CSE 332 Winter 2026
Lecture 10: hashing

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Dictionary (Map) ADT

- Contents:
  - Sets of key+value pairs
  - ~~Keys must be comparable~~
- Operations:
  - insert(key, value)
    - Adds the (key,value) pair into the dictionary
    - If the key already has a value, overwrite the old value
      - Consequence: Keys cannot be repeated
  - find(key)
    - Returns the value associated with the given key
  - delete(key)
    - Remove the key (and its associated value)

Dictionary Data Structures

Data Structure	Time to insert	Time to find	Time to delete
Unsorted Array	$\Theta(1)$	$\Theta(n)$	$\Theta(n)$
Unsorted Linked List	$\Theta(1)$	$\Theta(n)$	$\Theta(n)$
Sorted Array	$\Theta(n)$	$\Theta(\log n)$	$\Theta(n)$
Sorted Linked List	$\Theta(n)$	$\Theta(n)$	$\Theta(n)$
Heap	$\Theta(\log n)$	$\Theta(n)$	$\Theta(n)$
Binary Search Tree	$\Theta(\text{height})$	$\Theta(\text{height})$	$\Theta(\text{height})$
AVL Tree	$\Theta(\log n)$	$\Theta(\log n)$	$\Theta(\log n)$

BSTs and AVL Trees

- Binary Search Tree:
  - A binary tree where for each node, all keys in its left subtree are smaller and all keys in its right subtree are larger
  - Find:
    - If it matches, return the value.
    - If the search key is less than the current node, look left. If it’s greater, look right.
    - If we reach an empty spot, find was unsuccessful
  - Insert:
    - Do a find, if it was successful then update the value
    - If it was unsuccessful, add a new node to the empty spot we found.
  - Delete:
    - If the deleted node is a leaf, just remove it
    - If the deleted node had one child, replace it with that one child
    - If the deleted node had 2 children, replace it with the largest key to the left
- AVL Tree:
  - A binary search tree where for each node, the height of its left subtree and the height of its right subtree are off by at most 1.
  - Find:
    - Same as BST
  - Insert:
    - Do a BST insert, then rotate if tree is unbalanced (apply one LL, RR, LR, RL case)
  - Delete:
    - Do a BST delete, then rotate if the tree is unbalanced (apply LL, RR, LR, RL cases as needed from leaf to root)

Other Tree-based Dictionaries

- Red-Black Trees
  - Similar to AVL Trees in that we add shape rules to BSTs
  - More “relaxed” shape than an AVL Tree
    - Trees can be taller (though not asymptotically so)
    - Needs to move nodes less frequently
  - This is what Java’s TreeMap uses!
- Tries
  - Similar to a Huffman Tree
  - Requires keys to be sequences (e.g. Strings)
  - Combines shared prefixes among keys to save space
  - Often used for text-based searches
    - Web search
    - Genomes

Next topic: Hash Tables

Data Structure	Time to insert	Time to find	Time to delete
Unsorted Array	$\Theta(n)$	$\Theta(n)$	$\Theta(n)$
Unsorted Linked List	$\Theta(n)$	$\Theta(n)$	$\Theta(n)$
Sorted Array	$\Theta(n)$	$\Theta(\log n)$	$\Theta(n)$
Sorted Linked List	$\Theta(n)$	$\Theta(n)$	$\Theta(n)$
Binary Search Tree	$\Theta(\text{height})$	$\Theta(\text{height})$	$\Theta(\text{height})$
AVL Tree	$\Theta(\log n)$	$\Theta(\log n)$	$\Theta(\log n)$
Hash Table (Worst case)	$\Theta(n)$	$\Theta(n)$	$\Theta(n)$
Hash Table (Average)	$\Theta(1)$	$\Theta(1)$	$\Theta(1)$

