

CSE 332 Summer 2026

Lecture 17: Forkjoin

Michael Whitmeyer

<http://www.cs.uw.edu/332>

New Story

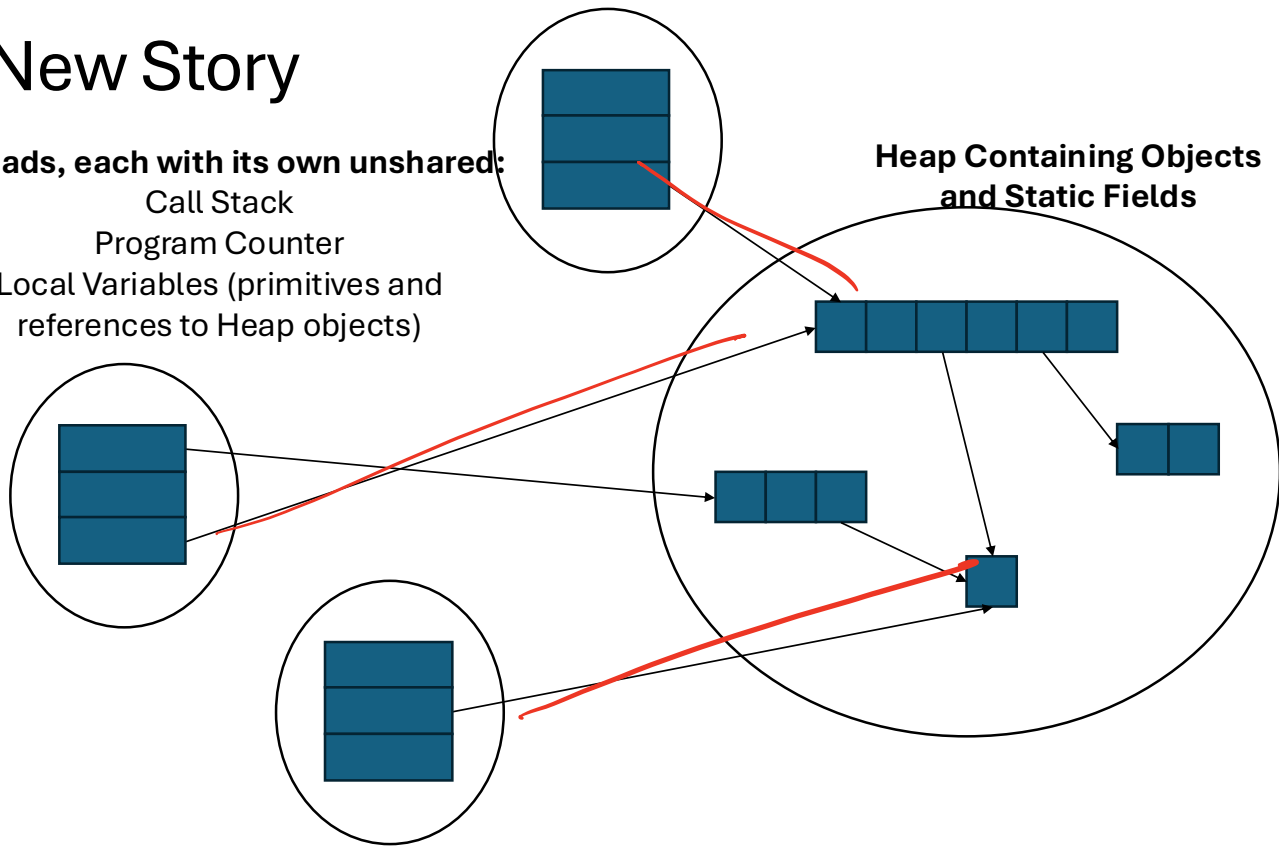
Threads, each with its own unshared:

Call Stack

Program Counter

Local Variables (primitives and
references to Heap objects)

**Heap Containing Objects
and Static Fields**



Accomplishing this in Java (**Java.lang.Thread**)

1. Create class C extending **java.lang.Thread**
 1. C must have a **run** method (acts as **main**)
 2. Call C's **start** method to
 1. create a new thread (i.e. parallel task, not Thread object in java)
 2. execute **run** in that separate, parallel thread
- Calling "**run**" directly causes the program to execute "**run**" sequentially

