CSE341: Programming Languages

Lecture 7
First-Class Functions

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What is functional programming?

“Functional programming” can mean a few different things:

1. Avoiding mutation in most/all cases (done and ongoing)
2. Using functions as values (this unit)

...  
• Style encouraging recursion and recursive data structures  
• Style closer to mathematical definitions  
• Programming idioms using laziness (later topic, briefly)  
• Anything not OOP or C? (not a good definition)

Not sure a definition of “functional language” exists beyond “makes functional programming easy / the default / required”
   - No clear yes/no for a particular language
First-class functions

• **First-class functions**: Can use them *wherever* we use values
  – Functions are values too
  – Arguments, results, parts of tuples, bound to variables, carried by datatype constructors or exceptions, …

```plaintext
fun double x = 2*x
fun incr x = x+1
val a_tuple = (double, incr, double(incre 7))
```

• Most common use is as an argument / result of another function
  – Other function is called a *higher-order function*
  – Powerful way to *factor out* common functionality
Function Closures

• *Function closure*: Functions can use bindings from outside the function definition (in scope where function is defined)
  – Makes first-class functions *much* more powerful
  – Will get to this feature in a bit, after simpler examples

• Distinction between terms *first-class functions* and *function closures* is not universally understood
  – Important conceptual distinction even if terms get muddled
Onward

The next week:
- How to use first-class functions and closures
- The precise semantics
- Multiple powerful idioms
Functions as arguments

- We can pass one function as an argument to another function
  - Not a new feature, just never thought to do it before

\[
\text{fun } f\ (g,\ldots) = \ldots g\ (\ldots) \ldots \\
\text{fun } h_1 \ldots = \ldots \\
\text{fun } h_2 \ldots = \ldots \\
\ldots f(h_1,\ldots) \ldots f(h_2,\ldots) \ldots
\]

- Elegant strategy for factoring out common code
  - Replace \( N \) similar functions with calls to 1 function where you pass in \( N \) different (short) functions as arguments

[See the code file for this lecture]
Example

Can reuse \texttt{n\_times} rather than defining many similar functions

- Computes \( f(f(...f(x))) \) where number of calls is \( n \)

\begin{verbatim}
fun n_times (f,n,x) = 
  if n=0
  then x
  else f (n_times(f,n-1,x))

fun double x = x + x
fun increment x = x + 1
val x1 = n_times(double,4,7)
val x2 = n_times(increment,4,7)
val x3 = n_times(tl,2,[4,8,12,16])

fun double_n_times (n,x) = n_times(double,n,x)
fun nth_tail (n,x) = n_times(tl,n,x)
\end{verbatim}
Relation to types

• Higher-order functions are often so “generic” and “reusable” that they have polymorphic types, i.e., types with type variables

• But there are higher-order functions that are not polymorphic

• And there are non-higher-order (first-order) functions that are polymorphic

• Always a good idea to understand the type of a function, especially a higher-order function
Types for example

fun n_times (f,n,x) = 
  if n=0
  then x
  else f (n_times(f,n-1,x))

- `val n_times : ('a -> 'a) * int * 'a -> 'a`
  - Simpler but less useful: `(int -> int) * int * int -> int`

- Two of our examples instantiated 'a with int
- One of our examples instantiated 'a with int list
- This polymorphism makes n_times more useful

- Type is inferred based on how arguments are used (later lecture)
  - Describes which types must be exactly something (e.g., `int`) and
    which can be anything but the same (e.g., `'a`)
Polymorphism and higher-order functions

- Many higher-order functions are polymorphic because they are so reusable that some types, “can be anything”

- But some polymorphic functions are not higher-order
  - Example: `len : 'a list -> int`

- And some higher-order functions are not polymorphic
  - Example: `times_until_0 : (int -> int) * int -> int`

```ml
fun times_until_zero (f, x) = 
  if x=0 then 0 else 1 + times_until_zero(f, f x)
```

Note: Would be better with tail-recursion
**Toward anonymous functions**

- Definitions unnecessarily at top-level are still poor style:
  
  ```haskell
  fun trip x = 3*x
  fun triple_n_times (f,x) = n_times(trip,n,x)
  ```

