



PAUL G. ALLEN SCHOOL
OF COMPUTER SCIENCE & ENGINEERING

CSE341: Programming Languages

Lecture 2

Functions, Pairs, Lists

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Autumn 2018

Function definitions

Functions: the most important building block in the whole course

- Like Java methods, have arguments and result
- But no classes, **this**, **return**, etc.

Example *function binding*:

```
(* Note: correct only if y>=0 *)  
  
fun pow (x : int, y : int) =  
  if y=0  
  then 1  
  else x * pow(x,y-1)
```

Note: The *body* includes a (recursive) *function call*: `pow(x,y-1)`

Example, extended

```
fun pow (x : int, y : int) =  
  if y=0  
  then 1  
  else x * pow(x,y-1)
```

```
fun cube (x : int) =  
  pow (x,3)
```

```
val sixtyfour = cube 4
```

```
val fortytwo = pow(2,2+2) + pow(4,2) + cube(2) + 2
```

Some gotchas

Three common “gotchas”

- Bad error messages if you mess up function-argument syntax
- The use of `*` in type syntax is not multiplication
 - Example: `int * int -> int`
 - In expressions, `*` is multiplication: `x * pow(x, y-1)`
- Cannot refer to later function bindings
 - That’s simply ML’s rule
 - Helper functions must come before their uses
 - Need special construct for *mutual recursion* (later)

Recursion

- If you're not yet comfortable with recursion, you will be soon 😊
 - Will use for most functions taking or returning lists
- “Makes sense” because calls to same function solve “simpler” problems
- Recursion more powerful than loops
 - We won't use a single loop in ML
 - Loops often (not always) obscure simple, elegant solutions

Function bindings: 3 questions

- Syntax: `fun x0 (x1 : t1, ... , xn : tn) = e`
 - (Will generalize in later lecture)
- Evaluation: **A function is a value!** (No evaluation yet)
 - Adds **x0** to environment so *later* expressions can *call* it
 - (Function-call semantics will also allow recursion)
- Type-checking:
 - Adds binding **x0 : (t1 * ... * tn) -> t** if:
 - Can type-check body **e** to have type **t** in the static environment containing:
 - “Enclosing” static environment (earlier bindings)
 - **x1 : t1, ..., xn : tn** (arguments with their types)
 - **x0 : (t1 * ... * tn) -> t** (for recursion)

More on type-checking

```
fun x0 (x1 : t1, ... , xn : tn) = e
```

- New kind of type: $(t_1 * \dots * t_n) \rightarrow t$
 - Result type on right
 - The overall type-checking result is to give x_0 this type in rest of program (unlike Java, not for earlier bindings)
 - Arguments can be used only in e (unsurprising)
- Because evaluation of a call to x_0 will return result of evaluating e , the return type of x_0 is the type of e
- The type-checker “magically” figures out t if such a t exists
 - Later lecture: Requires some cleverness due to recursion
 - More magic after hw1: Later can omit argument types too

Function Calls

A new kind of expression: 3 questions

Syntax: $e_0 (e_1, \dots, e_n)$

- (Will generalize later)
- Parentheses optional if there is exactly one argument

Type-checking:

If:

- e_0 has some type $(t_1 * \dots * t_n) \rightarrow t$
- e_1 has type t_1 , ..., e_n has type t_n

Then:

- $e_0(e_1, \dots, e_n)$ has type t

Example: `pow(x, y-1)` in previous example has type `int`

Function-calls continued

$e_0(e_1, \dots, e_n)$

Evaluation:

1. (Under current dynamic environment,) evaluate e_0 to a function $\text{fun } x_0 (x_1 : t_1, \dots, x_n : t_n) = e$
 - Since call type-checked, result *will be* a function
2. (Under current dynamic environment,) evaluate arguments to values v_1, \dots, v_n
3. Result is evaluation of e in an environment extended to map x_1 to v_1, \dots, x_n to v_n
 - (“An environment” is actually the environment where the function was defined, and includes x_0 for recursion)

Tuples and lists

So far: numbers, booleans, conditionals, variables, functions

- Now ways to build up data with multiple parts
- This is essential
- Java examples: classes with fields, arrays

Now:

- *Tuples*: fixed “number of pieces” that may have different types

Then:

- *Lists*: any “number of pieces” that all have the same type

Later:

- Other more general ways to create compound data

Pairs (2-tuples)

Need a way to *build* pairs and a way to *access* the pieces

Build:

- Syntax: $(e1, e2)$
- Evaluation: Evaluate $e1$ to $v1$ and $e2$ to $v2$; result is $(v1, v2)$
 - A pair of values is a value
- Type-checking: If $e1$ has type τ_a and $e2$ has type τ_b , then the pair expression has type $\tau_a * \tau_b$
 - A new kind of type