Parallel Divide and Conquer Pseudocode

$l \approx 1000's$

RecursiveSum(arr)

1. **If** $\text{len}(\text{arr}) < l$: return sum of elements in arr

base case

2. **Else:**

Sequential

1. Divide arr in half into arr1 and arr2

2. Conquer in parallel: call **RecursiveSum(arr1)** and

new thread

RecursiveSum(arr2) in ~~new threads~~

current thread

3. Wait for the recursive calls/threads to finish

4. Combine the sum of the two recursive calls (and return)

Divide and Conquer with Threads (optimized)

```
class SumThread extends java.lang.Thread {  
    public void run() { // override  
        if (hi - lo < SEQUENTIAL_CUTOFF) // "base case"  
            for (int i = lo; i < hi; i++) ans += arr[i];  
        else {  
            SumThread left = new SumThread(arr, lo, (hi + lo) / 2); // divide  
            SumThread right = new SumThread(arr, (hi + lo) / 2, hi); // divide  
            left.start(); // conquer in parallel in new thread  
            right.run(); // conquer in this thread  
            left.join(); // don't move this up a line - why?  
            //right.join();  
            ans = left.ans + right.ans; // combine  
        }  
    }  
}  
  
int sum(int[] arr) { // just make one thread!  
    SumThread t = new SumThread(arr, 0, arr.length);  
    t.run();  
    return t.ans; }
```

ForkJoin Framework

- This strategy is common enough that Java (and C++, and C#, and...) provides a library to do it for you!

What you would do in Threads	What to instead in ForkJoin
Subclass Thread	Subclass RecursiveTask<V>
Override run	Override compute
Store the answer in a field	Return a V from compute
Call start	Call fork
join synchronizes only	join synchronizes and returns the answer
Call run to execute sequentially	Call compute to execute sequentially
Have a topmost thread and call run	Create a pool and call invoke

Divide and Conquer with ForkJoin

```
class SumTask extends RecursiveTask<Integer> {  
    int lo; int hi; int[] arr; // fields to know what to do  
    SumTask(int[] a, int l, int h) { ... } // constructor  
    protected Integer compute() { // return answer  
        if (hi - lo < SEQUENTIAL_CUTOFF) { // base case  
            int ans = 0; // local var, not a field  
            for (int i = lo; i < hi; i++) {  
                ans += arr[i]; return ans; }  
        } else {  
            SumTask left = new SumTask(arr, lo, (hi + lo) / 2); // divide  
            SumTask right = new SumTask(arr, (hi + lo) / 2, hi); // divide  
            left.fork(); // conquer in parallel  
            int rightAns = right.compute(); // conquer in this thread  
            int leftAns = left.join(); // wait  
            return leftAns + rightAns; // combine  
        }  
    }  
}
```

Divide and Conquer with ForkJoin (continued)

```
static final ForkJoinPool POOL = new ForkJoinPool();  
static int parallelSum(int[] arr){  
    SumTask task = new SumTask(arr,0,arr.length)  
    return POOL.invoke(task); // invoke returns the value  
compute returns  
}
```

ForkJoin Pseudocode for Summing

GenericTask(arr)

1. **If** $\text{len}(\text{arr}) < \ell$: return sum of elements in arr

2. **Else:**

1. Divide arr in half into arr1 and arr2

2. Conquer in parallel: call **GenericTask**(arr1) in new thread and **GenericTask**(arr2) in this thread

3. Wait for the recursive calls/threads to finish

4. Combine the sum of the two recursive calls (and return)

fork()

compute()

join()

ForkJoin Pseudocode for ~~Summing~~

Finding Max

GenericTask(arr)

