# CSE 332 Summer 2026
# Lecture 9: Hashing 2

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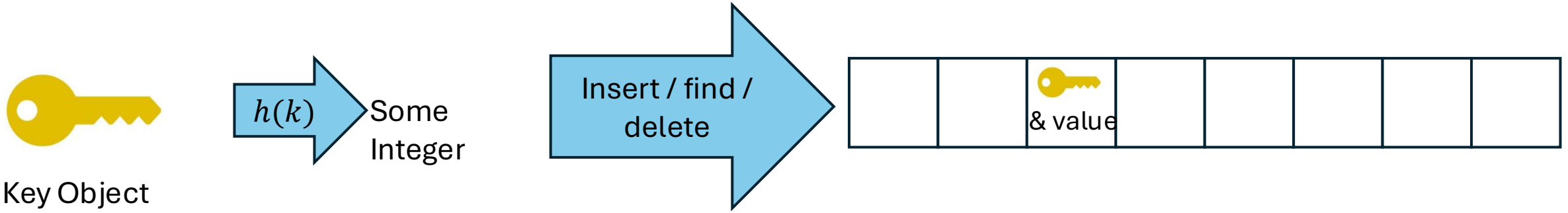
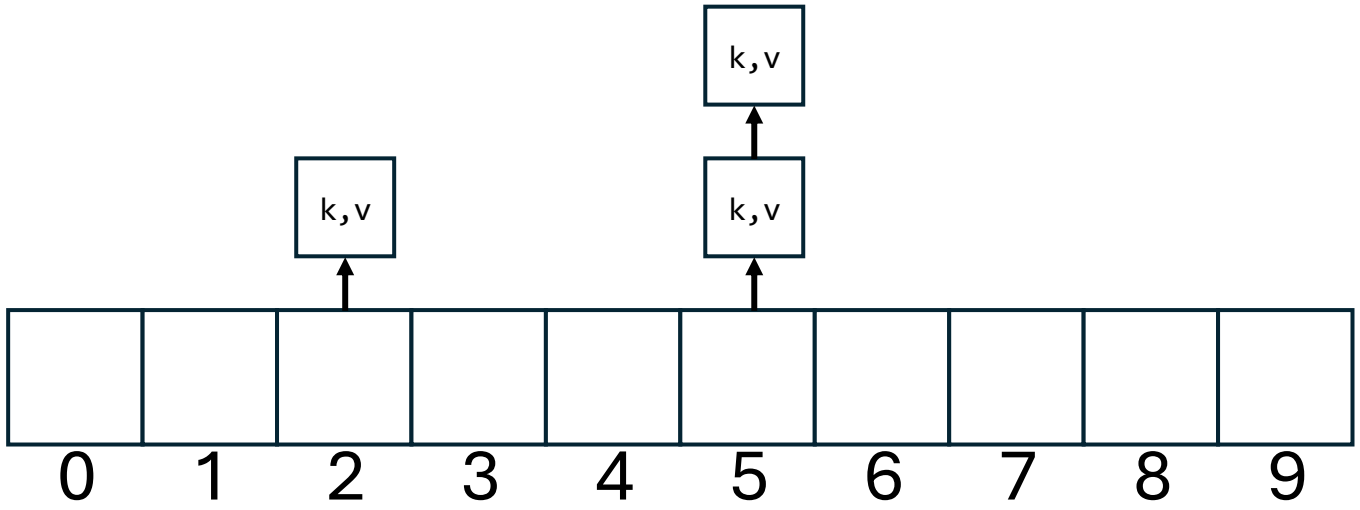
# Hash Tables

- Idea:
  - Have a small array to store information
  - Use a **hash function** to convert the key into an index
    - Hash function should “scatter” the keys, behave as if it randomly assigned keys to indices
  - Store key at the index given by the hash function
  - Do something if two keys map to the same place (should be very rare)
    - Collision resolution



# Separate Chaining Insert

- To insert `k, v`:
  - Compute the index using `i = h(k) % table.length`
  - Add the key-value pair to the data structure at `table[i]`



