Nested patterns

- We can nest patterns as deep as we want
  - Just like we can nest expressions as deep as we want
  - Often avoids hard-to-read, wordy nested case expressions

- So the full meaning of pattern-matching is to compare a pattern against a value for the “same shape” and bind variables to the “right parts”
  - More precise recursive definition coming after examples
Useful example: zip/unzip 3 lists

```ml
fun zip3 lists =
    case lists of
      (([],[]),[],[]) => []
    | (hd1::tl1,hd2::tl2,hd3::tl3) =>
        (hd1,hd2,hd3)::zip3(tl1,tl2,tl3)
    | _ => raise ListLengthMismatch

fun unzip3 triples =
    case triples of
      [] => ([],[],[])
    | (a,b,c)::tl =>
        let val (l1, l2, l3) = unzip3 tl
        in
            (a::l1,b::l2,c::l3)
        end
```

More examples in `.sml` files
Style

• Nested patterns can lead to very elegant, concise code
  – Avoid nested case expressions if nested patterns are simpler and avoid unnecessary branches or let-expressions
    • Example: unzip3 and nondecreasing
  – A common idiom is matching against a tuple of datatypes to compare them
    • Examples: zip3 and multsign

• Wildcards are good style: use them instead of variables when you do not need the data
  – Examples: len and multsign
(Most of) the full definition

The semantics for pattern-matching takes a pattern $p$ and a value $v$ and decides (1) does it match and (2) if so, what variable bindings are introduced.

Since patterns can nest, the definition is elegantly recursive, with a separate rule for each kind of pattern. Some of the rules:

- If $p$ is a variable $x$, the match succeeds and $x$ is bound to $v$
- If $p$ is $\_\_$, the match succeeds and no bindings are introduced
- If $p$ is $(p_1, \ldots, p_n)$ and $v$ is $(v_1, \ldots, v_n)$, the match succeeds if and only if $p_1$ matches $v_1$, ..., $p_n$ matches $v_n$. The bindings are the union of all bindings from the submatches
- If $p$ is $C \ p_1$, the match succeeds if $v$ is $C \ v_1$ (i.e., the same constructor) and $p_1$ matches $v_1$. The bindings are the bindings from the submatch.
- ... (there are several other similar forms of patterns)
Examples

- Pattern \( a::b::c::d \) matches all lists with \( \geq 3 \) elements

- Pattern \( a::b::c::[] \) matches all lists with 3 elements

- Pattern \( ((a,b),(c,d))::e \) matches all non-empty lists of pairs of pairs
Exceptions

An exception binding introduces a new kind of exception

```plaintext
exception MyFirstException
exception MySecondException of int * int
```

The `raise` primitive raises (a.k.a. throws) an exception

```plaintext
raise MyFirstException
raise (MySecondException (7,9))
```

A handle expression can handle (a.k.a. catch) an exception

- If doesn’t match, exception continues to propagate

```plaintext
e1 handle MyFirstException => e2
e1 handle MySecondException(x,y) => e2
```
Actually…

Exceptions are a lot like datatype constructors…

- Declaring an exception adds a constructor for type `exn`
- Can pass values of `exn` anywhere (e.g., function arguments)
  - Not too common to do this but can be useful
- `handle` can have multiple branches with patterns for type `exn`
Recursion

Should now be comfortable with recursion:

• No harder than using a loop (whatever that is 😊)

• Often much easier than a loop
  – When processing a tree (e.g., evaluate an arithmetic expression)
  – Examples like appending lists
  – Avoids mutation even for local variables

• Now:
  – How to reason about efficiency of recursion
  – The importance of tail recursion
  – Using an accumulator to achieve tail recursion
  – [No new language features here]
Call-stacks

While a program runs, there is a call stack of function calls that have started but not yet returned

- Calling a function \( f \) pushes an instance of \( f \) on the stack
- When a call to \( f \) finishes, it is popped from the stack

These stack-frames store information like the value of local variables and “what is left to do” in the function