1. If $\text{len}(\text{arr}) < \ell$: return ~~sum~~ ^{max} of elements in arr

2. Else:

1. Divide arr in half into arr1 and arr2

2. Conquer in parallel: call **GenericTask**(arr1) in new thread and **GenericTask**(arr2) in this thread

3. Wait for the recursive calls/threads to finish

4. Combine ~~the sum of the two~~ recursive calls (and return) ^{Take the max of the two}

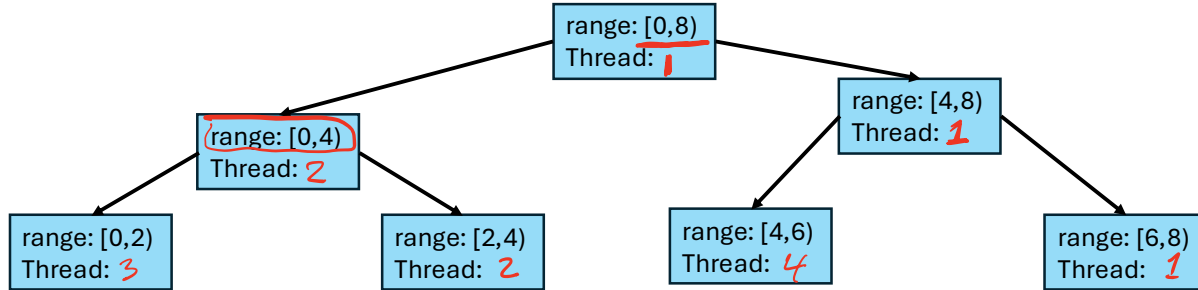
fork()

Save

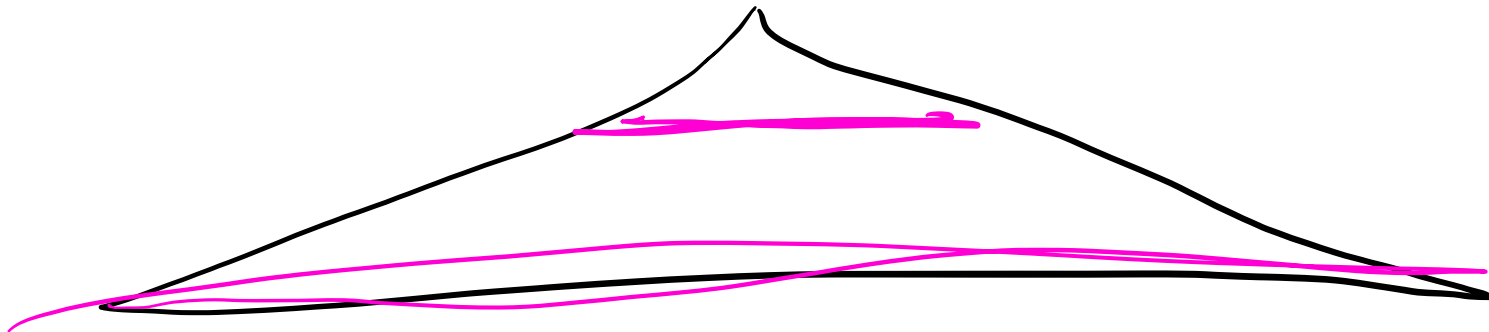
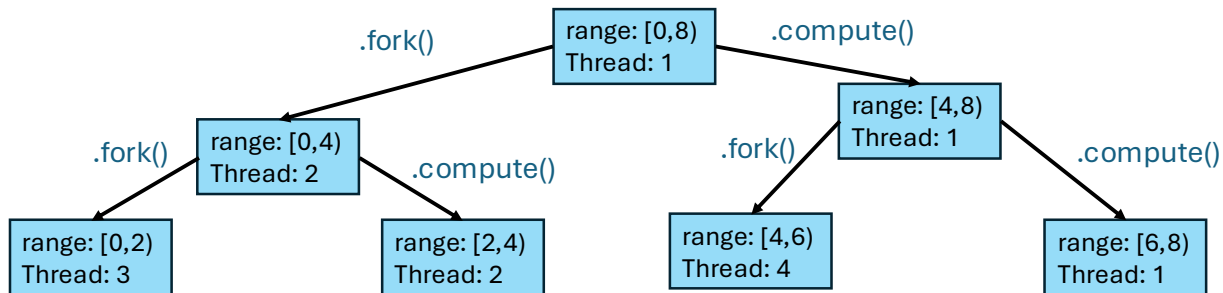
compute()

join()

ForkJoin Picture

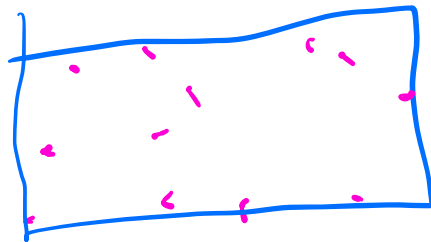


ForkJoin Picture



Other Problems that can be solved similarly

- Element Search
 - Is the value 17 in the array?
- Counting items with a certain property
 - How many elements of the array are divisible by 5?
- Checking if the array is sorted
- Find the smallest rectangle that covers all points in the array
- Find the first thing that satisfies a property
 - What is the leftmost item that is divisible by 20?