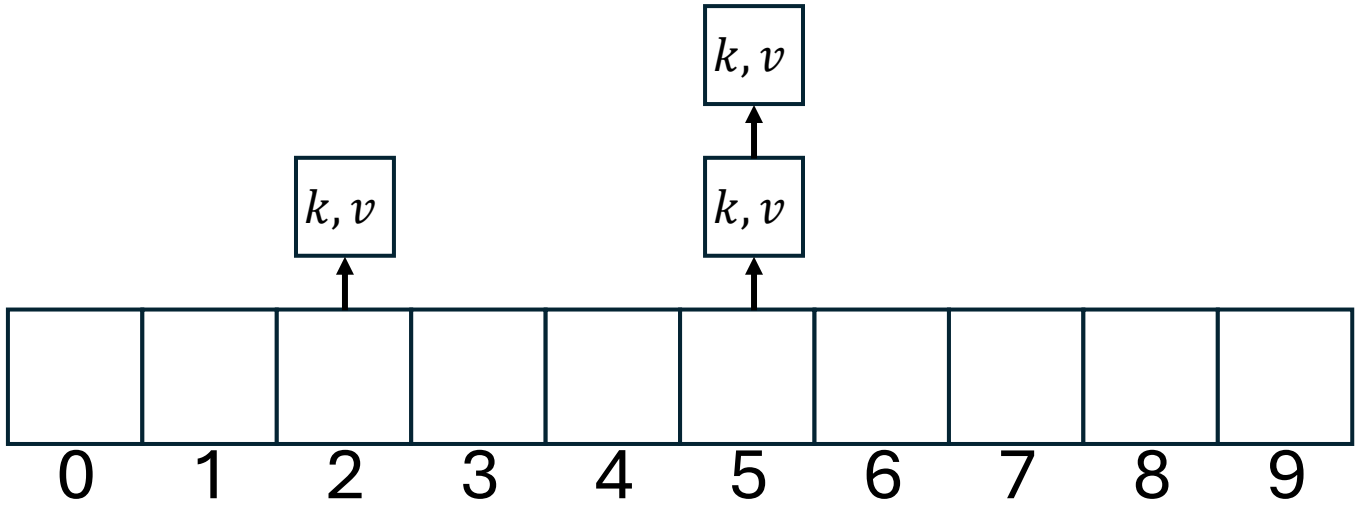
# **Separate Chaining** Runtime Analysis

- The **load factor** of a hash table represents the average number of items per "bucket"
  - $\lambda = \frac{n}{length}$
- Assume we have a hash table that uses a linked-list for separate chaining
  - What is the expected number of comparisons needed in an unsuccessful find?
  - What is the expected number of comparisons needed in a successful find?
- How can we make the expected running time $\Theta(1)$?

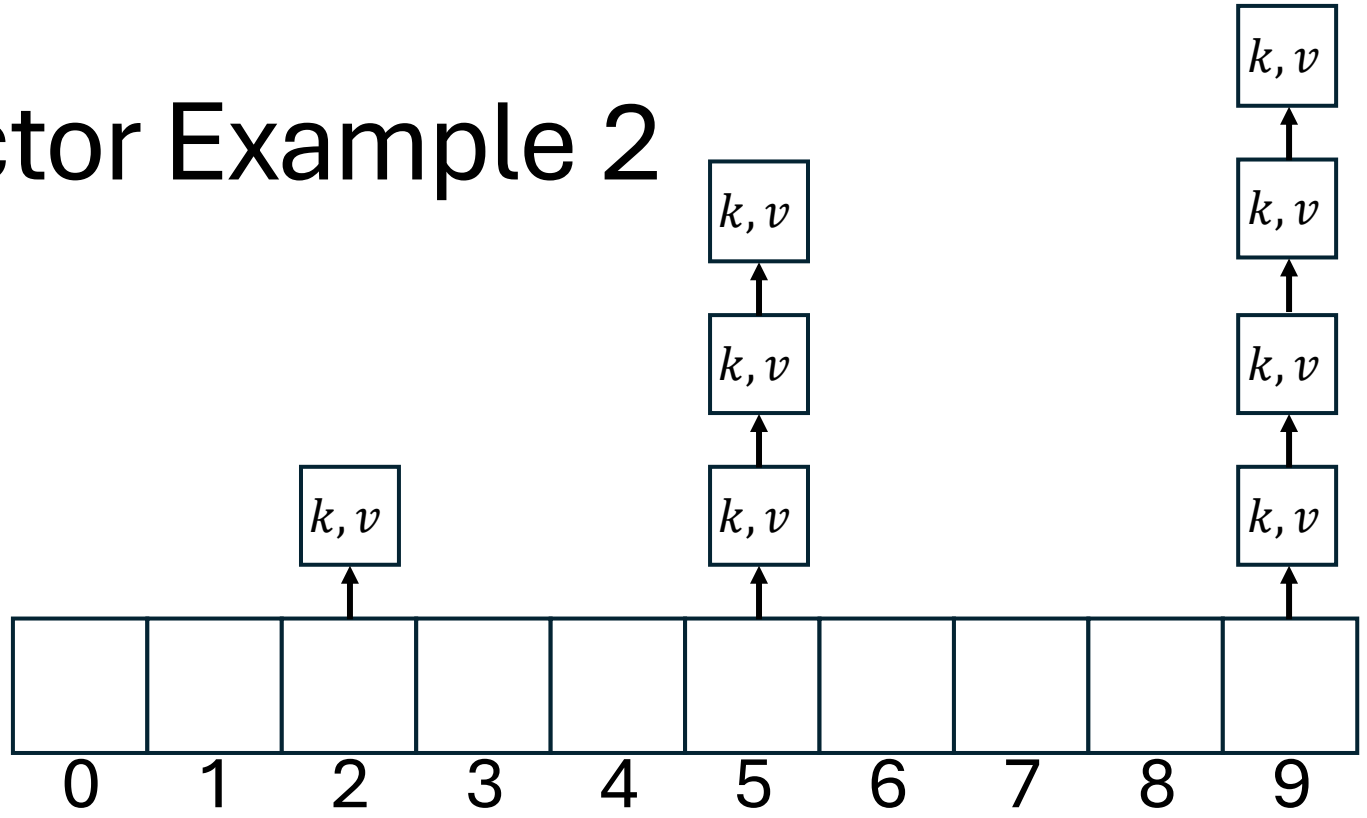
# **Separate Chaining** Runtime Analysis (answers)