Dictionary (Map) ADT

- Contents:
  - Sets of key+value pairs
  - ~~Keys must be comparable~~ Keys have a hash function
- Operations:
  - insert(key, value)
    - Adds the (key,value) pair into the dictionary
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  - find(key)
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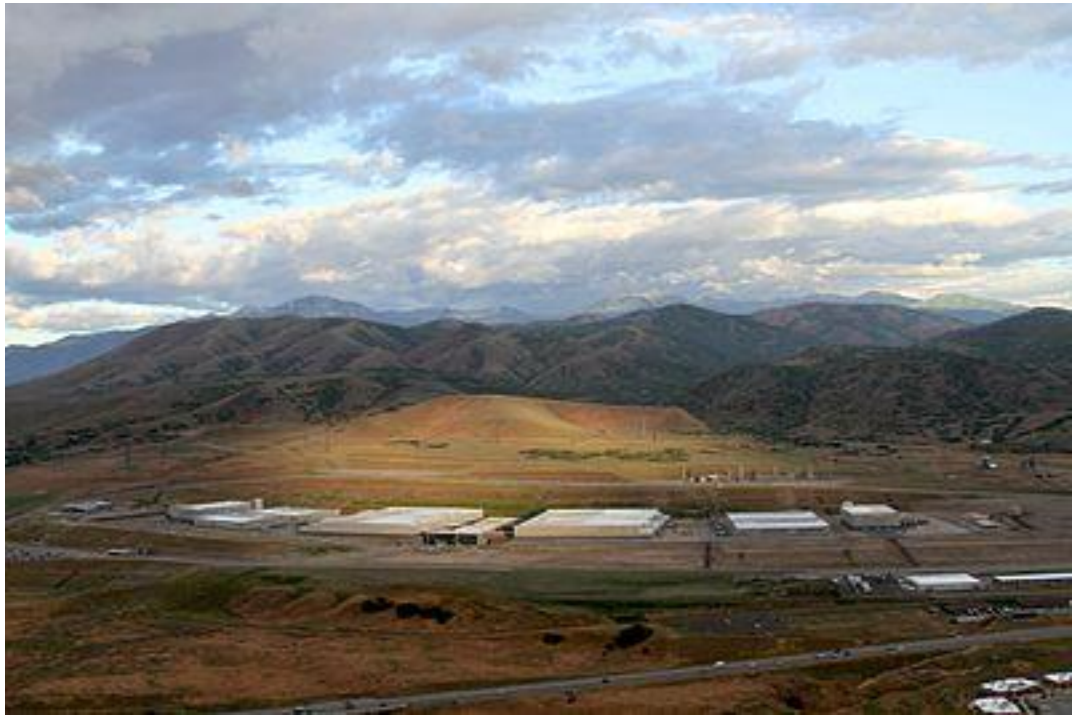
The Best Dictionary Data Structure!

- Think of every key as a number
- Give each key its own index in an array

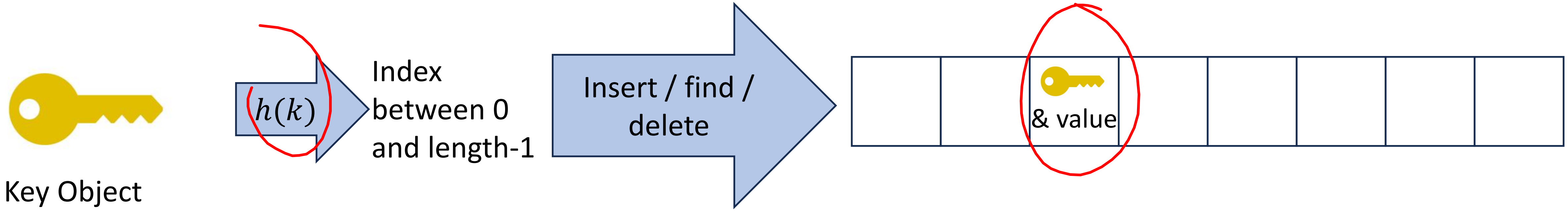
```
insert(key, value){
    arr[key]=value;
}
find(key){
    return arr[key];
}
delete(key){
    arr[key] = null;
}
```

Problem?



Hash Tables

- Idea:
  - Have a small array to store information
  - Use a **hash function** to convert the key into an index
    - Hash function should "scatter" the keys, behave as if it randomly assigned keys to indices
  - Store key at the index given by the hash function
  - Do something if two keys map to the same place (should be very rare)
    - Collision resolution



Example

0	1	2	3	4	5	6	7	8	9

- Key: Phone Number
- Value: People
- Table size: 10
- $h(phone)$ = number as an integer % 10
- $h(8675309) = 9$

What Influences Running time?

- How long hashing itself takes
- Likelihood of collisions
  - Size of the array vs number of values in the array
  - “quality” of our hash function
- What we do when we have a collision

