Last major topic: Subtyping

Build up key ideas from first principles
- In pseudocode because:
  - No time for another language
  - Simpler to first show subtyping without objects

Then:

- How does subtyping relate to types for OOP?
  - Brief sketch only

- What are the relative strengths of subtyping and generics?
- How can subtyping and generics combine synergistically?
A tiny language

- Can cover most core subtyping ideas by just considering *records with mutable fields*

- Will make up our own syntax
  - ML has records, but no subtyping or field-mutation
  - Racket and Ruby have no type system
  - Java uses class/interface names and rarely fits on a slide
Records (half like ML, half like Java)

Record creation (field names and contents):

\{f_1=e_1, f_2=e_2, \ldots, f_n=e_n\}

Evaluate $e_i$, make a record

Record field access:

$e.f$

Evaluate $e$ to record $v$ with an $f$ field, get contents of $f$ field

Record field update

$e_1.f = e_2$

Evaluate $e_1$ to a record $v_1$ and $e_2$ to a value $v_2$; Change $v_1$'s $f$ field (which must exist) to $v_2$; Return $v_2$
A Basic Type System

Record types: What fields a record has and type for each field

\{f_1:t_1, f_2:t_2, \ldots, f_n:t_n\}

Type-checking expressions:

- If \(e_1\) has type \(t_1\), \ldots, \(e_n\) has type \(t_n\),
  then \({f_1=e_1, \ldots, f_n=e_n}\) has type \({f_1:t_1, \ldots, f_n:t_n}\)

- If \(e\) has a record type containing \(f: t\),
  then \(e.f\) has type \(t\)

- If \(e_1\) has a record type containing \(f: t\) and \(e_2\) has type \(t\),
  then \(e_1.f = e_2\) has type \(t\)
This is safe

These evaluation rules and typing rules prevent ever trying to access a field of a record that does not exist

Example program that type-checks (in a made-up language):

```plaintext
fun distToOrigin (p:{x:real,y:real}) =
  Math.sqrt(p.x*p.x + p.y*p.y)

val pythag : {x:real,y:real} = {x=3.0, y=4.0}
val five : real = distToOrigin(pythag)
```
Motivating subtyping

But according to our typing rules, this program does not type-check
– It does nothing wrong and seems worth supporting

```
fun distToOrigin (p:{x:real,y:real}) = 
    Math.sqrt(p.x*p.x + p.y*p.y)

val c : {x:real,y:real,color:string} = 
    {x=3.0, y=4.0, color="green"}

val five : real = distToOrigin(c)
```
A good idea: allow extra fields

Natural idea: If an expression has type

\{f_1:t_1, f_2:t_2, \ldots, f_n:t_n\}

Then it can also have a type with some fields removed

This is what we need to type-check these function calls:

```plaintext
fun distToOrigin (p:{x:real,y:real}) = ... 
fun makePurple (p:{color:string}) = 
    p.color = "purple"
val c :{x:real,y:real,color:string} = 
    {x=3.0, y=4.0, color="green"}
val _ = distToOrigin(c)
val _ = makePurple(c)
```
Keeping subtyping separate

A programming language already has a lot of typing rules and we do not want to change them

– Example: The type of an actual function argument must equal the type of the function parameter

We can do this by adding “just two things to our language”

– Subtyping: Write \( t_1 <: t_2 \) for \( t_1 \) is a subtype of \( t_2 \)

– One new typing rule that uses subtyping:
  
  If \( e \) has type \( t_1 \) and \( t_1 <: t_2 \),
  
  then \( e \) (also) has type \( t_2 \)

Now all we need to do is define \( t_1 <: t_2 \)
Subtyping is not a matter of opinion

• Misconception: If we are making a new language, we can have whatever typing and subtyping rules we want

• Not if you want to prevent what you claim to prevent [soundness]
  – Here: No accessing record fields that do not exist

• Our typing rules were *sound* before we added subtyping
  – We should keep it that way

• Principle of *substitutability*: If \( t_1 <: t_2 \), then any value of type \( t_1 \) must be usable in every way a \( t_2 \) is
  – Here: Any value of subtype needs all fields any value of supertype has
Four good rules

For our record types, these rules all meet the substitutability test:

1. “Width” subtyping: A supertype can have a subset of fields with the same types

2. “Permutation” subtyping: A supertype can have the same set of fields with the same types in a different order

3. Transitivity: If $t_1 <: t_2$ and $t_2 <: t_3$, then $t_1 <: t_3$

4. Reflexivity: Every type is a subtype of itself

(4) may seem unnecessary, but it composes well with other rules in a full language and “does no harm”
**More record subtyping?**

[Warning: I am misleading you 😊]

Subtyping rules so far let us drop fields but not change their types.

