Last Topic of Unit

More careful look at what “two pieces of code are equivalent” means

- Fundamental software-engineering idea
- Made easier with
  - Abstraction (hiding things)
  - Fewer side effects

Not about any “new ways to code something up”

Equivalence

Must reason about “are these equivalent” all the time

- The more precisely you think about it the better
- Code maintenance: Can I simplify this code?
- Backward compatibility: Can I add new features without changing how any old features work?
- Optimization: Can I make this code faster?
- Abstraction: Can an external client tell I made this change?

To focus discussion: When can we say two functions are equivalent, even without looking at all calls to them?

- May not know all the calls (e.g., we are editing a library)

A definition

Two functions are equivalent if they have the same “observable behavior” no matter how they are used anywhere in any program

Given equivalent arguments, they:

- Produce equivalent results
- Have the same (non-)termination behavior
- Mutate (non-local) memory in the same way
- Do the same input/output
- Raise the same exceptions

Notice it is much easier to be equivalent if:

- There are fewer possible arguments, e.g., with a type system and abstraction
- We avoid side-effects: mutation, input/output, and exceptions

Example

Since looking up variables in ML has no side effects, these two functions are equivalent:

```plaintext
fun f x = x + x
val y = 2
fun f x = y * x
```

But these next two are not equivalent in general: it depends on what is passed for \( f \)

- Are equivalent if argument for \( f \) has no side-effects

```plaintext
fun g (f, x) = (f x) + (f x)  
\neq  
val y = 2
fun g (f, x) = y * (f x)
```

- Example: \( g \) divides by 0 and \( h \) mutates a top-level reference
- Great reason for “pure” functional programming

Another example

These are equivalent only if functions bound to \( g \) and \( h \) do not raise exceptions or have side effects (printing, updating state, etc.)

- Again: pure functions make more things equivalent

```plaintext
fun f x = let
    val y = g x
    val s = h x
    in
    (y, s)
end

fun f x = let
    val y = g x
    val s = h x
    in
    (y, s)
end
```

- Example: \( g \) writes to a reference that \( h \) reads from
One that really matters
Once again, turning the left into the right is great but only if the functions are pure:

\[
\text{map } (f \circ g) \, \text{xs} = \text{map } f \, (\text{map } g \, \text{xs})
\]

Syntactic sugar
Using or not using syntactic sugar is always equivalent - by definition, else not syntactic sugar

Example:

But be careful about evaluation order

Standard equivalences
Three general equivalences that always work for functions - In (any?) decent language

1. Consistently rename bound variables and uses

But notice you can’t use a variable name already used in the function body to refer to something else

2. Use a helper function or do not

But notice you need to be careful about environments

3. Unnecessary function wrapping

But notice that if you compute the function to call and that computation has side-effects, you have to be careful

One more
If we ignore types, then ML let-bindings can be syntactic sugar for calling an anonymous function:

\[
\text{let } \text{val } x = e_1 \text{ in } \text{fn } x \Rightarrow e_2 \text{ end} = (\text{fn } x \Rightarrow e_2) \, e_1
\]

- These both evaluate \(e_1\) to \(v_1\), then evaluate \(e_2\) in an environment extended to map \(x\) to \(v_1\)
- So exactly the same evaluation of expressions and result

But in ML, there is a type-system difference:

- \(x\) on the left can have a polymorphic type, but not on the right
- \(x\) can always go from right to left
- If \(x\) need not be polymorphic, can go from left to right
What about performance?

According to our definition of equivalence, these two functions are equivalent, but we learned one is awful

- (Actually we studied this before pattern-matching)

```plaintext
fun max xs = case xs of [] => raise Empty | x:xs' => x | x::xs' => if x > max xs' then x else max xs'
```

Different definitions for different jobs

- **PL Equivalence (341):** given same inputs, same outputs and effects
  - Good: Lets us replace bad `max` with good `max`
  - Bad: Ignores performance in the extreme
- **Asymptotic equivalence (332):** Ignore constant factors
  - Good: Focus on the algorithm and efficiency for large inputs
  - Bad: Ignores “four times faster”
- **Systems equivalence (333):** Account for constant overheads, performance tune
  - Good: Faster means different and better
  - Bad: Beware overtuning on “wrong” (e.g., small) inputs; definition does not let you “swap in a different algorithm”

Claim: Computer scientists implicitly (?) use all three every (?) day