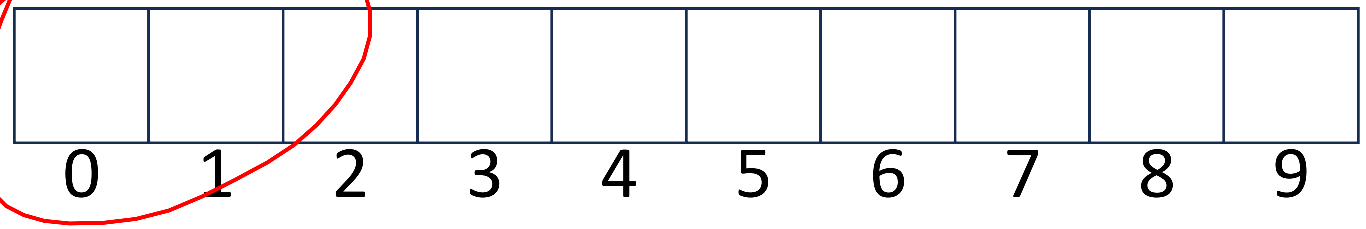
Properties of a “Good” Hash

- Definition: A hash function maps objects to integers

- **Consistent**
  - Objects considered “equal” should hash to the same value
  - Deterministic: running the hash function on the same object twice should yield the same result
- **Uniform**
  - Should be able to use every index in a fixed-size array
  - Should use every index at roughly equal rates
- **Effective**
  - It should be difficult to find two objects which hash to the same value
  - Given on object, it should be hard to find a different object which hashes to the same value
  - “Avalanche effect”: making a small change to the object yields big changes in the value it hashes to
- **Efficient**
  - Time to calculate the hash should be very small

A Bad Hash (and phone number trivia)

- $h(phone)$ = the first digit of the phone number
  - Assume 10-digit format
  - No US phone numbers start with 1 or 0
  - If we're sampling from this class, 2 is by far the most likely
- Consistent? Yes!
- Uniform? No!
- Effective? No!
- Efficient? Yes!



Compare These Hash Functions (for strings)

- Let $s = s_0 s_1 s_2 \dots s_{m-1}$ be a string of length m
  - Let $a(s_i)$ be the ascii encoding of the character s_i
- $h_1(s) = a(s_0)$

prime

- $h_2(s) = \left(\sum_{i=0}^{m-1} a(s_i)\right)$
- $h_3(s) = \left(\sum_{i=0}^{m-1} a(s_i) \cdot 37^i\right)$

$x = (a(s_i) \cdot x) 3 >$

- $h_4(s) = \left(2 \cdot \sum_{i=0}^{m-1} a(s_i) \cdot 37^i\right)$

Compare These Hash Functions (for strings)

- Let $s = s_0 s_1 s_2 \dots s_{m-1}$ be a string of length m
  - Let $a(s_i)$ be the ascii encoding of the character s_i
- $h_1(s) = a(s_0)$
  - Is: consistent, efficient
- $h_2(s) = \left(\sum_{i=0}^{m-1} a(s_i)\right)$
  - Is: consistent, efficient, and possibly uniform
- $h_3(s) = \left(\sum_{i=0}^{m-1} a(s_i) \cdot 37^i\right)$
  - Is: Consistent, efficient, uniform, and effective
- $h_4(s) = \left(2 \cdot \sum_{i=0}^{m-1} a(s_i) \cdot 37^i\right)$
  - Is: Consistent, efficient, effective

Ideal Insert procedure

Supposing we have a “good” hash function:

```
insert(key, value){
    h = key.hash();
    arr[h % table.length] = value;
}
```

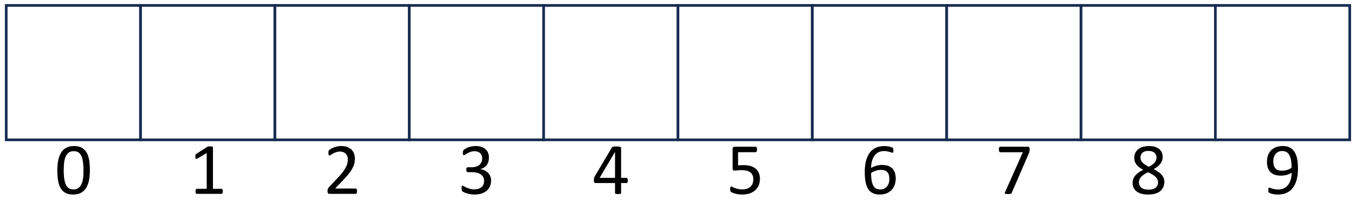
arr

Problem: It’s possible that two different keys map to the same index!