Due to recursion, multiple stack-frames may be calls to the same function
fun fact n = if n=0 then 1 else n*fact(n-1)
val x = fact 3
Example Revised

fun fact n =
    let fun aux(n,acc) =
        if n=0
        then acc
        else aux(n-1,acc*n)
    in
        aux(n,1)
    end
val x = fact 3

Still recursive, more complicated, but the result of recursive calls is the result for the caller (no remaining multiplication)
### The call-stacks

<table>
<thead>
<tr>
<th>fact 3</th>
<th>fact 3: __</th>
<th>fact 3: __</th>
<th>fact 3: __</th>
</tr>
</thead>
<tbody>
<tr>
<td>aux (3,1)</td>
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<td>aux (2,3)</td>
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</table>

Etc…
An optimization

It is unnecessary to keep around a stack-frame just so it can get a callee’s result and return it without any further evaluation.

ML recognizes these tail calls in the compiler and treats them differently:

- Pop the caller before the call, allowing callee to reuse the same stack space
- (Along with other optimizations,) as efficient as a loop

Reasonable to assume all functional-language implementations do tail-call optimization.
What really happens

fun fact n = 
  let fun aux(n,acc) = 
    if n=0 
    then acc 
    else aux(n-1,acc*n) 
  in 
  aux(n,1) 
end 

val x = fact 3
Moral of tail recursion

• Where reasonably elegant, feasible, and important, rewriting functions to be *tail-recursive* can be much more efficient
  – Tail-recursive: recursive calls are tail-calls

• There is a *methodology* that can often guide this transformation:
  – Create a helper function that takes an *accumulator*
  – Old base case becomes initial accumulator
  – New base case becomes final accumulator
Methodology already seen

fun fact n = 
  let fun aux(n,acc) = 
    if n=0 
      then acc 
      else aux(n-1,acc*n) 
  in 
    aux(n,1) 
  end 

val x = fact 3
Another example

fun sum xs =
  case xs of
    [] => 0
  | x::xs' => x + sum xs'

fun sum xs =
  let fun aux(xs,acc) =
    case xs of
      [] => acc
    | x::xs' => aux(xs',x+acc)
  in
    aux(xs,0)
  end
And another

fun rev xs =
case xs of
  [[]] => []
  | x::xs' => (rev xs') @ [x]

fun rev xs =
  let fun aux(xs,acc) =
    case xs of
      [[]] => acc
      | x::xs' => aux(xs',x::acc)
    in
      aux(xs,[])
    end
Actually much better

fun rev xs = 
case xs of
  [] => []
| x::xs' => (rev xs') @ [x]

• For fact and sum, tail-recursion is faster but both ways linear time
• Non-tail recursive rev is quadratic because each recursive call uses append, which must traverse the first list
  – And 1+2+…+(length-1) is almost length*length/2
  – Moral: beware list-append, especially within outer recursion
• Cons constant-time (and fast), so accumulator version much better
Always tail-recursive?

There are certainly cases where recursive functions cannot be evaluated in a constant amount of space.

Most obvious examples are functions that process trees.

In these cases, the natural recursive approach is the way to go:
  - You could get one recursive call to be a tail call, but rarely worth the complication.

Also beware the wrath of premature optimization:
  - Favor clear, concise code.
  - But do use less space if inputs may be large.
**What is a tail-call?**

The “nothing left for caller to do” intuition usually suffices

- If the result of \( f \ x \) is the “immediate result” for the enclosing function body, then \( f \ x \) is a tail call

But we can define “tail position” recursively

- Then a “tail call” is a function call in “tail position”

...
Precise definition

A tail call is a function call in tail position

- If an expression is not in tail position, then no subexpressions are
- In `fun f p = e`, the body `e` is in tail position
- If `if e1 then e2 else e3` is in tail position, then `e2` and `e3` are in tail position (but `e1` is not). (Similar for case-expressions)
- If `let b1 ... bn in e end` is in tail position, then `e` is in tail position (but no binding expressions are)
- Function-call arguments `e1 e2` are not in tail position
- ...

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