- The **load factor** of a hash table represents the average number of items per "bucket"
  - $\lambda = \frac{n}{length}$
- Assume we have a hash table that uses a linked-list for separate chaining
  - What is the expected number of comparisons needed in an unsuccessful find?
    - $\lambda$
  - What is the expected number of comparisons needed in a successful find?
    - $\frac{\lambda}{2}$
- How can we make the expected running time $\Theta(1)$?
  - Pick a constant value, resize the array whenever $\lambda$ exceeds that constant

# Load Factor Example 1

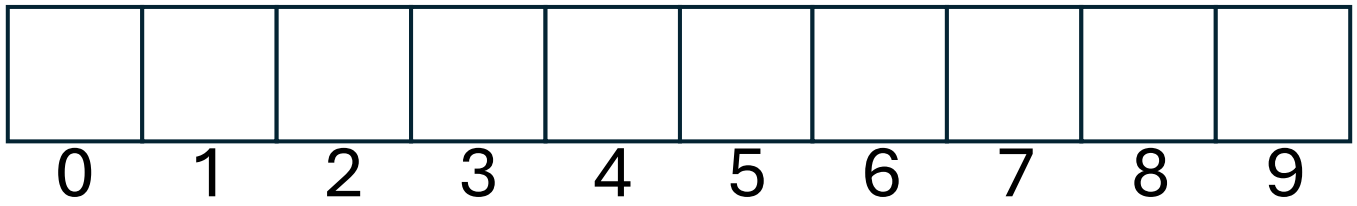


# Load Factor Example 2



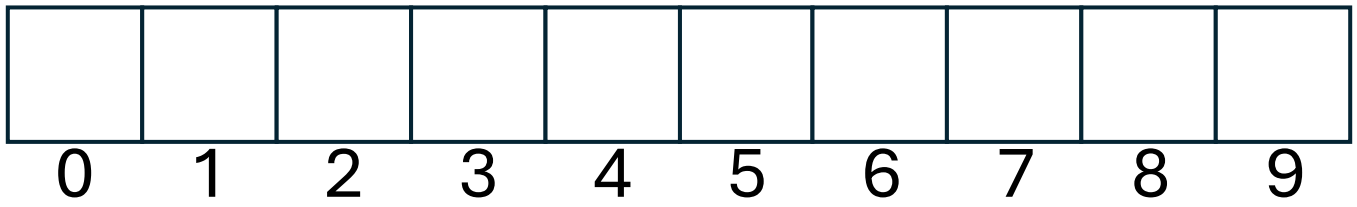
# Collision Resolution: **Linear Probing**

- When there's a collision, use the next open space in the table



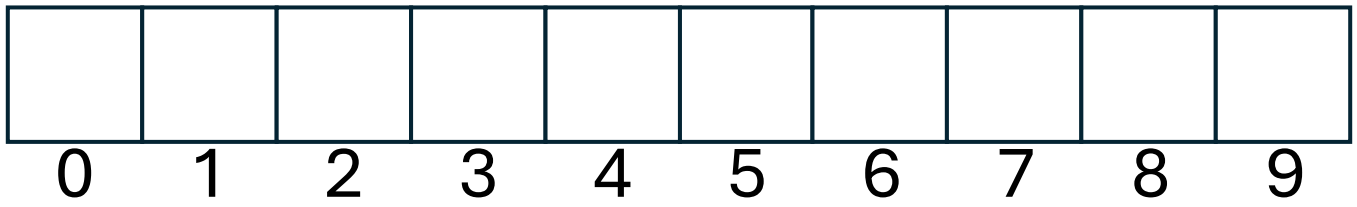
# Linear Probing: Insert Procedure

- To insert `k,v`
  - Calculate `i = h(k) % table.length`
  - If `table[i]` is occupied then try index `(i+1) % table.length`
  - If that is occupied try index `(i+2) % table.length`
  - If that is occupied try index `(i+3) % table.length`
  - …