This is called a “collision”

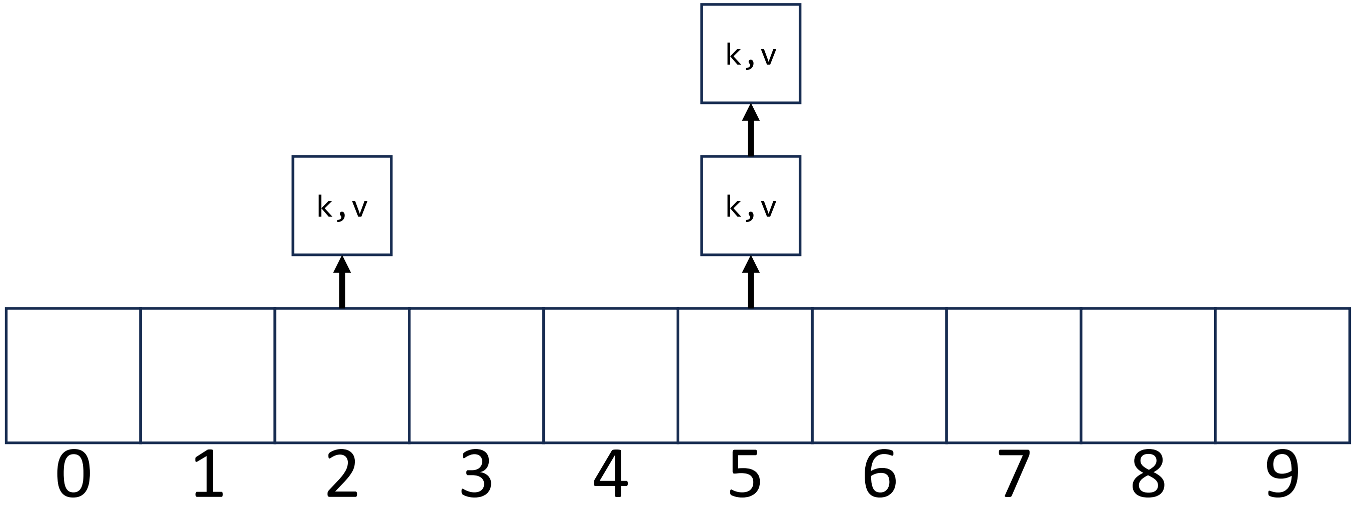
Collision Resolution

- A Collision occurs when we want to insert something into an already-occupied position in the hash table
- 2 main strategies:
  - Separate Chaining
    - Use a secondary data structure to contain the items
      - E.g. each index in the hash table is itself a linked list
  - Open Addressing
    - Use a different spot in the table instead
      - Linear Probing
      - Quadratic Probing
      - Double Hashing



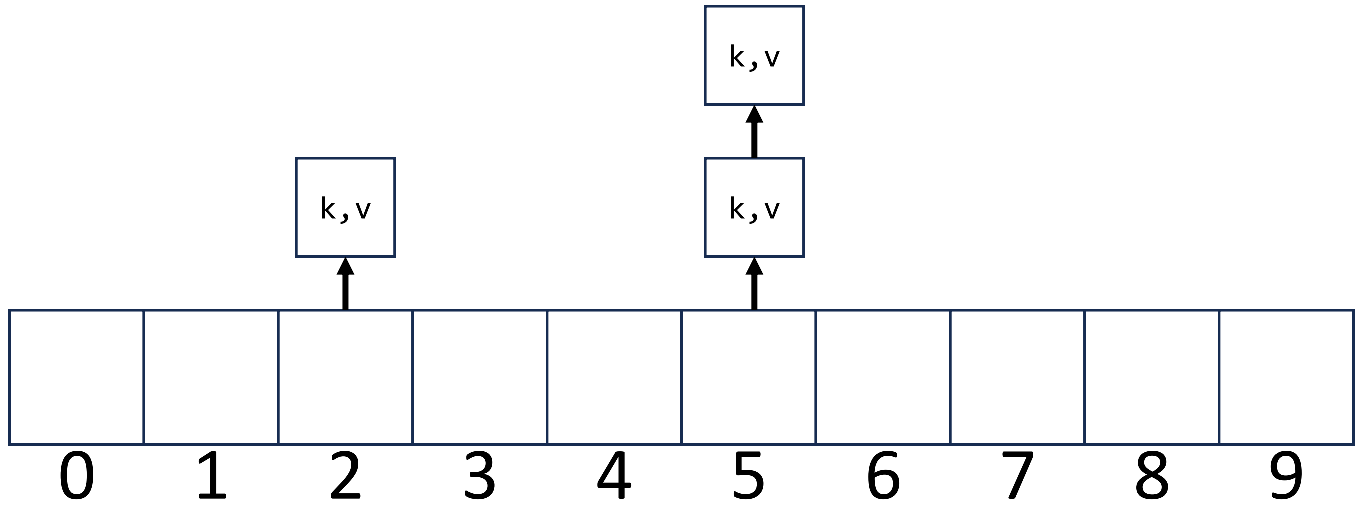
Separate Chaining Insert

- To insert `k, v`:
  - Compute the index using `i = h(k) % table.length`
  - Add the key-value pair to the data structure at `table[i]`



