Derivation Trees

• R1: \text{rev}([],[]).

• R2: \text{rev}([X|Xs],Ys) :-
  \text{rev}(Xs,Rs), \text{append}(Rs, [X], Ys).

• A1: \text{append}([],Ys,Ys).

• A2: \text{append}([X|Xs],Ys,[X|Zs]) :-
  \text{append}(Xs,Ys,Zs).
rev([1,2], Rs)
rev([1,2], Rs)

R1

Fail ([] = [1,2])
Fail ([] = [1,2])

R1

rev([1,2], Rs)

R2

X = 1, Xs = [2]

rev([2], Rs),
append(Rs1,[1],Rs)
rev([1,2], Rs)

R1

Fail ([] = [1,2])

R2

X = 1, Xs = [2]

rev([2], Rs), append(Rs1,[1],Rs)

R1

Fail ([] = [2])
\begin{itemize}
  \item \texttt{rev([1,2], Rs)}
  \item \texttt{R1}
  \item \texttt{Fail ([] = [1,2])}
  \item \texttt{R1}
  \item \texttt{Fail ([] = [2])}
  \item \texttt{R2}
  \item \texttt{X = 1, Xs = [2]}
  \item \texttt{rev([2], Rs),
      append(Rs1,[1],Rs)}
  \item \texttt{R2}
  \item X = 2, Xs = []
  \item \texttt{rev([], Rs2),
      append(Rs2,[2],Rs1)
      append(Rs1,[1],Rs)}
\end{itemize}
rev([1,2], Rs)

R1

Fail ([] = [1,2])

R1

Fail ([] = [2])

R2

X = 1, Xs = [2]

rev([2], Rs),
append(Rs1,[1],Rs)

R2

X = 2, Xs = []

rev([], Rs2),
append(Rs2,[2],Rs1)
append(Rs1,[1],Rs)

R1

Yay ([] = [])
rev([1,2], Rs)

R1
Fail ([] = [1,2])
  R1
Fail ([] = [2])
  R1
Yay ([] = [])
  R2
Fail ([X|Xs] = [])

R2
X = 1, Xs = [2]
  rev([2], Rs),
  append(Rs1,[1],Rs)

R2
X = 2, Xs = [], Rs2 = []
  rev([], Rs2),
  append(Rs2,[2],Rs1)
  append(Rs1,[1],Rs)
rev([1,2], Rs)
R1
Fail ([] = [1,2])
R2
X = 1, Xs = [2]
rev([2], Rs),
append(Rs1,[1],Rs)
R1
Fail ([] = [2])
R2
X = 2, Xs = [], Rs2 = []
rev([], Rs2),
append([], [2], Rs1)
append(Rs1, [1], Rs)
rev([1,2], Rs)

R1

Fail ([] = [1,2])

R1

Fail ([] = [2])

R2
X = 1, Xs = [2]

rev([2], Rs),
append(Rs1,[1],Rs)

R2
X = 2, Xs = [], Rs2 = [], Rs1 = [2]

rev([], Rs2),
append([],[2],Rs1)
append(Rs1,[1],Rs)

A1

Yay ([] = [])
\[
\begin{align*}
R1 & \quad \text{rev}([1,2], R) \\
\text{Fail} & \quad ([] = [1,2])
\end{align*}
\]

\[
\begin{align*}
R1 & \quad \text{rev}([2], R), \\
& \quad \text{append}(R, [1], R) \\
\end{align*}
\]

\[
\begin{align*}
R2 & \quad X = 1, Xs = [2] \\
\text{Fail} & \quad ([] = [2])
\end{align*}
\]

\[
\begin{align*}
R2 & \quad \text{rev}([], R), \\
& \quad \text{append}([], [2], R) \\
& \quad \text{append}(R, [1], R)
\end{align*}
\]

\[
\begin{align*}
R1 & \quad X = 2, Xs = [], R2 = [], R1 = [2] \\
\text{A1} & \quad \text{Yay} ([] = []) \\
\text{A2} & \quad \text{Fail} ([X|Xs] = [])
\end{align*}
\]
\[ \text{rev}([1,2], Rs) \]

\[ \text{R1} \]

\[ \text{Fail} \quad ([] = [1,2]) \]

\[ \text{R1} \]

\[ \text{Fail} \quad ([] = [2]) \]

\[ \text{R2} \]

\[ \text{X} = 1, \text{Xs} = [2] \]

\[ \text{rev}([2], Rs), \]

\[ \text{append(Rs1,[1],Rs)} \]

\[ \text{R2} \]

\[ \text{X} = 2, \text{Xs} = [], \text{Rs2} = [], \text{Rs1} = [2] \]

\[ \text{rev}([], \text{Rs2}), \]

\[ \text{append([],2,Rs1)} \]

\[ \text{append}(2,[1],\text{Rs}) \]
Fail ([] = [1,2])