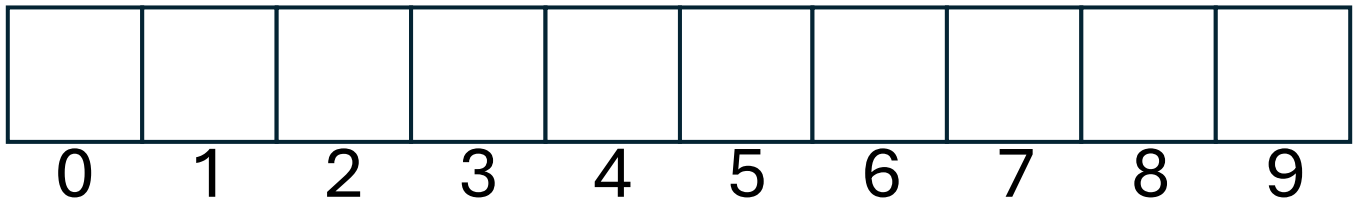
# Linear Probing: How to find?

- What do you think?



# Linear Probing: Find

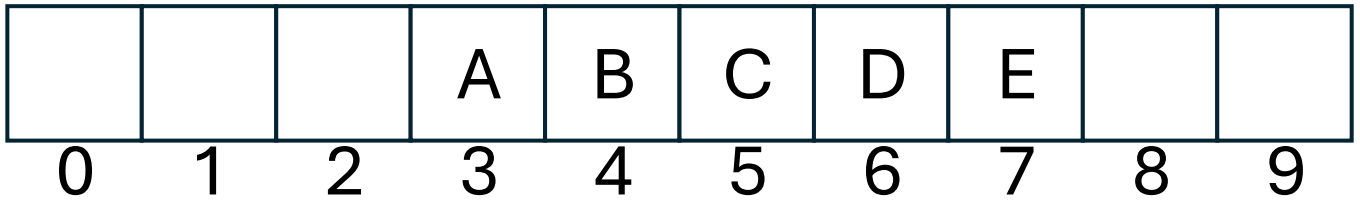
- To find key k
  - Calculate `i = h(k) % table.length`
  - If `table[i]` is occupied but doesn't have k, check `(i+1) % table.length`
  - If that is occupied and doesn't contain k, check `(i+2) % table.length`
  - If that is occupied and doesn't contain k, check `(i+3) % table.length`
  - Repeat until you either find $k$ or else you reach an empty cell in the table



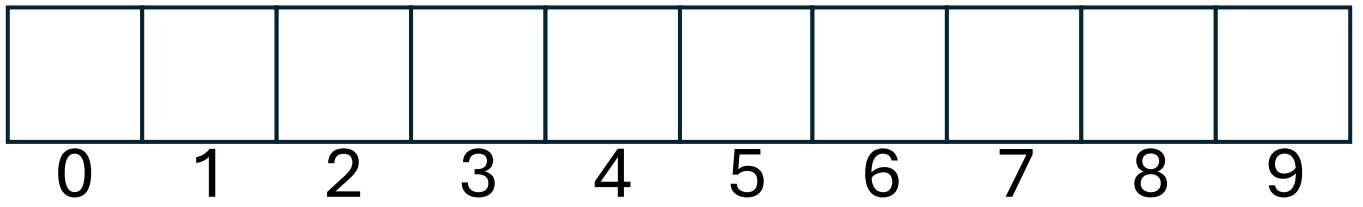
# Linear Probing: Delete is Hard (Example 1)

- Suppose we insert A, B, C, D, and E in that order, where all map to 3
- How should we delete B?

