CSE341: Programming Languages
Lecture 7
First-Class Functions

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What is functional programming?

“Functional programming” can mean a few different things:

1. Avoiding mutation in most/all cases (done and ongoing)
2. Using functions as values (this unit)

... 

- Style encouraging recursion and recursive data structures
- Style closer to mathematical definitions
- Programming idioms using laziness (later topic, briefly)
- Anything not OOP or C? (not a good definition)

Not sure a definition of “functional language” exists beyond “makes functional programming easy / the default / required”

- No clear yes/no for a particular language
First-class functions

- *First-class functions*: Can use them wherever we use values
  - Functions are values too
  - Arguments, results, parts of tuples, bound to variables, carried by datatype constructors or exceptions, ...

```latex
fun double x = 2*x
fun incr x = x+1
val a_tuple = (double, incr, double(incr 7))
```

- Most common use is as an argument / result of another function
  - Other function is called a *higher-order function*
  - Powerful way to *factor out* common functionality
Function Closures

- **Function closure**: Functions can use bindings from outside the function definition (in scope where function is defined)
  - Makes first-class functions *much* more powerful
  - Will get to this feature in a bit, after simpler examples

- Distinction between terms *first-class functions* and *function closures* is not universally understood
  - Important conceptual distinction even if terms get muddled
Onward

The next week:

- How to use first-class functions and closures
- The precise semantics
- Multiple powerful idioms
Functions as arguments

• We can pass one function as an argument to another function
  – Not a new feature, just never thought to do it before

fun f (g,…) = … g (…) …
fun h1  … = …
fun h2  … = …
… f(h1,…) … f(h2,…) …

• Elegant strategy for factoring out common code
  – Replace $N$ similar functions with calls to 1 function where you pass in $N$ different (short) functions as arguments

[See the code file for this lecture]
Example

Can reuse n\_times rather than defining many similar functions
  
  - Computes \( f(f(...f(x))) \) where number of calls is \( n \)

\[
\text{fun } n\_\text{times}(f,n,x) = \\
  \text{if } n=0 \\
  \text{then } x \\
  \text{else } f(\text{n\_times}(f,n-1,x))
\]

fun double x = x + x
fun increment x = x + 1
val x1 = n\_times(double,4,7)
val x2 = n\_times(increment,4,7)
val x3 = n\_times(tl,2,[4,8,12,16])

fun double\_n\_times (n,x) = n\_times(double,n,x)
fun nth\_tail (n,x) = n\_times(tl,n,x)
Relation to types

• Higher-order functions are often so “generic” and “reusable” that they have polymorphic types, i.e., types with type variables

• But there are higher-order functions that are not polymorphic

• And there are non-higher-order (first-order) functions that are polymorphic

• Always a good idea to understand the type of a function, especially a higher-order function
Types for example

fun n_times (f,n,x) = 
  if n=0 
  then x 
  else f (n_times(f,n-1,x))

- val n_times : ('a -> 'a) * int * 'a -> 'a 
  - Simpler but less useful: (int -> int) * int * int -> int

- Two of our examples instantiated 'a with int
- One of our examples instantiated 'a with int list
- This polymorphism makes n_times more useful

- Type is inferred based on how arguments are used (later lecture) 
  - Describes which types must be exactly something (e.g., int) and which can be anything but the same (e.g., 'a)
Polymorphism and higher-order functions

• Many higher-order functions are polymorphic because they are so reusable that some types, “can be anything”

• But some polymorphic functions are not higher-order
  – Example: \( \text{len} : 'a \text{ list} \rightarrow \text{int} \)

• And some higher-order functions are not polymorphic
  – Example: \( \text{times}_\text{until}_\text{0} : (\text{int} \rightarrow \text{int}) \times \text{int} \rightarrow \text{int} \)

\[
\text{fun times}_\text{until}_\text{zero} (f,x) = \\
\quad \text{if } x = 0 \text{ then } 0 \text{ else } 1 + \text{times}_\text{until}_\text{zero}(f, f x)
\]