R1

rev([1,2], Rs)

R2

X = 1, Xs = [2]

rev([2], Rs),
append(Rs1, [1], Rs)

Fail ([] = [2])

R1

X = 1, Xs = [2]

rev([2], Rs),
append(Rs1, [1], Rs)

R2

X = 2, Xs = [], Rs2 = [], Rs1 = [2]

rev([], Rs2),
append([], [2], Rs1)
append([2], [1], Rs)

A1

Fail ([] = [2])
\[
\text{rev([1,2], Rs)} \\
\text{R1} \\
\text{Fail ([]} = [1,2])) \\
\text{R1} \\
\text{Fail ([]} = [2]) \\
\text{R2} \\
\text{X = 1, Xs = [2]} \\
\text{rev([2], Rs),} \\
\text{append(Rs1,[1],Rs)} \\
\text{R1} \\
\text{Fail ([]} = [1, 2]) \\
\text{A1} \\
\text{rev([], Rs2),} \\
\text{append([], [2], Rs1)} \\
\text{append([2],[1],Rs)} \\
\text{Fail ([]} = [2]) \\
\text{A2} \\
\text{X = 2, Xs = [], Ys = [1], Rs = [X|Zs]} \\
\text{append([], [1], Zs)}
\[ \text{rev([1,2], Rs)} \]

\[ \text{R1} \]

\[ \text{Fail ([] = [1,2])} \]

\[ \text{R1} \]

\[ \text{Fail ([] = [2])} \]

\[ \text{rev([2], Rs)}, \text{append(Rs1,[1],Rs)} \]

\[ \text{R2} \]

\[ \text{X = 1, Xs = [2]} \]

\[ \text{rev([2], Rs)}, \text{append(Rs1,[1],Rs)} \]

\[ \text{A1} \]

\[ \text{Fail ([] = [2])} \]

\[ \text{rev([], Rs2)}, \text{append([], [2], Rs1)} \]

\[ \text{append([2],[1],Rs)} \]

\[ \text{A2} \]

\[ \text{X = 2, Xs = [], Rs2 = [], Rs1 = [2]} \]

\[ \text{rev([], Rs2)}, \text{append([], [2], Rs1)} \]

\[ \text{append([2],[1],Rs)} \]

\[ \text{A1} \]

\[ \text{Fail ([] = [2])} \]

\[ \text{X = 2, Xs = [], Ys = [1], Rs = [X|Zs], Zs = [1]} \]

\[ \text{append([], [1], Zs)} \]

\[ \text{A1} \]

\[ \text{Yay ([] = [])} \]
rev([1,2], Rs)

R1
Fail ([] = [1,2])
R2

X = 1, Xs = [2]
rev([2], Rs),
append(Rs1,[1],Rs)
append(Rs, Rs1)
R1
Fail ([] = [2])
R2

X = 2, Xs = [], Rs2 = [], Rs1 = [2]
rev([], Rs2),
append([], [2], Rs1)
append([2], [], Rs)
A1
Fail ([] = [2])
A2

X = 2, Xs = [], Ys = [1], Rs = [2,1]
append([], [1], Zs)
A1
Yay ([] = [])

Rs = [2,1]
\[
\text{rev}([1,2], \text{Rs})
\]

**R1**

\[
\text{Fail} ([] = [1,2])
\]

**R2**

\[
X = 1, Xs = [2]
\]

\[
\text{rev}([2], \text{Rs}),
\text{append}(\text{Rs1}, [1], \text{Rs})
\]

**R1**

\[
\text{Fail} ([] = [2])
\]

**R2**

\[
X = 2, Xs = [], \text{Rs2} = [], \text{Rs1} = [2]
\]

\[
\text{rev}([], \text{Rs2}),
\text{append}([], [2], \text{Rs1})
\text{append}([2], [1], \text{Rs})
\]

**A1**

\[
\text{Fail} ([] = [2])
\]

**A2**

\[
X = 2, Xs = [], Ys = [1], \text{Rs} = [2,1]
\]

\[
\text{append}([], [1], Zs)
\]

**A1**

**A2**

\[
\text{Yay} ([] = [])
\]

\[
\text{Fail} ([X|Xs] = [])
\]

\[
\text{Rs} = [2,1]
\]
rev([1,2], Rs)

R1

Fail ([] = [1,2])

R2

X = 1, Xs = [2]

rev([2], Rs),
append(Rs1,[1],Rs)

R1

Fail ([] = [2])

R2

X = 2, Xs = [], Rs2 = [], Rs1 = [2]

rev([], Rs2),
append([],[2],Rs1),
append([],[1],Rs)

A1

Fail ([] = [2])

A2

X = 2, Xs = [], Ys = [1], Rs = [2,1]

append([], [1], Zs)

A1

Yay ([] = [])

A2

Fail ([X|Xs] = [])

Rs = [2,1]