Before:




After:



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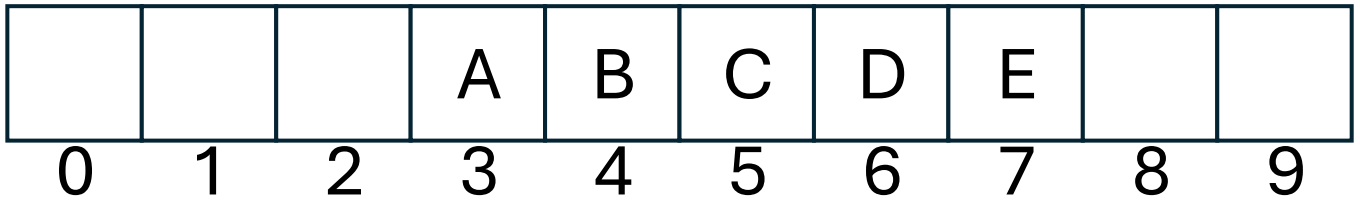
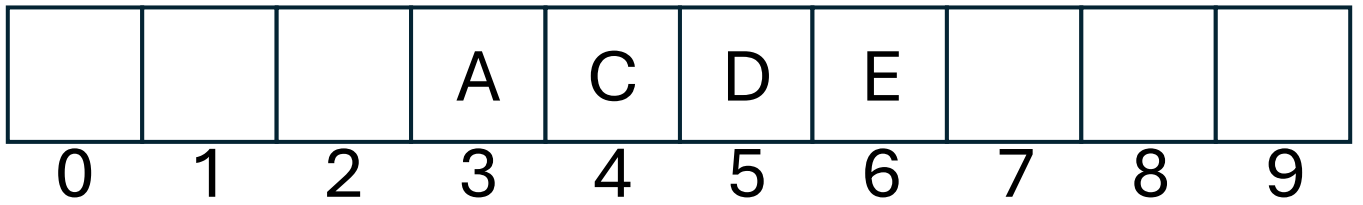



We need future probing to work correctly, so we must fill in the hole.

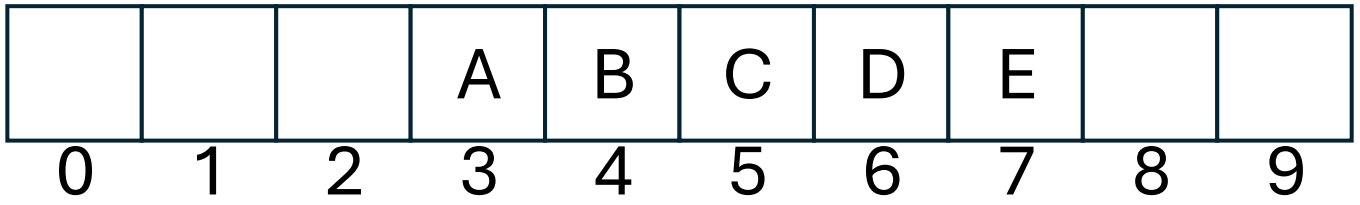
After:



# Linear Probing: Delete is Hard (Example 2)

- Suppose we insert A, B, C, D, and E in that order, where A,B,E all map to 3 and C,D map to 5.
- How should we delete B?

Before:

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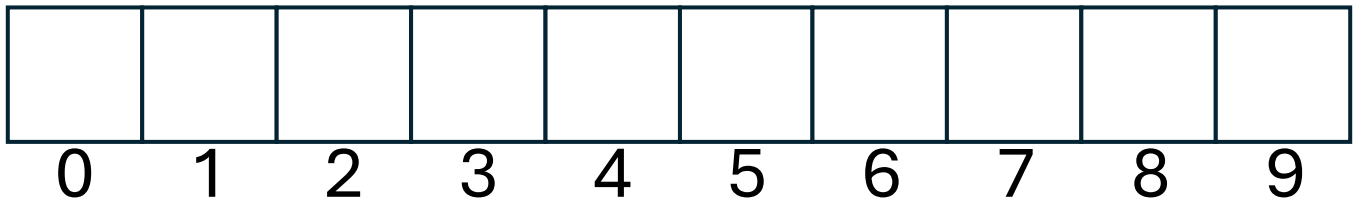