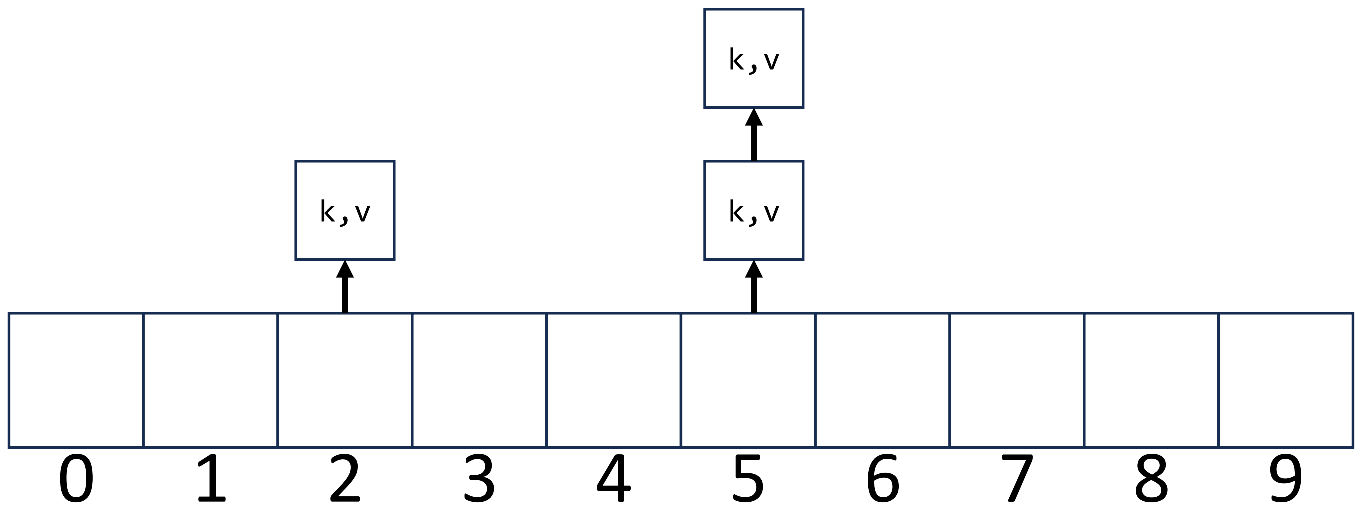
Separate Chaining Find

- To find k:
  - Compute the index using `i = h(k) % table.length`
  - Call find with the key on the data structure at `table[i]`



Separate Chaining Delete

- To delete k:
  - Compute the index using `i = h(k) % table.length`
  - Call delete with the key on the data structure at `table[i]`



