CSE 341
Lecture 4

merge sort; basic efficiency issues

Ullman 3.3 - 3.4

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Exercise

- Write a function `mergeSort` that accepts a list and uses the merge sort algorithm to produce a list with the same elements in non-decreasing order.
  - `mergeSort([5,2,8,4,9,6])` produces `[2,4,5,6,8,9]`

(a tricky recursive algorithm for us to practice...)

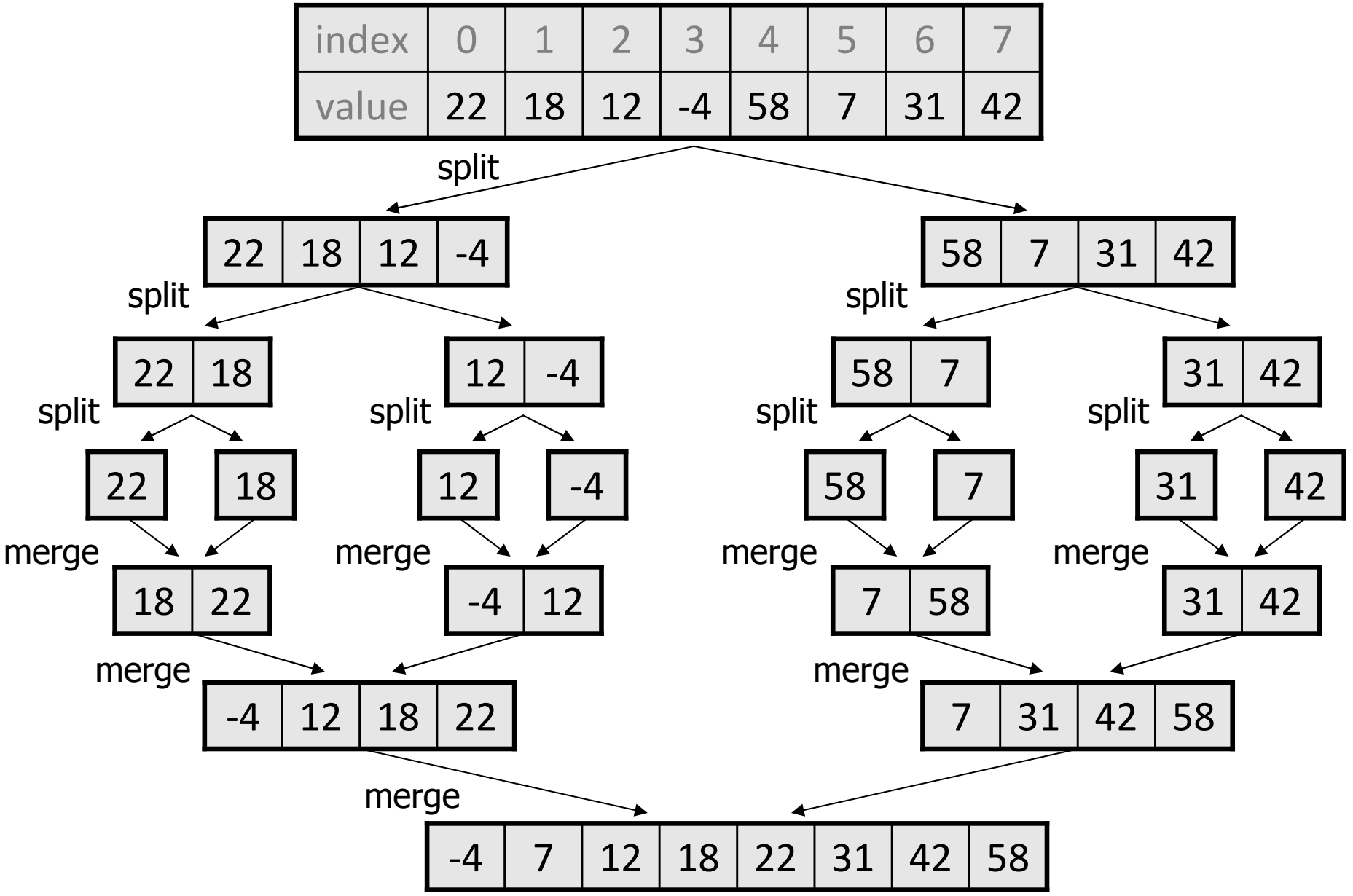
Merge sort

- **merge sort**: Repeatedly divides data in half, sorts each half, and combines the sorted halves into a sorted whole.