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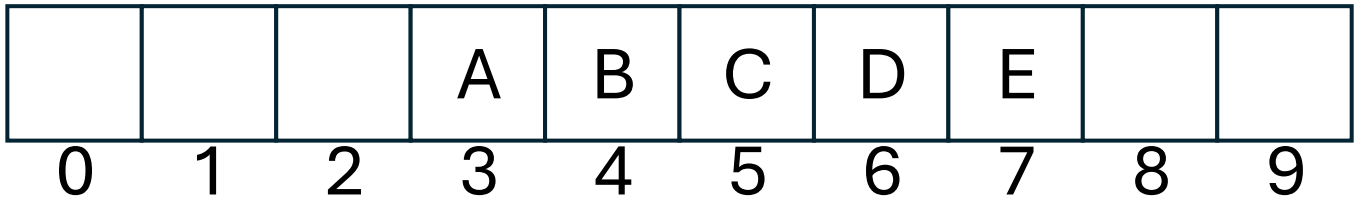
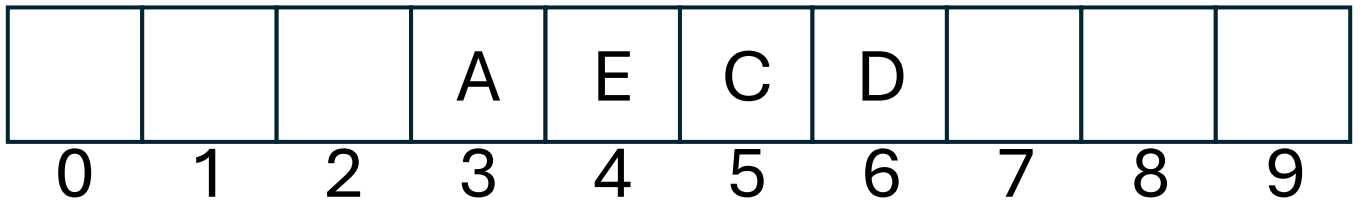



We need future probing to work correctly, so we must fill in the hole.

We cannot move C or D over because they map to an index later in the probe path

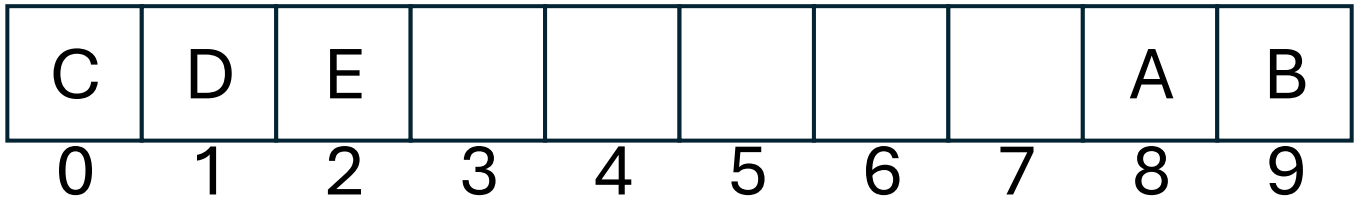
After:



# Linear Probing: Delete is Hard (Example 3)

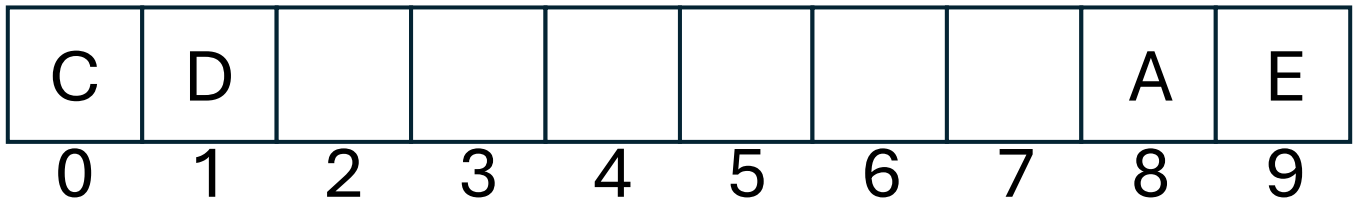
- Suppose we insert A, B, C, D, and E in that order, where A,B,E map to 8 and C,D map to 0.
- How should we delete B?

Before:



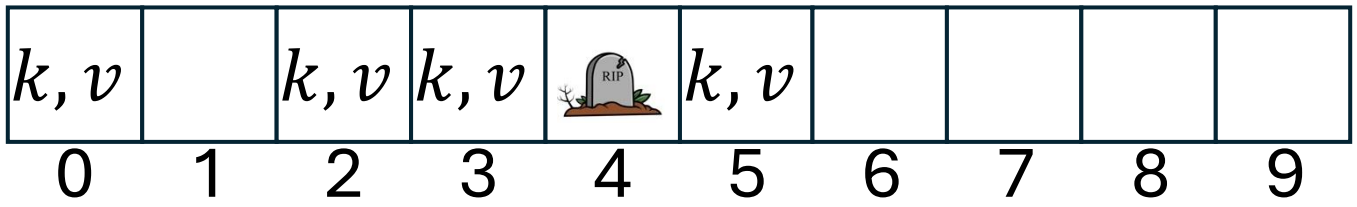

We cannot move C or D over because they map to an index later in the probe path, even though the index number happens to be smaller.

