CSE 341 Assignment 4

January 31, 2006

Due: Thursday, February 2nd, at 11pm. No late assignments will be accepted. Submit your assignment using the online turnin link on the course web.

Calculus

This assignment explores using higher-order functions to solve a real problem, namely calculating the derivatives and integrals of functions operating on real numbers. The algorithms to do this numerically are outlined here in case your calculus is rusty.

Recall that the derivative of a function $f(x)$ with respect to the variable x is defined as

$$f'(x) = \frac{d}{dx} f(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

To approximate a derivative numerically, simply choose some Δx close to 0 instead of taking the limit.

Approximating an integral is somewhat more complicated. For a function $f(x)$ over the interval $[a, b]$, divide the interval into n subintervals of width $\Delta x = \frac{(b-a)}{n}$, and choose a point from each interval x_i. The definite integral is:

$$\int_a^b f(x)\, dx = \lim_{x \to \infty} \sum_{i=1}^{n} f(x_i) \Delta x$$

As n approaches infinity, the result approaches the true integral, so we can pick a very large n, iterate through values of x between a and b in increments of Δx, find the value of f at each x and multiply by Δx, and then sum the terms. There are other methods, such as the trapezoid method, that give more accurate results; if you know them, you may use them instead of the algorithm here. The definition of an indefinite integral is simple once the definite integral is defined. Let $F(b)$ be the indefinite integral of $f(x)$. It is defined as:

$$F(b) = \int f(x) dx. = \int_0^b f(x) dx$$

A Taylor series is a series expansion of a function about a point. A one-dimensional Taylor series is an expansion of function $f(x)$ about a point $x = a$ is given by

$$f(x) = f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \frac{f^{(3)}(a)}{3!}(x-a)^3 + \ldots + \frac{f^{(n)}(a)}{n!}(x-a)^n + \ldots$$

Which can be rearranged to

$$f(x) = \sum_{k=0}^{n} \frac{(x-a)^k f^{(k)}(a)}{k!} + R_n$$

where R_n is the remainder term known as the Lagrange remainder, which is given by

$$R_n = \int_a^x f^{n+1}(a)\frac{(x-a)^n}{n!}dt$$

We will use the following constants to make writing these functions easier:

```
val bigNumber = 100000.0
val smallNumber = 1.0 / bigNumber
```

For additional information regarding any of these equations and concepts, you should go to www.mathworld.com, which has detailed information on each of the calculus topics covered here.

You will now implement several functions related to the problems described above. For all these functions, the function f that you will be provided with be of type `real → real`. Implement the following functions:

1. Curried function `derivative f x` which evaluates to the derivative of $f(x)$ when provided with some `real` x.
2. Curried function `def_integral f a b` which returns the integral of $f(x)$ between a and b. For convenience, you may use the special syntax `fun def_integral f a b = ...`.
3. Function `integral f` which returns the indefinite integral of f and makes use of `def_integral`
4. Curried function `derivative_i f i` that returns f^i
5. Curried function `taylor f a n x` that returns the taylor expansion about point $x = a$ calculated up to the n-th derivative

Sample Log Execution

- fun h x = Math.pow(x, 9.0) + Math.pow(x, 3.0)
val h = fn : real → real
- val g = derivative h
val g = fn : real → real
- g 2.0
val it = 2316.00005377 : real
- val g = derivative_i h 3
val g = fn : real → real
- g 0.0
val it = 6.0 : real
*-def_integral (fn x ⇒ 2.0*x-Math.pow(x,˜2.0)) 1.0 2.0*
val it = 2.49986249853 : real
-val g = taylor Math.sin 0.0
val g = fn : int → real → real
- g 3 (Math.pi/2.0)
val it = 1.00003447921: real
*- integral (fn x ⇒ x*x) 3.0*
val it = 8.998650045 : real