Example: A circle has a center field holding another record.

```plaintext
fun circleY (c:{center:{x:real,y:real}, r:real}) =
  c.center.y

val sphere:{center:{x:real,y:real,z:real}, r:real} =
  {center={x=3.0,y=4.0,z=0.0}, r=1.0}

val _ = circleY(sphere)
```

For this to type-check, we need:

{center:{x:real,y:real,z:real}, r:real} <: {center:{x:real,y:real}, r:real}
Do not have this subtyping – could we?

\[
\{\text{center}:{x:real,y:real,z:real}, \ r:real\} \\
<: \\
\{\text{center}:{x:real,y:real}, \ r:real\}
\]

- No way to get this yet: we can drop center, drop r, or permute order, but cannot “reach into a field type” to do subtyping

- So why not add another subtyping rule... “Depth” subtyping:
  \[
  \text{If } ta <: tb, \text{ then } \{f1:t1, \ldots, f:ta, \ldots, fn:tn\} <: \\
  \{f1:t1, \ldots, f:tb, \ldots, fn:tn\}
  \]

- Depth subtyping (along with width on the field's type) lets our example type-check
Stop!

• It is nice and all that our new subtyping rule lets our example type-check

• But it is not worth it if it breaks soundness
  – Also allows programs that can access missing record fields

• Unfortunately, it breaks soundness 😞
Mutation strikes again

If $ta \leq tb$, then \[
\{f_1:t_1, \ldots, f:ta, \ldots, fn:tn\} <:
\{f_1:t_1, \ldots, f:tb, \ldots, fn:tn\}
\]

```
fun setToOrigin (c:{center:{x:real,y:real}, r:real})=
  c.center = {x=0.0, y=0.0}
val sphere:{center:{x:real,y:real,z:real}, r:real} =
  {center={x=3.0, y=4.0, z=0.0}, r=1.0}
val _ = setToOrigin(sphere)
val _ = sphere.center.z (* kaboom! (no z field) *)
```
Moral of the story

• In a language with records/objects with getters and setters, depth subtyping is unsound
  – Subtyping cannot change the type of fields

• If fields are immutable, then depth subtyping is sound!
  – Yet another benefit of outlawing mutation!
  – Choose two of three: setters, depth subtyping, soundness

• Remember: subtyping is not a matter of opinion
Picking on Java (and C#)

Arrays should work just like records in terms of depth subtyping
  – But in Java, if \( t_1 <: t_2 \), then \( t_1[] <: t_2[] \)
  – So this code type-checks, surprisingly

```java
class Point { … }  
class ColorPoint extends Point { … }  
…  
void m1(Point[] pt_arr) {
    pt_arr[0] = new Point(3,4);
}
String m2(int x) {
    ColorPoint[] cpt_arr = new ColorPoint[x];
    for(int i=0; i < x; i++)
        cpt_arr[i] = new ColorPoint(0,0,"green");
    m1(cpt_arr); // !
    return cpt_arr[0].color; // !
}
```
Why did they do this?

- More flexible type system allows more programs but prevents fewer errors
  - Seemed especially important before Java/C# had generics
- Good news: despite this “inappropriate” depth subtyping
  - `e.color` will never fail due to there being no `color` field
  - Array reads `e1[e2]` always return a (subtype of) `t` if `e1` is a `t[]`
- Bad news: to get the good news
  - `e1[e2]=e3` can fail even if `e1` has type `t[]` and `e3` has type `t`
  - Array stores check the run-time class of `e1`'s elements and do not allow storing a supertype
  - No type-system help to avoid such bugs / performance cost
So what happens

void m1(Point[] pt_arr) {
    pt_arr[0] = new Point(3,4); // can throw
}

String m2(int x) {
    ColorPoint[] cpt_arr = new ColorPoint[x];
    ...
    m1(cpt_arr); // "inappropriate" depth subtyping
    ColorPoint c = cpt_arr[0]; // fine, cpt_arr
    // will always hold (subtypes of) ColorPoints
    return c.color; // fine, a ColorPoint has a color
}

• Causes code in m1 to throw an `ArrayStoreException`
  – Even though logical error is in m2
  – At least run-time checks occur only on array stores, not on field accesses like c.color
**null**

- Array stores probably the most *surprising* choice for flexibility over static checking

- But **null** is the most *common* one in practice
  - **null** is not an object; it has *no* fields or methods
  - But Java and C# let it have *any* object type (backwards, huh?!)
  - So, in fact, we do *not* have the static guarantee that evaluating `e in e.f or e.m(…)` produces an object that has an `f` or `m`
  - The “or **null**” caveat leads to run-time checks and errors, as you have surely noticed

- Sometimes **null** is convenient (like ML's option types)
  - But also having “cannot be **null**” types would be nice
Now functions

• Already know a caller can use subtyping for arguments passed
  – Or on the result

• More interesting: When is one function type a subtype of another?
  – Important for higher-order functions: If a function expects an argument of type \( t_1 \rightarrow t_2 \), can you pass a \( t_3 \rightarrow t_4 \) instead?
  