After:



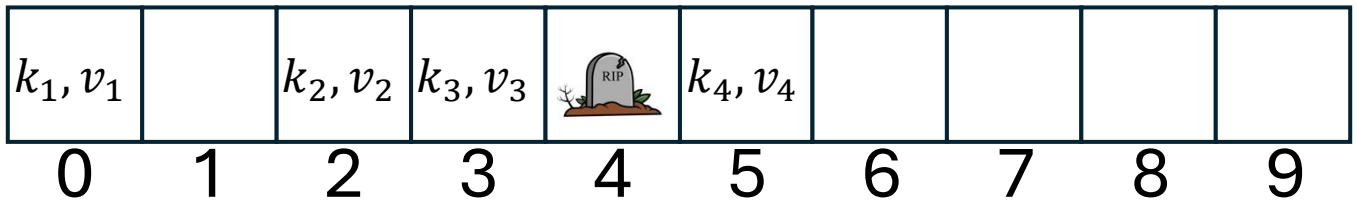
# Linear Probing: Delete

- **Option 1 (harder)**: Plug the hole with other items in a way that makes probes behave correctly
  - Rough idea: to delete at index $i$, starting from $i$ and going until we find an empty index, if probe path of the key at index $j$ has index $i$ before $j$ then move the key-value pair at $j$ to index $i$, then repeat on index $j$.
- **Option 2 (easier)**: "Tombstone" deletion. Leave a special object that indicates an something was deleted from there
  - The tombstone does not act as an open space when finding (so keep looking after its reached)
  - When inserting you can replace a tombstone with a new item



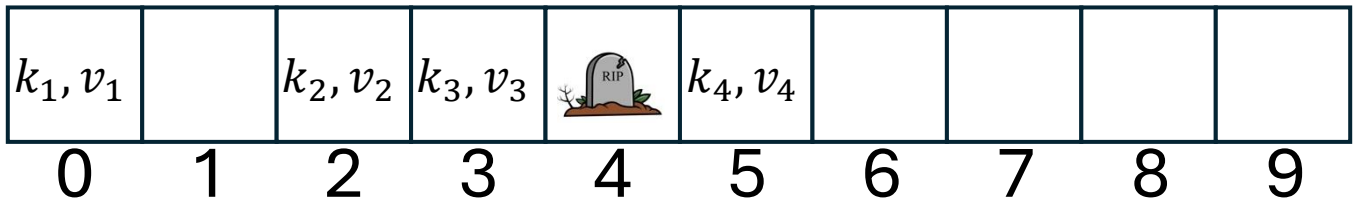
# Linear Probing + Tombstone: Find

- To find key k
  - Calculate `i = h(k) % table.length`
  - While `table[i]` has a key other than k, set `i = (i+1) % table.length`
  - If you come across k return `table[i]`
  - If you come across an empty index, the find was unsuccessful
    - Tombstones do not count as empty!



# Linear Probing + Tombstone: Insert

- To insert `k,v`
  - Calculate `i = h(k) % table.length`
  - While `table[i]` has a key other than k, set `i = (i+1) % table.length`
    - If `table[i]` has a tombstone, set `dest = i`
      - That is where we will insert if the find is unsuccessful
    - If you come across k, set `table[i] = k,v`
    - If you come across an empty index, the find was unsuccessful
      - Set `table[x] = k,v` if we saw a tombstone
    - Set `table[x] = k,v` otherwise

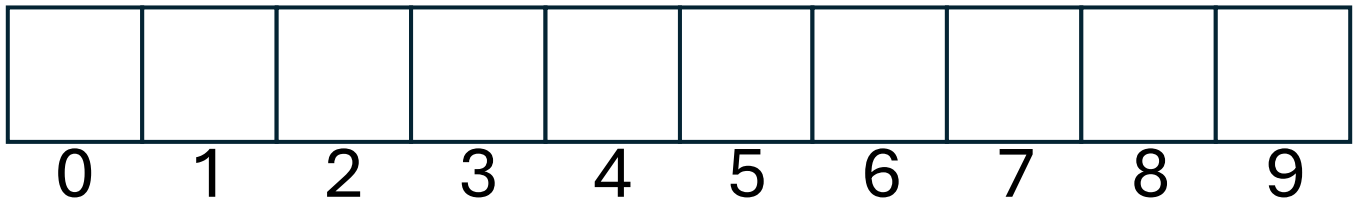


# Downsides of Linear Probing

- What happens when $\lambda$ approaches 1?
  - Get longer and longer contiguous blocks
  - A collision is guaranteed to grow a block
    - Larger blocks experience more collisions
    - Feedback loop!
- What happens when $\lambda$ exceeds 1?
  - Impossible!
  - You can’t insert more stuff

