Please do not turn the page until 8:30.

Rules:

- The exam is closed-book, closed-note, etc. except for two sides of one 8.5x11in piece of paper.
- Please stop promptly at 10:20.
- There are 126 points, distributed unevenly among 7 questions (most all with multiple parts):
- The exam is printed double-sided.
- Put your name on every page.

Advice:

- Read questions carefully. Understand a question before you start writing.
- Write down thoughts and intermediate steps so you can get partial credit. But clearly indicate what is your final answer.
- We will be scanning the exam to grade it. If you put an answer to a question in a non-intuitive spot, leave us a note. When in doubt, labeling where your answer is will never hurt.
- The questions are not necessarily in order of difficulty. Skip around!. Make sure you get to all the questions, but do not do the exam in order.
- If you have questions, ask.
- Relax. You are here to learn.
1. (21 points) Suppose you want to define data structures a bit differently in Racket. Instead of just \texttt{cons} cells, you want \texttt{tcons} (or \textit{triple cons}) cells, with 3 parts.

You might accomplish this with a struct definition like so:

\begin{verbatim}
(struct tcons (fst snd trd) #:transparent)
\end{verbatim}

(a) Suppose that a proper \texttt{tlist} made of \texttt{tcons} cells is just like a normal list made of \texttt{cons} cells, except that \texttt{fst} and \texttt{snd} both hold list elements, and \texttt{trd} holds the rest of the list. Implement \texttt{tlist?}, which returns true if and only if its argument is a proper \texttt{tlist}.

(b) Implement \texttt{map-tlist}, which will apply a given function to every element in a proper \texttt{tlist}, and return a resulting \texttt{tlist}.

(c) Implement \texttt{flatten}, which will take \textit{any} structure represented with possibly arbitrarily nested \texttt{tcons} cells, and flatten it out. For instance:

\begin{verbatim}
(tcons (tcons 1 2 3) 4 (tcons 5 6 (tcons 7 8 9)))
\end{verbatim}

should be flattened out to a result \begin{verbatim}
(list 1 2 3 4 5 6 7 8 9)
\end{verbatim}. Note that the result should be a \texttt{list} and \textit{not} a \texttt{tlist}. 
2. (8 points)

Consider the following definition in Racket:

\[(\text{define (foo x y)}\)
\n\ (\text{begin})
\n\ (\text{print (mcar x)})
\n\ (\text{print (mcar y)})
\n\ (\text{set-mcar! x (+ (mcar x) 1)})
\n\ (\text{set-mcar! y (- (mcar y) 1)})
\n\ (\text{print (mcar x)})
\n\ (\text{print (mcar y)})\)\]

Write a Racket expression which contains a call to \texttt{foo}, which results in all four print statements printing the number 2.
3. (Racket Streams) (23 points)

As in class, we define a stream to be a thunk which returns a pair when called, the cdr of which is itself a stream.

(a) Write a function `sparse-stream` which takes three arguments: default which is a single element, and two stream arguments `elems` and `indices`. `sparse-stream` should return a stream, which yields elements from `elems` in order, spaced out with repetitions of the default element. The number of default elements to insert before the next element from `elems` is given by the number retrieved from the `indices` stream.

For example, suppose the stream

\[ \text{nats} = 0,1,2,3,4,5 \ldots \]

is already defined. Then, `(sparse-stream -8 nats nats)` should result in a stream which looks like:

\[ 0,1,-8,2,-8,3,-8,4,-8,5,-8,6 \ldots \]

(b) Write a function `id-stream` which takes one stream argument, and results in a stream which is identical to the argument. Use one call of `sparse-stream` to implement `id-stream`: do not simply return the argument.

(c) Suppose you have a result which you know was made by `sparse-stream`, but you’ve lost the original `indices` argument. Write a function `recover-indices` which takes two arguments: a default element and a stream s. Assuming that the stream s is the result of a call to `sparse-stream`, and that the original `elems` argument to the call to `sparse-stream` never yielded the default element, your result should be identical to the original `indices` argument to `sparse-stream`. 

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Name: ________________________________
4. (21 points) Below is the definition of TPL (Tiny Programming Language) abstract syntax. TPL is identical to MUPL, but with fewer expressions.

(struct var (string) #:transparent) ;; a variable, e.g., (var "foo")
(struct int (num) #:transparent) ;; a constant number, e.g., (int 17)
(struct add (e1 e2) #:transparent) ;; add two expressions
(struct fun (nameopt formal body) #:transparent) ;; a recursive(?) 1-argument function
(struct call (funexp actual) #:transparent) ;; function call
(struct mlet (var e body) #:transparent) ;; a local binding (let var = e in body)
(struct apair (e1 e2) #:transparent) ;; make a new pair

(a) Implement an optimization function precompute-add which will take a TPL program, and return a TPL program which will always evaluate to the same answer, but has any addition expressions which consist of two constants replaced by their answer. For instance, the program (add (int 3) (int 2)) should be replaced by (int 5) when optimized by precompute-add.