Note: Would be better with tail-recursion
Toward anonymous functions

• Definitions unnecessarily at top-level are still poor style:

```latex
fun trip x = 3*x
fun triple_n_times (f,x) = n_times(trip,n,x)
```

• So this is better (but not the best):

```latex
fun triple_n_times (f,x) =
  let fun trip y = 3*y
  in
  n_times(trip,n,x)
  end
```

• And this is even smaller scope
  – It makes sense but looks weird (poor style; see next slide)

```latex
fun triple_n_times (f,x) =
  n_times(let fun trip y = 3*y in trip end, n, x)
```
Anonymous functions

• This does not work: A function binding is not an expression

```
fun triple_n_times (f,x) =
n_times((fun trip y = 3*y), n, x)
```

• This is the best way we were building up to: an expression form for anonymous functions

```
fun triple_n_times (f,x) =
n_times((fn y => 3*y), n, x)
```

  – Like all expression forms, can appear anywhere
  – Syntax:
    - `fn` not `fun`
    - `=>` not `=`
    - no function name, just an argument pattern
Using anonymous functions

• Most common use: Argument to a higher-order function
  – Don’t need a name just to pass a function

• But: Cannot use an anonymous function for a recursive function
  – Because there is no name for making recursive calls
  – If not for recursion, \texttt{fun} bindings would be syntactic sugar
    for \texttt{val} bindings and anonymous functions

\begin{verbatim}
  fun triple x = 3*x
  val triple = fn y => 3*y
\end{verbatim}


A style point

Compare:

\[
\text{if } x \text{ then true else false}
\]

With:

\[
(f n \ x \Rightarrow f \ x)
\]

So don’t do this:

\[
n\_\text{times}((f n \ y \Rightarrow \text{tl } y),3,xs)
\]

When you can do this:

\[
n\_\text{times}(\text{tl},3,xs)
\]
Map

fun map (f, xs) =
  case xs of
    [] => []
  | x::xs' => (f x)::(map(f, xs'))

val map : ('a -> 'b) * 'a list -> 'b list

Map is, without doubt, in the “higher-order function hall-of-fame”
  – The name is standard (for any data structure)
  – You use it all the time once you know it: saves a little space, but more importantly, communicates what you are doing
  – Similar predefined function: List.map
    • But it uses currying (coming soon)
Filter

fun filter (f,xs) = 
  case xs of 
    [] => [] 
    | x::xs' => if f x 
      then x::(filter(f,xs')) 
      else filter(f,xs')

val filter : ('a -> bool) * 'a list -> 'a list

Filter is also in the hall-of-fame
  – So use it whenever your computation is a filter
  – Similar predefined function: List.filter
    • But it uses currying (coming soon)
Generalizing

Our examples of first-class functions so far have all:
– Taken one function as an argument to another function
– Processed a number or a list

But first-class functions are useful anywhere for any kind of data
– Can pass several functions as arguments
– Can put functions in data structures (tuples, lists, etc.)
– Can return functions as results
– Can write higher-order functions that traverse your own data structures

Useful whenever you want to abstract over “what to compute with”
– No new language features
Returning functions

• Remember: Functions are first-class values
  – For example, can return them from functions

• Silly example:

```haskell
fun double_or_triple f = 
  if f 7
  then fn x => 2*x
  else fn x => 3*x
```

Has type \((\text{int} \to \text{bool}) \to \text{int} \to \text{int}\)

But the REPL prints \((\text{int} \to \text{bool}) \to \text{int} \to \text{int}\)
because it never prints unnecessary parentheses and
\(t_1 \to t_2 \to t_3 \to t_4\) means \((t_1\to(t_2\to(t_3\to t_4)))\)
Other data structures

• Higher-order functions are not just for numbers and lists.

• They work great for common recursive traversals over your own data structures (datatype bindings) too.

• Example of a higher-order predicate:
  
  – Are all constants in an arithmetic expression even numbers?
  
  – Use a more general function of type
    
    \((\text{int} \rightarrow \text{bool}) \times \text{exp} \rightarrow \text{bool}\)
  
  – And call it with \((\text{fn}\ x \Rightarrow x \mod 2 = 0)\)