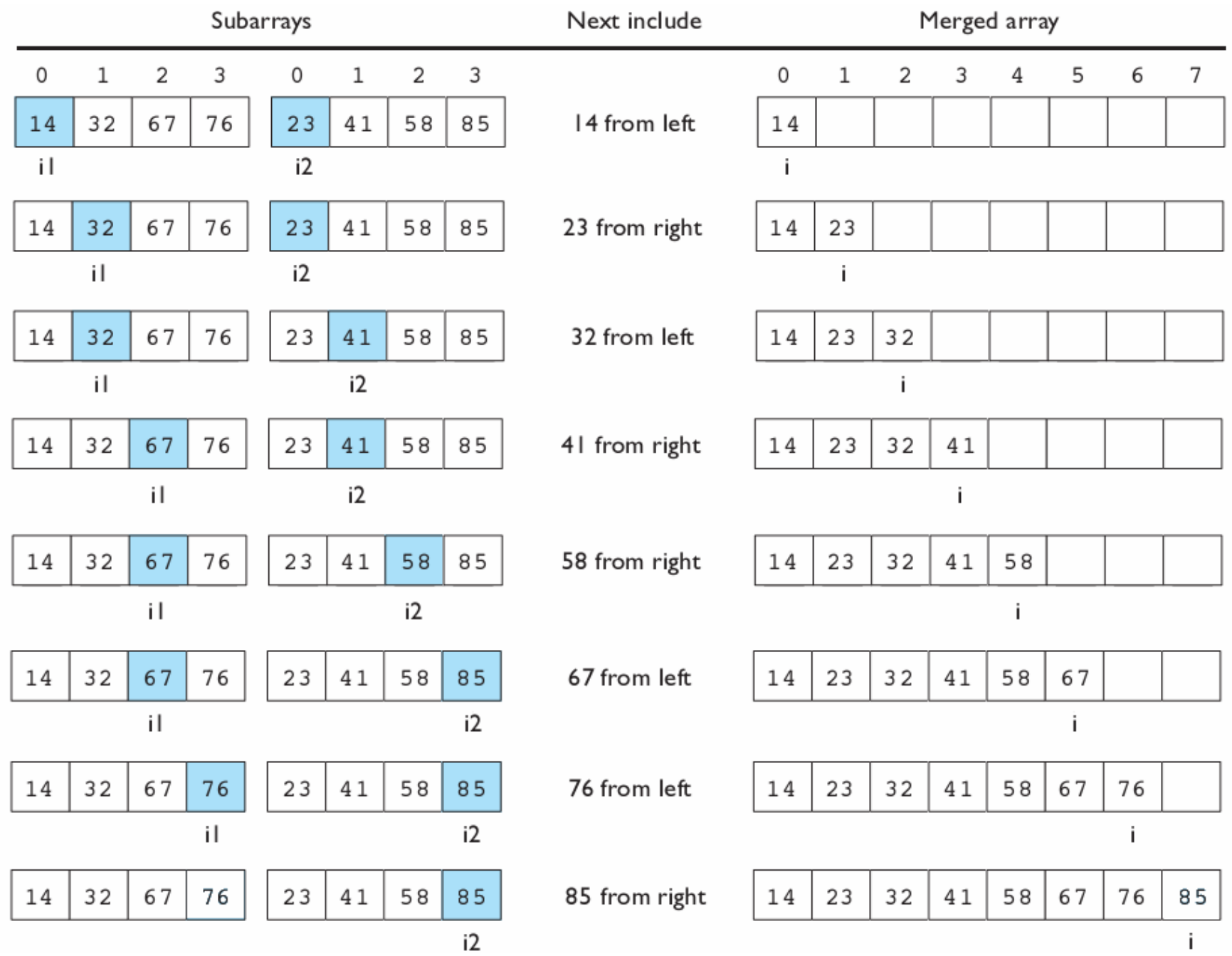
  The algorithm:
  - Divide the list into two roughly equal halves.
  - Sort the left half.
  - Sort the right half.
  - Merge the two sorted halves into one sorted list.

  - Often implemented recursively.
  - An example of a "divide and conquer" algorithm.
    - Invented by John von Neumann in 1945

Merge sort example



Merging sorted halves



Merge halves code (Java)

```
// Merges the left/right elements into a sorted result.
// Precondition: left/right are sorted
public static void merge(int[] result, int[] left, int[] right) {
    int i1 = 0;   // index into left array
    int i2 = 0;   // index into right array

    for (int i = 0; i < result.length; i++) {
        if (i2 >= right.length ||
           (i1 < left.length && left[i1] <= right[i2])) {
            result[i] = left[i1];    // take from left
            i1++;
        } else {
            result[i] = right[i2];   // take from right
            i2++;
        }
    }
}
```

Merge sort code 2

```
// Rearranges the elements of a into sorted order using
// the merge sort algorithm (recursive).
public static void mergeSort(int[] a) {
    if (a.length >= 2) {
        // split array into two halves
        int[] left  = Arrays.copyOfRange(a, 0, a.length/2);
        int[] right = Arrays.copyOfRange(a, a.length/2, a.length);

        // sort the two halves
        mergeSort(left);
        mergeSort(right);

        // merge the sorted halves into a sorted whole
        merge(a, left, right);
    }
}
```

Suggested helpers

- Write a function `split` that accepts a list and produces a tuple of two lists representing its even and odd indexes.
  - `split([12, ~3, 0, 19, 1])` produces `([12, 0, 1], [~3, 19])`

- Write a function `merge` that accepts two sorted lists and produces a new merged sorted list.
  - `merge([4, 9, 11], [~3, 2, 10])` produces `[~3, 2, 4, 9, 10, 11]`