(b) Suppose you want to add a type system to TPL. Implement the simplest complete type checker for TPL (a type checker takes a TPL program and returns a bool). For type checking, #t means that the program typechecks, and #f means that it does not.

(c) Write an expression in TPL which, when evaluated, acts like stream, giving the natural numbers in order (i.e. 0,1,2,3,4...). Since we can’t call a function with 0 arguments, instead assume that a stream is called with munit in order to get the next result.
5. **(12 points)**

You are developing a kitchen simulation program in Ruby with classes to model various different kinds of physical containers: Jars, Bottles, Cups, Mugs, Glasses, Cannisters, Bags, Boxes, Cans, Trays, Cartons, Bowls, and more. In development, you’ve implemented quite the complicated system, which is perhaps too complicated to really be useful. The last time you ran the program, you ended up with boxes full of hot coffee, and a bunch of granulated sugar in bottles. None of this makes sense.

In order to understand what your errant program is currently doing, you decided to extract some debug information from all of the different classes which implement your different containers.

Your code is structured such that there is one `Container` class, and each specific container (e.g. Mug) is a subclass of `Container`.

(a) Write a mixin `Debug` which provides one method: `describe`. This method should take no arguments, and return a string which is the name of the runtime class of the object which `describe` is called on, as well as the result of the `contents` method (which returns what, if anything, is contained in a container).

(b) Which class(es) should include the mixin `Debug` such that every container will be able to describe itself accurately?

(c) You manage to track down what you were doing with boxes and bottles: a simple typo on one line. However, you are now running into another issue with some of your containers, but this seems limited to those meant for drinking (e.g. Mug or Glass). Unfortunately, you don’t have a `DrinkingContainer` class anywhere. Without adding any classes or changing the existing class hierarchy in any way, describe your strategy for changing the definition of `describe` for drinking containers, so that it will print out the type and volume of liquid contained inside (accessible by the `total_volume` and `percent_full` methods. Make sure to mention how you will make sure your new `describe` will be called, instead of your old definition. Explain in at most 2 sentences.
6. (25 points) For this question, recall the game Rock Paper Scissors! If you’ve never played it before, all you need to know is that paper beats (covers) rock, rock beats (smashes) scissors, and scissors beats (cuts) paper.

Implement the classic Rock Paper Scissors! game using double dispatch in Ruby. Each object of the Rock, Paper, and Scissors classes should provide a play method, which takes one argument, and returns the winner. In the case of a tie, either result may be returned.

Example:

```ruby
r = Rock.new
p = Paper.new
p.play(r) #result should be p
r.play(p) #result should be p
```

Your code below:

```ruby
class Rock
end

class Scissors
end

class Paper
end
```
(continued) Now, implement the same functionality in SML. Your code should be one pattern match only, and you should use the minimum number of cases possible.

```sml
datatype move = Rock | Paper | Scissors

(* play : move -> move -> move *)
fun play left_move right_move =
```

Now, suppose that you want to extend your game to the classic Rock Paper Scissors Lizard Spock! Each move wins against 2 others, and loses against 2 others, but the probability that you and your opponent pick the same play is diminished. For reference, Scissors cuts Paper covers Rock crushes Lizard poisons Spock smashes Scissors decapitates Lizard eats Paper disproves Spock vaporizes Rock crushes Scissors. Would you rather extend your double dispatch implementation in Ruby, or your functional implementation in SML? Pick one, and explain why.
7. (Subtyping) (16 points) Consider a language similar to the language we saw in lecture, with (1) records with *mutable* fields, (2) higher order functions, and (3) subtyping. Indicate whether the following subtypings are sound. No need for an explanation. We use \( A <: B \) to indicate that \( A \) is a subtype of \( B \).

(a) \{f1 : int\} <: \{f1 : int\}
(b) \{f1 : int, f3 : string\} <: \{f3 : string, f2 : bool, f1 : int\}
(c) \{f1 : \{x : int, y : int\}, f2 : \{x : int\} \} <: \\
\{f2 : \{x : int, y : int\}, f1 : \{x : int, z : int\}\}
(d) {} <: \{x : int, y : int, z : int\}
(e) \{x : int, y : int\} -> \{x : int, y : int\} <: \\
\{x : int, y : int\} -> \{x : int\}
(f) \{x : int, y : int\} -> \{x : int, y : int\} <: \\
\{x : int\} -> \{x : int, y : int\}
(g) \{a : \{x : int\} -> \{r : int\}\} <: \{a : \{x : int, y : int\} -> \{r : int\}\}
(h) \{x : int\} -> \{y : int\} -> \{x : int, y : int, z : int\} <: \\
\{x : int, y : int\} -> \{x : int, y : int\} -> \{x : int, y : int\}