Pairs (2-tuples)

Need a way to *build* pairs and a way to *access* the pieces

Access:

- Syntax: **#1 e** and **#2 e**
- Evaluation: Evaluate e to a pair of values and return first or second piece
 - Example: If e is a variable x , then look up x in environment
- Type-checking: If e has type $\tau_a * \tau_b$, then **#1 e** has type τ_a and **#2 e** has type τ_b

Examples

Functions can take and return pairs

```
fun swap (pr : int*bool) =  
  (#2 pr, #1 pr)
```

```
fun sum_two_pairs (pr1 : int*int, pr2 : int*int) =  
  (#1 pr1) + (#2 pr1) + (#1 pr2) + (#2 pr2)
```

```
fun div_mod (x : int, y : int) =  
  (x div y, x mod y)
```

```
fun sort_pair (pr : int*int) =  
  if (#1 pr) < (#2 pr)  
  then pr  
  else (#2 pr, #1 pr)
```

Tuples

Actually, you can have *tuples* with more than two parts

– A new feature: a generalization of pairs

- (e_1, e_2, \dots, e_n)
- `ta * tb * ... * tn`
- `#1 e, #2 e, #3 e, ...`

Homework 1 uses triples of type `int*int*int` a lot

Nesting

Pairs and tuples can be nested however you want

- Not a new feature: implied by the syntax and semantics

```
val x1 = (7, (true, 9)) (* int * (bool*int) *)
```

```
val x2 = #1 (#2 x1)      (* bool *)
```

```
val x3 = (#2 x1)        (* bool*int *)
```

```
val x4 = ((3, 5), ((4, 8), (0, 0)))  
          (* (int*int)*((int*int)*(int*int)) *)
```

Lists

- Despite nested tuples, the type of a variable still “commits” to a particular “amount” of data

In contrast, a list:

- Can have any number of elements
- But all list elements have the same type

Need ways to *build* lists and *access* the pieces...

Building Lists

- The empty list is a value:

`[]`

- In general, a list of values is a value; elements separated by commas:

`[v1, v2, ..., vn]`

- If e_1 evaluates to v and e_2 evaluates to a list $[v_1, \dots, v_n]$, then $e_1 :: e_2$ evaluates to $[v, \dots, v_n]$

`e1 :: e2` (* pronounced "cons" *)

Accessing Lists

Until we learn pattern-matching, we will use three standard-library functions

- `null e` evaluates to `true` if and only if `e` evaluates to `[]`
- If `e` evaluates to `[v1, v2, ..., vn]` then `hd e` evaluates to `v1`
 - (raise exception if `e` evaluates to `[]`)
- If `e` evaluates to `[v1, v2, ..., vn]` then `tl e` evaluates to `[v2, ..., vn]`
 - (raise exception if `e` evaluates to `[]`)
 - Notice result is a list

Type-checking list operations

Lots of new types: For any type t , the type t `list` describes lists where all elements have type t

– Examples: `int list` `bool list` `int list list`
`(int * int) list` `(int list * int) list`

- So `[]` can have type t `list` for *any* type
 - SML uses type `'a list` to indicate this (“quote a” or “alpha”)
- For `e1::e2` to type-check, we need a t such that `e1` has type t and `e2` has type t `list`. Then the result type is t `list`
- `null` : `'a list -> bool`
- `hd` : `'a list -> 'a`
- `tl` : `'a list -> 'a list`

Example list functions

```
fun sum_list (xs : int list) =  
  if null xs  
  then 0  
  else hd(xs) + sum_list(tl(xs))
```

```
fun countdown (x : int) =  
  if x=0  
  then []  
  else x :: countdown (x-1)
```

```
fun append (xs : int list, ys : int list) =  
  if null xs  
  then ys  
  else hd (xs) :: append (tl(xs), ys)
```

Recursion again

Functions over lists are usually recursive

- Only way to “get to all the elements”
- What should the answer be for the empty list?
- What should the answer be for a non-empty list?
 - Typically in terms of the answer for the tail of the list!

Similarly, functions that produce lists of potentially any size will be recursive

- You create a list out of smaller lists

Lists of pairs

Processing lists of pairs requires no new features. Examples:

```
fun sum_pair_list (xs : (int*int) list) =
  if null xs
  then 0
  else #1(hd xs) + #2(hd xs) + sum_pair_list(tl xs)

fun firsts (xs : (int*int) list) =
  if null xs
  then []
  else #1(hd xs) :: firsts(tl xs)

fun seconds (xs : (int*int) list) =
  if null xs
  then []
  else #2(hd xs) :: seconds(tl xs)

fun sum_pair_list2 (xs : (int*int) list) =
  (sum_list (firsts xs)) + (sum_list (seconds xs))
```