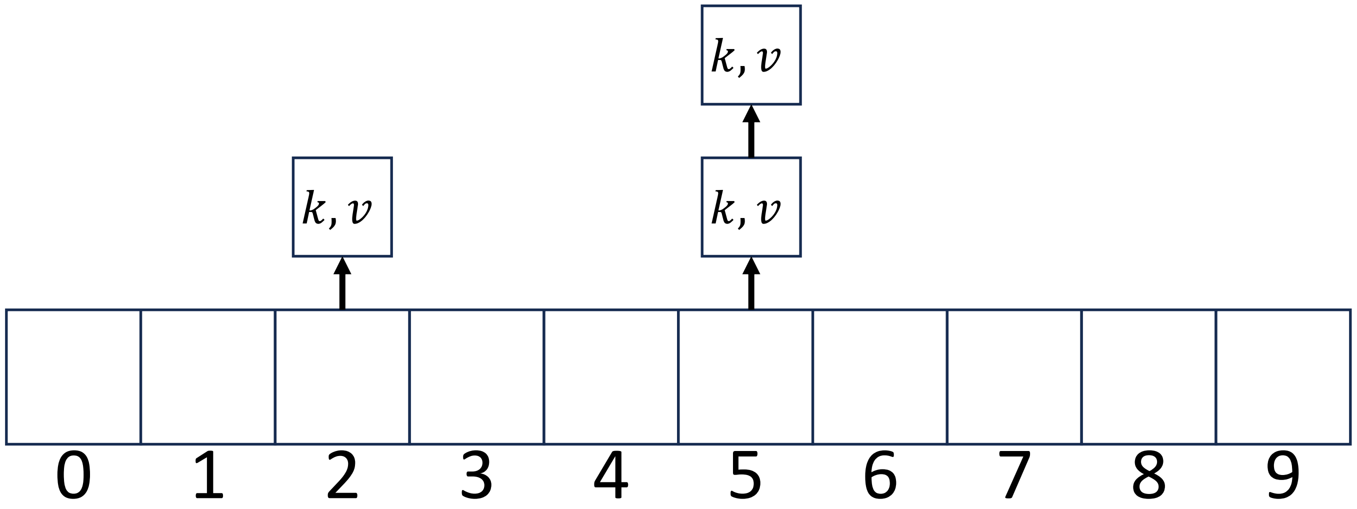
Formal Running Time Analysis

- The **load factor** of a hash table represents the average number of items per “bucket”
  - $\lambda = \frac{n}{length}$
- Assume we have a hash table that uses a linked-list for separate chaining
  - What is the expected number of comparisons needed in an unsuccessful find?
  - What is the expected number of comparisons needed in a successful find?
- How can we make the expected running time $\Theta(1)$?