Helper solutions

```
(* Splits a list into 2 sublists of its even/odd indexes. *)
fun split([]) = ([], [])
|   split([x]) = ([x], [])
|   split(first :: second :: rest) =
        let val (l1, l2) = split(rest)
        in (first :: l1, second :: l2)
        end;

(* Merges sorted L1 and L2 into a sorted whole. *)
fun merge([], L2) = L2
|   merge(L1, []) = L1
|   merge(L1 as first1 :: rest1, L2 as first2 :: rest2) =
        if   first1 < first2
        then first1 :: merge(rest1, L2)
        else first2 :: merge(L1, rest2);
```

Merge sort solution

```
(* Rearranges the elements of the given list to be in
   non-decreasing order using the merge sort algorithm. *)
fun mergeSort([]) = []
|   mergeSort([value]) = [value]
|   mergeSort(lst) =
        let
            val (left, right) = split(lst)
        in
            merge(mergeSort(left), mergeSort(right))
        end;
```

Efficiency exercise

- Write a function called reverse that accepts a list and produces the same elements in the opposite order.
  - `reverse([6, 2, 9, 7])` produces `[7,9,2,6]`

- Write a function called `range` that accepts a maximum integer value *n* and produces the list [1, 2, 3, ..., *n*-1, *n*]. Produce an empty list for all numbers less than 1.
  - Example: `range(5)` produces `[1,2,3,4,5]`

Flawed solutions

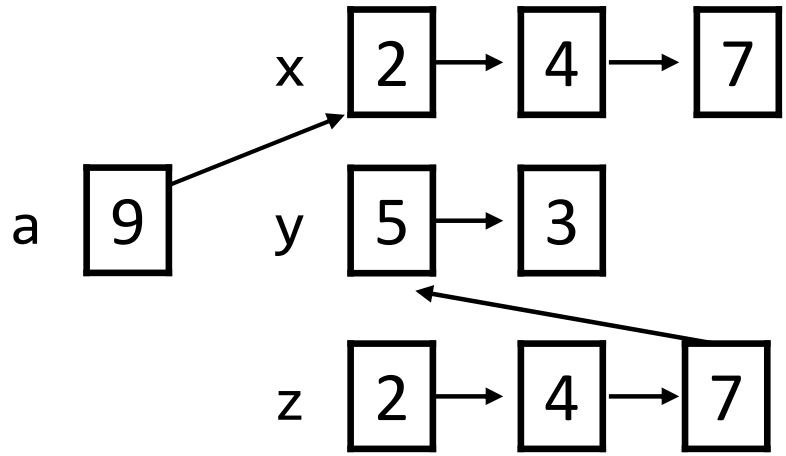
- These solutions are correct; but they have a problem...

```
fun reverse([]) = []
|   reverse(first :: rest) =
            reverse(rest) @ [first];

fun range(0) = []
|   range(n) = range(n - 1) @ [n];
```

Efficiency of the @ operator

```
val x = [2, 4, 7];
val y = [5, 3];
val a = 9 :: x;
val z = x @ y;
```



- The `::` operator is fast: O(1)
  - simply creates a link from the first element to front of right
- The @ operator is slow: O(*n*)
  - must walk/copy the left list and then append the right one
  - using @ in a recursive function *n* times : function is O(n^2)

Fixing inefficient recursion

- How can we improve the inefficient `range` code?

```
fun range(0) = []
|   range(n) = range(n - 1) @ [n];
```

  - *Hint:* Replace `@` with `::` as much as possible.

Better solution

```
fun range(n) =
    let
        fun helper(min, max) =
            if min = max then [min]
            else min :: helper(min + 1, max)
    in
        helper(1, n)
    end;
```

More about efficiency

- The `fibonacci` function we wrote previously is also inefficient, for a different reason.
  - It makes an exponential number of recursive calls.
  - Example: fibonacci(5)
    - fibonacci(4)
      - fibonacci(3)
        - » fibonacci(2)
        - » fibonacci(1)
      - fibonacci(2)
    - fibonacci(3)
      - fibonacci(2)
      - fibonacci(1)
  - How can we fix it to make fewer (O(*n*)) calls?

Iterative Fibonacci in Java

```
// Returns the nth Fibonacci number.
// Precondition: n >= 1
public static int fibonacci(int n) {
    if (n == 1 || n == 2) {
        return 1;
    }
    int curr = 1;    // the 2 most recent Fibonacci numbers
    int prev = 1;

    // k stores what fib number we are on now
    for (int k = 2; k < n; k++) {
        int next = curr + prev;   // advance to next
        prev = curr;              // Fibonacci number
        curr = next;
    }
    return curr;
}
```

Efficient Fibonacci in ML

```
(* Returns the nth Fibonacci number.
   Precondition: n >= 1 *)
fun fib(1) = 1
|   fib(2) = 1
|   fib(n) =
        let
            fun helper(k, prev, curr) =
                if k = n then curr
                else helper(k + 1, curr, prev + curr)
        in
            helper(2, 1, 1)
        end;
```