mylist -> int` that produces the sum of all the elements in the argument.

Part (b) of this question is on the next page.
(b) Port the SML code, including `sum`, to Ruby, converting the code to use good object-oriented style but otherwise using the same algorithms. Your code should follow all these guidelines:

- Have 3 classes: `Mylist` and two subclasses of it, `Empty` and `NonEmpty`.
- One of your classes should have an `initialize` method.
- Your code should have no mutation (so aliasing versus copying is not an issue).
- You should not use `nil`, `is_a?`, or any standard-library classes (e.g., `Array`).
- Do not define an exception `Bad`; in Ruby, `raise` can just take a string directly for an error message.

The sample solution is 40–45 lines but every line is short and many are `end`. 
9. **(13 points) Subtyping** This problem considers a language like in lecture containing (1) records with mutable fields, (2) functions, and (3) subtyping. Like in lecture, subtyping for records includes width subtyping and permutation subtyping but not depth subtyping, and subtyping for functions includes contravariant arguments and covariant results.

Consider these nine type definitions:

```haskell
type t1 = { h : int}
type t2 = { h : int, f : int}
type t3 = { f : int, g : {a : int}, h : int}
type t4 = { f : int, g : {a : int, b: int}, h : int}
type t5 = { f : int, g : {a : int, b: int, c: int}, h : int}
type t6 = { h : int} -> { h : int}
type t7 = { h : int} -> { f : int, g : {a : int}, h : int}
type t8 = { f : int, g : {a : int}, h : int} -> { h : int}
type t9 = { f : int, g : {a : int}, h : int} -> { f : int, g : {a : int}, h : int}
```

(a) Which of the nine types above are subtypes of `t1`?

(b) Which of the nine types above are subtypes of `t2`?

(c) Which of the nine types above are subtypes of `t3`?

(d) Which of the nine types above are subtypes of `t4`?

(e) Which of the nine types above are subtypes of `t5`?

(f) Which of the nine types above are subtypes of `t6`?

(g) Which of the nine types above are subtypes of `t7`?

(h) Which of the nine types above are subtypes of `t8`?

(i) Which of the nine types above are subtypes of `t9`?
This is an extra sheet of paper in case you need more room for any of the questions. If you use it, please write “see extra page” on the page with the question. In either case, please do NOT remove it from your exam.