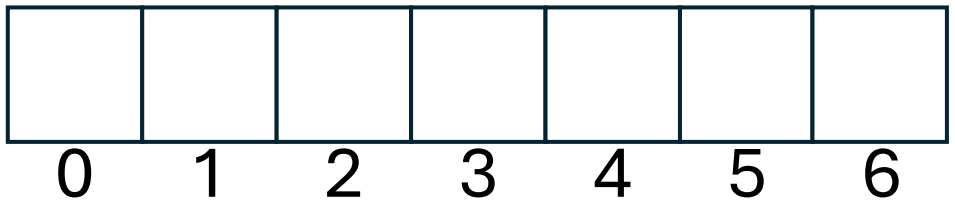
# Quadratic Probing: Insert Procedure

- To insert `k,v`
  - Calculate `i = h(k) % table.length`
  - If `table[i]` is occupied then try `(i+1²) % table.length`
  - If that is occupied try `(i+2²) % table.length`
  - If that is occupied try `(i+3²) % table.length`
  - If that is occupied try `(i+4²) % table.length`
  - …



# Quadratic Probing: Example

- Insert:
  - 76
  - 40
  - 48
  - 5
  - 55
  - 47



# Using Quadratic Probing

- If you probe `table.length` times, you start repeating indices
- If `table.length` is prime and $\lambda < \frac{1}{2}$ then you're guaranteed to find an open spot in at most `table.length`$/2$ probes

- Helps with the clustering problem of linear probing, but does not help if many things hash to the same value

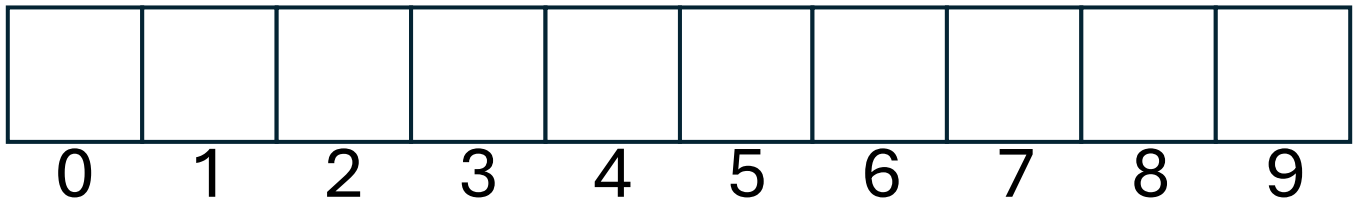
# Quadratic Load Factor Lemma

**Fact**: if $a \cdot b \equiv 0 \ (mod \ p)$ then either $a \equiv 0$ or $b \equiv 0$ (mod $p$)

- **Lemma:** $1^2, 2^2, \ldots, \left\lfloor \frac{p}{2} \right\rfloor^2$ are distinct mod $p$.
- **Proof:**

# Double Hashing: Insert Procedure

- Given h and g are both good hash functions
- To insert k, v
  - Calculate `i = h(k) % table.length`
  - If `table[i]` is occupied then try `(i+g(k)) % table.length`
  - If that is occupied try `(i+2*g(k)) % table.length`
  - If that is occupied try `(i+3*g(k)) % table.length`
  - If that is occupied try `(i+4*g(k)) % table.length`
  - …



# Rehashing

- If your load factor $\lambda$ gets too large, copy everything over to a larger hash table
  - To do this: make a new, larger array
  - Re-insert all items into the new hash table by reapplying the hash function
    - We need to reapply the hash function because items should map to a different index
  - New array should be "roughly" double the length (but probably still want it to be prime)
- What does "too large" mean?
  - For separate chaining, typically we want $\lambda < 2$
  - For open addressing, typically we want $\lambda < \frac{1}{2}$

# Sorting

- Rearrangement of items into some defined sequence
  - Usually: reordering a list from smallest to largest according to some metric
- Why sort things?
  - Enable things like binary search
    - It makes some algorithms faster
  - Nicer for human algorithms too
  - Data organization

# More Formal Definition

- Input:
  - An array $A$ of items
  - A comparison function for these items
    - Given two items $x$ and $y$, we can determine whether $x < y$, $x > y$, or $x = y$
- Output:
  - A permutation of $A$, call it $B$, such that if $i \leq j$ then $B[i] \leq B[j]$
    - Permutation: a sequence of the same items but perhaps in a different order

# Sorting “Landscape”

- There is no singular best algorithm for sorting
- Some are faster, some are slower
- Some use more memory, some use less
- Some are super extra fast if your data matches particular assumptions
- Some have other special properties that make them valuable
- No sorting algorithm can have only all the “best” attributes