What is functional programming?

“Functional programming” can mean a few different things:

1. Avoiding mutation in most/all cases (done and ongoing)
2. Using functions as values (this unit)

... 

- Style encouraging recursion and recursive data structures
- Style closer to mathematical definitions
- Programming idioms using laziness (later topic, briefly)
- Anything not OOP or C? (not a good definition)

Not sure a definition of “functional language” exists beyond “makes functional programming easy / the default / required”

First-class functions

- **First-class functions**: Can use them wherever we use values
  - Functions are values too
  - Arguments, results, parts of tuples, bound to variables, carried by datatype constructors or exceptions, ...

```plaintext
fun double x = 2*x
fun incr x = x+1
val a_tuple = (double, incr, double(incr 7))
```

- Most common use is as an argument / result of another function
  - Other function is called a higher-order function
  - Powerful way to factor out common functionality

Function Closures

- **Function closure**: Functions can use bindings from outside the function definition (in scope where function is defined)
  - Makes first-class functions much more powerful
  - Will get to this feature in a bit, after simpler examples

- Distinction between terms first-class functions and function closures is not universally understood
  - Important conceptual distinction even if terms get muddled
Onward

The next week:
- How to use first-class functions and closures
- The precise semantics
- Multiple powerful idioms

Functions as arguments

- We can pass one function as an argument to another function
  - Not a new feature, just never thought to do it before

  ```
  fun f (g, ...) = ... g (...) ...
  fun h1 ... = ...
  fun h2 ... = ...
  ... f(h1,...) ... f(h2,...) ...
  ```

- Elegant strategy for factoring out common code
  - Replace $N$ similar functions with calls to 1 function where you pass in $N$ different (short) functions as arguments

  [See the code file for this lecture]

Example

Can reuse `n_times` rather than defining many similar functions
- Computes $f(f(...f(x)))$ where number of calls is $n$

  ```
  fun n_times (f,n,x) = 
    if n=0 
      then x 
      else f (n_times(f,n-1,x))
  fun double x = x + x
  fun increment x = x + 1
  val x1 = n_times(double,4,7)
  val x2 = n_times(increment,4,7)
  val x3 = n_times(tl,2,[4,8,12,16])
  fun double_n_times (n,x) = n_times(double,n,x)
  fun nth_tail (n,x) = n_times(tl,n,x)
  ```

Relation to types

- Higher-order functions are often so “generic” and “reusable” that they have polymorphic types, i.e., types with type variables

- But there are higher-order functions that are not polymorphic

- And there are non-higher-order (first-order) functions that are polymorphic

- Always a good idea to understand the type of a function, especially a higher-order function
Types for example

```ocaml
fun n_times (f, n, x) = 
  if n=0 then x 
  else f (n_times(f, n-1, x))
```

- `val n_times : ('a -> 'a) * int * 'a -> 'a
- Simpler but less useful: (int -> int) * int * int -> int
- Two of our examples instantiated 'a with int
- One of our examples instantiated 'a with int list
- This polymorphism makes n_times more useful
- Type is inferred based on how arguments are used (later lecture)
- Describes which types must be exactly something (e.g., int) and which can be anything but the same (e.g., 'a)

Polymorphism and higher-order functions

- Many higher-order functions are polymorphic because they are so reusable that some types, "can be anything"
- But some polymorphic functions are not higher-order
- Example: `len : 'a list -> int
- And some higher-order functions are not polymorphic
- Example: `times_until_zero : (int -> int) * int -> int

```ocaml
fun times_until_zero (f, x) = 
  if x=0 then 0 else 1 + times_until_zero(f, f x)
```

Note: Would be better with tail-recursion

Toward anonymous functions

- Definitions unnecessarily at top-level are still poor style:

```ocaml
fun trip x = 3*x 
fun triple_n_times (f, x) = n_times(trip, n, x)
```

- So this is better (but not the best):

```ocaml
fun triple_n_times (f, x) = 
  let fun trip y = 3*y 
  in 
    n_times(trip, n, x) 
  end
```

- And this is even smaller scope
  - It makes sense but looks weird (poor style; see next slide)

```ocaml
fun triple_n_times (f, x) = 
  n_times(let fun trip y = 3*y in trip end, n, x)
```

Anonymous functions

- This does not work: A function binding is not an expression

```ocaml
fun triple_n_times (f, x) = 
  n_times((fun trip y = 3*y), n, x)
```

- This is the best way we were building up to: an expression form for anonymous functions

```ocaml
fun triple_n_times (f, x) = 
  n_times((fn y => 3*y), n, x)
```

- Like all expression forms, can appear anywhere
- Syntax:
  - fn not fun
  - => not =
  - no function name, just an argument pattern
**Using anonymous functions**

- Most common use: Argument to a higher-order function
  - Don’t need a name just to pass a function

- But: Cannot use an anonymous function for a recursive function
  - Because there is no name for making recursive calls
  - If not for recursion, `fun` bindings would be syntactic sugar for `val` bindings and anonymous functions

```haskell
fun triple x = 3*x
val triple = fn y => 3*y
```

**A style point**

Compare:

```haskell
if x then true else false
```

With:

```haskell
(fn x => f x)
```

So don’t do this:

```haskell
n_times((fn y => tl y),3,xs)
```

When you can do this:

```haskell
n_times(tl,3,xs)
```

**Map**

```haskell
fun map (f,xs) = 
  case xs of 
  [] => [] 
  | x::xs' => (f x)::(map(f,xs'))
val map : ('a -> 'b) * 'a list -> 'b list
```

Map is, without doubt, in the "higher-order function hall-of-fame"

- The name is standard (for any data structure)
- You use it *all the time* once you know it: saves a little space, but more importantly, *communicates what you are doing*
- Similar predefined function: `List.map`
  - But it uses currying (coming soon)

**Filter**

```haskell
fun filter (f,xs) = 
  case xs of 
  [] => [] 
  | x::xs' => if f x 
  then x::(filter(f,xs')) 
  else filter(f,xs')
val filter : ('a -> bool) * 'a list -> 'a list
```

Filter is also in the hall-of-fame

- So use it whenever your computation is a filter
- Similar predefined function: `List.filter`
  - But it uses currying (coming soon)
Generalizing

Our examples of first-class functions so far have all:
– Taken one function as an argument to another function
– Processed a number or a list

But first-class functions are useful anywhere for any kind of data
– Can pass several functions as arguments
– Can put functions in data structures (tuples, lists, etc.)
– Can return functions as results
– Can write higher-order functions that traverse your own data structures

Useful whenever you want to abstract over “what to compute with”
– No new language features

Returning functions

• Remember: Functions are first-class values
  – For example, can return them from functions

• Silly example:

  fun double_or_triple f =
  if f 7
  then fn x => 2*x
  else fn x => 3*x

  Has type (int -> bool) -> (int -> int)

But the REPL prints (int -> bool) -> int -> int
because it never prints unnecessary parentheses and
(t1 -> t2 -> t3 -> t4) means t1->(t2->(t3->t4))

Other data structures

• Higher-order functions are not just for numbers and lists

• They work great for common recursive traversals over your own data structures (datatype bindings) too

• Example of a higher-order predicate:
  – Are all constants in an arithmetic expression even numbers?
  – Use a more general function of type
    (int -> bool) * exp -> bool
  – And call it with (fn x => x mod 2 = 0)