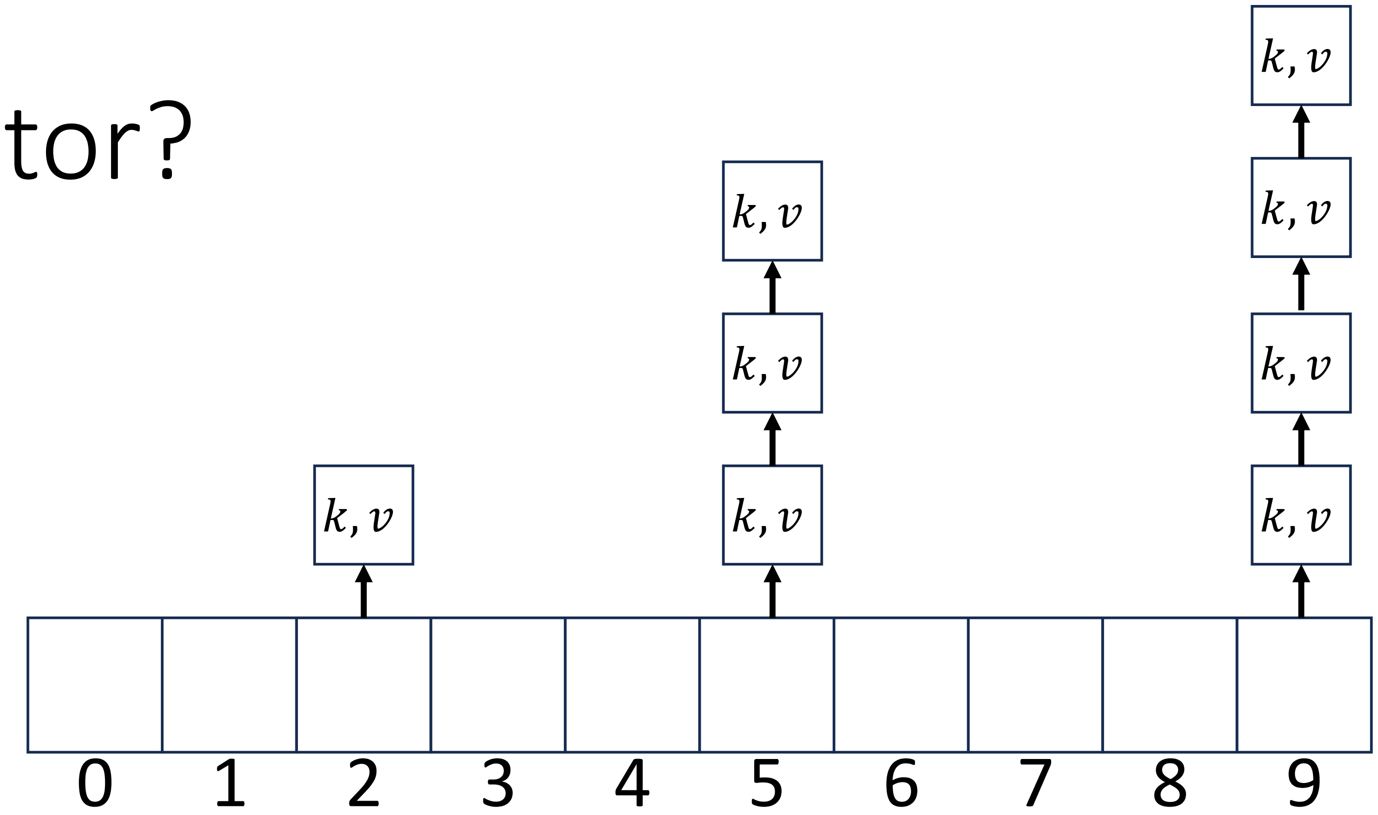
Formal Running Time Analysis

- The **load factor** of a hash table represents the average number of items per "bucket"
  - $\lambda = \frac{n}{length}$
- Assume we have a hash table that uses a linked-list for separate chaining
  - What is the expected number of comparisons needed in an unsuccessful find?
    - λ
  - What is the expected number of comparisons needed in a successful find?
    - $\frac{\lambda}{2}$
- How can we make the expected running time $\Theta(1)$?
  - Pick a constant value, resize the array whenever λ exceeds that constant
    - We'll talk about which constant we should pick later

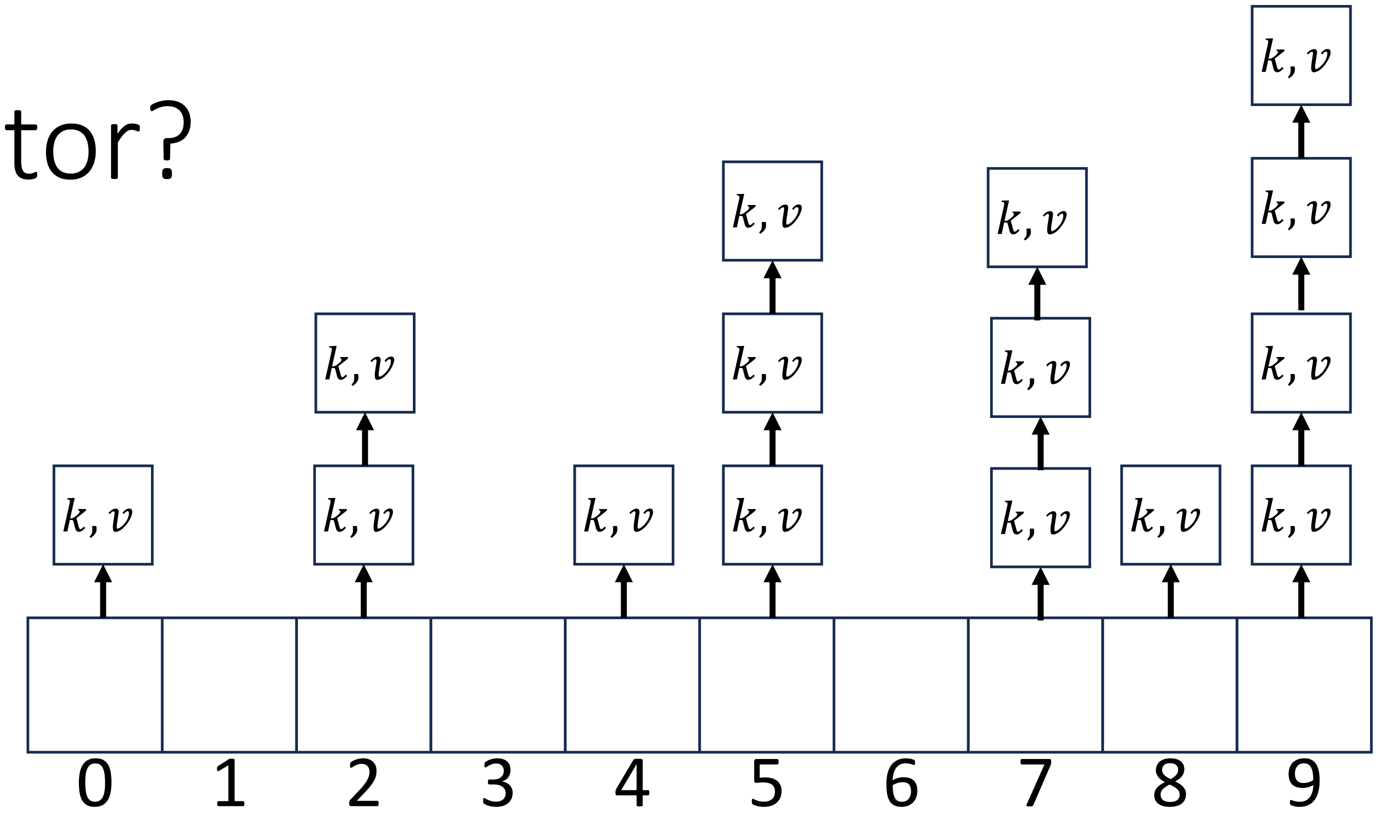
Load Factor?



Load Factor?



Load Factor?



