Closures

How can functions be evaluated in old environments that aren’t around anymore?
- The language implementation keeps them around as necessary

Can define the semantics of functions as follows:
- A function value has two parts
  - The code (obviously)
  - The environment that was current when the function was defined
- This is a “pair” but unlike ML pairs, you cannot access the pieces
- All you can do is call this “pair”
- This pair is called a function closure
- A call evaluates the code part in the environment part (extended with the function argument)

Example

Demonstrates lexical scope even without higher-order functions:

```
(* 1 *) val x = 1
(* 2 *) fun f y = x + y
(* 3 *) val a = 2
(* 4 *) val b = 3
(* 5 *) val c = f (x + y)
```

- Line 2 defines a function that, when called, evaluates body `x+y` in environment where `x` maps to 1 and `y` maps to the argument
- Call on line 5:
  - Looks up `f` to get the function defined on line 2
  - Evaluates `x+y` in current environment, producing 5
  - Calls the function with 5, which evaluates the body in the old environment, producing 6

Coming up:

Now you know the rule: lexical scope

Next steps:
- (Silly) examples to demonstrate how the rule works with higher-order functions
- Why the other natural rule, dynamic scope, is a bad idea
- Powerful idioms with higher-order functions that use this rule
  - Passing functions to iterators like `filter`
  - Next lecture: Several more idioms

Very important concept

- We know function bodies can use any bindings in scope
- But now that functions can be passed around: In scope where?

Where the function was defined
(not where it was called)

- This semantics is called lexical scope
- There are lots of good reasons for this semantics (why)
  - Discussed after explaining what the semantics is (what)
  - Later in course: implementing it (how)
- Must “get this” for homework, exams, and competent programming
The rule stays the same

A function body is evaluated in the environment where the function was defined (created)
- Extended with the function argument
Nothing changes to this rule when we take and return functions
- But "the environment" may involve nested let-expressions, not just the top-level sequence of bindings
Makes first-class functions much more powerful
- Even if may seem counterintuitive at first

Example: Returning a function

```plaintext
(1+) val x = 1
(2+) fun f y =
(2a+) let val x = y+1
(2b+) in fn z => x+y+z end
(3+) val x = 3
(4+) val y = x+4
(5+) val y = 5
(6+) val z = 4+6
```

- Trust the rule: Evaluating line 4 binds to g to a closure:
  - Code: "take z and have body x+y+z"
  - Environment: "y maps to 4, x maps to 5 (shadowing), ..."
- So this closure will always add 9 to its argument
- So line 6 binds 15 to z

Example: Passing a function

```plaintext
(1+) fun f g = (* call arg with 2 *)
(1a+) let val x = 3
(1b+) in g 2 end
(2+) val x = 4
(3+) fun h y = x + y
(4+) val x = f h
```

- Trust the rule: Evaluating line 3 binds to a closure:
  - Code: "take y and have body x+y"
  - Environment: "x maps to 4, g maps to a closure, ..."
- So this closure will always add 4 to its argument
- So line 4 binds 6 to x
- Line 1a is as stupid and irrelevant as it should be

Why lexical scope

- Lexical scope: use environment where function is defined
- Dynamic scope: use environment where function is called

Decades ago, both might have been considered reasonable, but now we know lexical scope makes much more sense

Here are three precise, technical reasons
- Not a matter of opinion

Why lexical scope?

1. Function meaning does not depend on variable names used
   
   Example: Can change body of f to use q everywhere instead of x
     - Lexical scope: it cannot matter
     - Dynamic scope: depends how result is used

   ```plaintext
   fun f y =
   let val x = y+1
   in fn x => x+y+x end
   ```

   Example: Can remove unused variables:
   - Dynamic scope: but maybe some g uses it (weird)

   ```plaintext
   fun f y =
   let val x = y+1
   in fn x => x+y+x end
   ```

Why lexical scope?

2. Functions can be type-checked and reasoned about where defined

Example: Dynamic scope tries to add a string and an unbound variable to 6

```plaintext
val x = 1
fun f y =
  let val x = y+1
  in fn z => x+z + x end
val x = "hi"
val g = f 7
val z = g 4
```
Why lexical scope?

3. Closures can easily store the data they need
   – Many more examples and idioms to come

```haskell
fun greaterThanX x = fn y => y > x
fun filter [f, xs] = case xs of
  [] => []
| x:xs => if f x then x :: (filter(f, xs))
               else filter(f, xs)
fun noNegatives xs = filter(greaterThanX -1, xs)
fun allGreater(xs, n) = filter(fn x => x > n, xs)
```

Does dynamic scope exist?

- Lexical scope for variables is definitely the right default
  – Very common across languages
- Dynamic scope is occasionally convenient in some situations
  – So some languages (e.g., Racket) have special ways to do it
  – But most do not bother
- If you squint some, exception handling is more like dynamic scope:
  – raise e transfers control to the current innermost handler
  – Does not have to be syntactically inside a handle expression
    (and usually is not)

When things evaluate

Things we know:
- A function body is not evaluated until the function is called
- A function body is evaluated every time the function is called
- A variable binding evaluates its expression when the binding is evaluated, not every time the variable is used

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- Not so worried about performance, but good example to emphasize the semantics of functions

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Recomputation

These both work and rely on using variables in the environment

```haskell
fun allShorterThan1(xs, s) = filter(fn x => String.size x < String.size s, xs)
fun allShorterThan2(xs, s) = let val i = String.size s
                          in filter(fn x => String.size x < i, xs) end
```

Another famous function: Fold

fold (and synonyms: reduce, inject, etc.) is another very famous iterator over recursive structures

Accumulates an answer by repeatedly applying f to answer so far
- fold(f,acc,[x1,x2,x3,x4]) computes f(f(f(f(acc,x1),x2),x3),x4)

```haskell
fun fold (f, acc, xs) = case xs of
  [] => acc
| x:xs => fold(f, f(acc,x), xs)
val fold = fn : ('a * 'b -> 'a) * 'a * 'b list -> 'a
```

Why iterators again?

- These “iterator-like” functions are not built into the language
  – Just a programming pattern
- Though many languages have built-in support, which often allows stopping early without resorting to exceptions
- This pattern separates recursive traversal from data processing
  – Can reuse same traversal for different data processing
  – Can reuse same data processing for different data structures
  – In both cases, using common vocabulary concisely communicates intent
Examples with fold

These are useful and do not use “private data”

```haskell
fun f1 xs = fold((fn (x,y) => x+y), 0, xs)
fun f2 xs = fold((fn (x,y) => x andalso y>=0), true, xs)
```

These are useful and do use “private data”

```haskell
fun f3 (xs,hi,lo) = fold((fn (x,y) => x + (if y >= lo andalso y <= hi then 1 else 0)), 0, xs)
fun f4 (g, xs) = fold((fn (x,y) => x andalso g y), true, xs)
```

Iterators made better

- Functions like `map`, `filter`, and `fold` are much more powerful thanks to closures and lexical scope
- Function passed in can use any “private” data in its environment
- Iterator “doesn’t even know the data is there” or what type it has