  – Coming next: Important for understanding methods
    • (An object type is a lot like a record type where “method positions” are immutable and have function types)
Example

fun distMoved (f : {x:real,y:real}->{x:real,y:real},
    p : {x:real,y:real}) =
    let val p2 : {x:real,y:real} = f p
    val dx : real = p2.x – p.x
    val dy : real = p2.y – p.y
    in Math.sqrt(dx*dx + dy*dy) end

fun flip p = {x = ~p.x, y=~p.y}
val d = distMoved(flip, {x=3.0, y=4.0})

No subtyping here yet:
  – flip has exactly the type distMoved expects for f
  – Can pass distMoved a record with extra fields for p,
    but that's old news
Return-type subtyping

fun distMoved (f : {x:real,y:real}->{x:real,y:real},
               p : {x:real,y:real}) =
    let val p2 : {x:real,y:real} = f p
       val dx : real = p2.x – p.x
       val dy : real = p2.y – p.y
    in Math.sqrt(dx*dx + dy*dy) end

fun flipGreen p = {x = ~p.x, y=~p.y, color="green"}
val d = distMoved(flipGreen, {x=3.0, y=4.0})

• Return type of flipGreen is {x:real,y:real,color:string},
  but distMoved expects a return type of {x:real,y:real}

• Nothing goes wrong: If ta <: tb, then t->ta <: t->tb
  – A function can return “more than it needs to”
  – Jargon: “Return types are covariant”
This is wrong

fun distMoved (f : {x:real,y:real}->{x:real,y:real},
    p : {x:real,y:real}) =
  let val p2 : {x:real,y:real} = f p
  val dx : real = p2.x – p.x
  val dy : real = p2.y – p.y
  in Math.sqrt(dx*dx + dy*dy) end

fun flipIfGreen p = if p.color = "green" (*kaboom!*)
    then {x = ~p.x, y=~p.y}
    else {x = p.x, y=p.y}
val d = distMoved(flipIfGreen, {x=3.0, y=4.0})

• Argument type of flipIfGreen is
  {x:real,y:real,color:string}, but it is called with a
  {x:real,y:real}

• Unsound! ta <: tb does NOT allow ta->t <: tb->t
The other way works!

fun distMoved (f : {x:real,y:real}->{x:real,y:real}, p : {x:real,y:real}) = 
  let val p2 : {x:real,y:real} = f p 
     val dx : real = p2.x – p.x 
     val dy : real = p2.y – p.y 
  in Math.sqrt(dx*dx + dy*dy) end

fun flipX_Y0 p = {x = ~p.x, y=0.0} 
val d = distMoved(flipX_Y0, {x=3.0, y=4.0})

• Argument type of flipX_Y0 is \{x:real\}, but it is called with a \{x:real,y:real\}, which is fine

• If \(tb <: ta\), then \(ta -> t <: tb -> t\)
  – A function can assume “less than it needs to” about arguments
  – Jargon: “Argument types are \(contravariant\)”
Can do both

fun distMoved (f : {x:real,y:real}->{x:real,y:real}, p : {x:real,y:real}) =
  let val p2 : {x:real,y:real} = f p
  val dx : real = p2.x – p.x
  val dy : real = p2.y – p.y
  in Math.sqrt(dx*dx + dy*dy) end

fun flipXMakeGreen p = {x = ~p.x, y=0.0, color="green"}  
val d = distMoved(flipXMakeGreen, {x=3.0, y=4.0})

• flipXMakeGreen has type
  {x:real} -> {x:real,y:real,color:string}
• Fine to pass a function of such a type as function of type
  {x:real,y:real} -> {x:real,y:real}
• If t3 <: t1 and t2 <: t4, then t1 -> t2 <: t3 -> t4
Conclusion

• If $t_3 <: t_1$ and $t_2 <: t_4$, then $t_1 -> t_2 <: t_3 -> t_4$
  – Function subtyping contravariant in argument(s) and covariant in results

• Also essential for understanding subtyping and methods in OOP

• Most unintuitive concept in the course
  – Smart people often forget and convince themselves covariant arguments are okay
  – These people are always mistaken
  – At times, you or your boss or your friend may do this
  – Remember: A guy with a PhD in PL *jumped up and down* insisting that function/method subtyping is always contravariant in its argument -- covariant is unsound