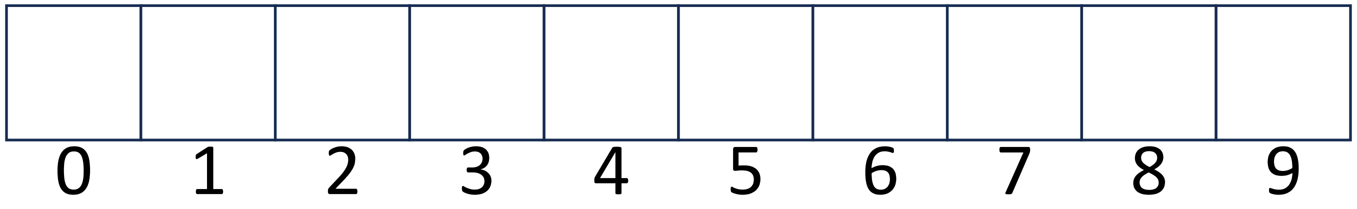
Collision Resolution: Linear Probing

- When there’s a collision, use the next open space in the table

0	1	2	3	4	5	6	7	8	9

Linear Probing: Insert Procedure

- To insert `k,v`
  - Calculate `i = h(k) % table.length`
  - If `table[i]` is occupied then try index `(i+1) % table.length`
  - If that is occupied try index `(i+2) % table.length`
  - If that is occupied try index `(i+3) % table.length`
  - …



Linear Probing: Find

0	1	2	3	4	5	6	7	8	9

Linear Probing: Find

- To find key k
  - Calculate `i = h(k) % table.length`
  - If `table[i]` is occupied but doesn't have k, check `(i+1) % table.length`
  - If that is occupied and doesn't contain k, check `(i+2) % table.length`
  - If that is occupied and doesn't contain k, check `(i+3) % table.length`
  - Repeat until you either find k or else you reach an empty cell in the table

0	1	2	3	4	5	6	7	8	9

Linear Probing: Delete

- Suppose A, B, C, D, and E all hashed to 3
- Now let's delete B

Before:

			A	B	C	D	E		
0	1	2	3	4	5	6	7	8	9

After:

0	1	2	3	4	5	6	7	8	9

Linear Probing: Delete

- Suppose A, B, and E all hashed to 3, and C and D hashed to 5
- Now let's delete B

Before:

			A	B	C	D	E		
0	1	2	3	4	5	6	7	8	9

After:

0	1	2	3	4	5	6	7	8	9

Linear Probing: Delete

- Suppose A and E hashed to 3, and B,C, and D hashed to 4
- Now let’s delete B

Before:

			A	B	C	D	E		
0	1	2	3	4	5	6	7	8	9

After:

0	1	2	3	4	5	6	7	8	9

Linear Probing: Delete

- Let’s do this together!

0	1	2	3	4	5	6	7	8	9

Linear Probing: Delete

- To delete key k, where `h(k) % table.length = i`
  - Assume it is present
- Beginning at index `i`, probe until we find k(call this location index `j`)
- Mark `j` as empty (e.g. null), then…
  - Challenge: we need to make sure future finds could be successful
  - What if there were values that mapped to index `i` that appeared after `j`?
  - What if there were items that hashed to a value between `i` and `j` and appeared after `j` due to probing?

0	1	2	3	4	5	6	7	8	9

Linear Probing: Delete

- **Option 1 (harder)**: Plug the hole with other items in a way that makes probes behave correctly
- **Option 2 (easier)**: “Tombstone” deletion. Leave a special object that indicates an something was deleted from there
  - The tombstone does not act as an open space when finding (so keep looking after its reached)
  - When inserting you can replace a tombstone with a new item

k, v		k, v	k, v		k, v				
0	1	2	3	4	5	6	7	8	9

Linear Probing + Tombstone: Find

- To find key k
  - Calculate `i = h(k) % table.length`
  - While `table[i]` has a key other than k, set `i = (i+1) % table.length`
  - If you come across k return `table[i]`
  - If you come across an empty index, the find was unsuccessful

0	1	2	3	4	5	6	7	8	9

Linear Probing + Tombstone: Insert

- To insert `k,v`
  - Calculate `i = h(k) % table.length`
  - While `table[i]` has a key other than k, set `i = (i+1) % table.length`
    - If `table[i]` has a tombstone, set `x = i`
      - That is where we will insert if the find is unsuccessful
  - If you come across k, set `table[i] = k,v`
  - If you come across an empty index, the find was unsuccessful
    - Set `table[x] = k,v` if we saw a tombstone
    - Set `table[x] = k,v` otherwise

0	1	2	3	4	5	6	7	8	9