- So this is better (but not the best):
  
  ```haskell
  fun triple_n_times (f,x) =
  let fun trip y = 3*y
  in
  n_times(trip,n,x)
  end
  ```

- And this is even smaller scope
  - It makes sense but looks weird (poor style; see next slide)

  ```haskell
  fun triple_n_times (f,x) =
  n_times(let fun trip y = 3*y in trip end, n, x)
  ```
Anonymous functions

• This does not work: A function binding is not an expression

```scala
fun triple_n_times (f,x) = 
n_times((fun trip y = 3*y), n, x)
```

• This is the best way we were building up to: an expression form for anonymous functions

```scala
fun triple_n_times (f,x) = 
n_times((fn y => 3*y), n, x)
```

  – Like all expression forms, can appear anywhere
  – Syntax:
    • `fn` not `fun`
    • `=>` not `=
    • no function name, just an argument pattern
Using anonymous functions

• Most common use: Argument to a higher-order function
  – Don’t need a name just to pass a function

• But: Cannot use an anonymous function for a recursive function
  – Because there is no name for making recursive calls
  – If not for recursion, `fun` bindings would be syntactic sugar for `val` bindings and anonymous functions

\[
\text{fun } \text{triple } x = 3 \times x
\]
\[
\text{val } \text{triple } = \text{fn } y \Rightarrow 3 \times y
\]
A style point

Compare:

\[ \text{if } x \text{ then true else false} \]

With:

\[ (\text{fn } x \Rightarrow f \ x) \]

So don’t do this:

\[ \text{n\_times}((\text{fn } y \Rightarrow \text{tl } y),3,xs) \]

When you can do this:

\[ \text{n\_times(tl,3,xs)} \]
Map

fun map (f,xs) =
  case xs of
  [] => []
  | x::xs' => (f x)::(map(f,xs'))

val map : ('a -> 'b) * 'a list -> 'b list

Map is, without doubt, in the “higher-order function hall-of-fame”
  – The name is standard (for any data structure)
  – You use it all the time once you know it: saves a little space, but more importantly, communicates what you are doing
  – Similar predefined function: List.map
    • But it uses currying (coming soon)
Filter

fun filter (f,xs) =
  case xs of
    [] => []
    | x::xs' => if f x
       then x::(filter(f,xs'))
       else filter(f,xs')

val filter : ('a -> bool) * 'a list -> 'a list

Filter is also in the hall-of-fame
  – So use it whenever your computation is a filter
  – Similar predefined function: List.filter
    • But it uses currying (coming soon)
Generalizing

Our examples of first-class functions so far have all:
– Taken one function as an argument to another function
– Processed a number or a list

But first-class functions are useful anywhere for any kind of data
– Can pass several functions as arguments
– Can put functions in data structures (tuples, lists, etc.)
– Can return functions as results
– Can write higher-order functions that traverse your own data structures

Useful whenever you want to abstract over “what to compute with”
– No new language features
Returning functions

• Remember: Functions are first-class values
  – For example, can return them from functions

• Silly example:

```plaintext
fun double_or_triple f = 
  if f 7
  then fn x => 2*x
  else fn x => 3*x
```

Has type \((\texttt{int} \rightarrow \texttt{bool}) \rightarrow (\texttt{int} \rightarrow \texttt{int})\)

But the REPL prints \((\texttt{int} \rightarrow \texttt{bool}) \rightarrow \texttt{int} \rightarrow \texttt{int}\)
because it never prints unnecessary parentheses and
\(t1 \rightarrow t2 \rightarrow t3 \rightarrow t4\) means \(t1 \rightarrow (t2 \rightarrow (t3 \rightarrow t4))\)
Other data structures

• Higher-order functions are not just for numbers and lists

• They work great for common recursive traversals over your own data structures (datatype bindings) too

• Example of a higher-order *predicate*:
  
  – Are all constants in an arithmetic expression even numbers?
  
  – Use a more general function of type
    
    `(int -> bool) * exp -> bool`
  
    – And call it with `(fn x => x mod 2 = 0)`