$[(x_1, y_1), (x_2, y_2)]$

Reductions/Folds

[. . .] $\mapsto$ 1 thing.

- All examples of a category of computation called a reduction
 - We “reduce” all elements in an array to a single item
 - Requires operation done among elements is **associative**
 - $(x + y) + z = x + (y + z)$
 - $\min(\min(x,y), z) = \min(x, \min(y, z))$
 - The “single item” can itself be complex
 - E.g. create a histogram of results from an array of trials

Map

reduction: $[\dots] \mapsto 1 \text{ thing}$

map: $[\dots] \mapsto [\dots]$
($[\dots]$) same length.

- **New task:** Apply a function (map) to each element of an array
- Examples:
 - Vector addition:
 - $\text{sum}[i] = \text{arr1}[i] + \text{arr2}[i]$
 - Function application:
 - $\text{out}[i] = f(\text{arr}[i]);$
- **Observation:** maps also parallelizable in the same way!
 - No need for combination step

ForkJoin Pseudocode for Doubling each element

GenericTask(arr)

1. **If** $\text{len}(\text{arr}) < \ell$: double each element in arr
(sequentially)

2. **Else:**

1. Divide arr in half into arr1 and arr2

2. Conquer in parallel: call **GenericTask**(arr1) in new
thread and **GenericTask**(arr2) in this thread

3. Wait for the recursive calls/threads to finish

4. Return

fork()

compute()

join()

ForkJoin Pseudocode for ~~Double each element~~

Applying map f

GenericTask(arr) Apply f to

1. **If** $\text{len}(\text{arr}) < \ell$: ~~double~~ each element in arr
(sequentially)

2. **Else:**

1. Divide arr in half into arr1 and arr2

2. Conquer in parallel: call **GenericTask**(arr1) in new
thread and **GenericTask**(arr2) in this thread

3. Wait for the recursive calls/threads to finish

4. Return

fork()

compute()

join()

Map with ForkJoin

```
class AddVecs extends RecursiveAction {  
    int lo; int hi; int[] arr; // fields to know what to do  
    AddVecs(int[] a, int[] b, int[] sum, int l, int h) { ... }  
    protected void compute() { // return answer  
        if (hi - lo < SEQUENTIAL_CUTOFF) { // base case  
            for (int i = lo; i < hi; i++) {  
                sum[i] = a[i] + b[i];  
            }  
        } else {  
            AddTask left = new AddVecs(a, b, sum, lo, (hi + lo) / 2); // divide  
            AddTask right = new AddVecs(a, b, sum, (hi + lo) / 2, hi); // divide  
            left.fork(); // fork a thread and calls compute (conquer)  
            right.compute(); // call compute directly (conquer)  
            left.join(); // wait for thread to finish  
            return; // combine  
        }  
    }  
}
```

Map with ForkJoin (continued)

```
static final ForkJoinPool POOL = new ForkJoinPool();  
int[] add(int[] a, int[] b){  
    int[] ans = new int[a.length];  
    AddVecs task = new AddVecs(a, b, ans, 0, a.length)  
    POOL.invoke(task);  
    return ans;  
}
```

Maps and Reductions

- “Workhorse” constructs in parallel programming
- Many problems can be written in terms of maps and reductions
- With practice, writing them will become second nature
 - Like how over time for loops and if statements have gotten easier

Map/Reduction Example

- **Task:** Multiply together the lengths of all the odd-length strings in array

Map/Reduction Example

- **Task:** Multiply together the lengths of all the odd-length strings in array
 1. Apply map to convert the array of strings into an array of their lengths
 2. Then do a map on that array so each value maps to 1 if it's even and itself if it's odd
 3. Then do a reduction to multiply together that final result
- **Note:** You could do this in a single ForkJoin RecursiveTask
 - but “deconstructing” useful since some languages designed specifically for parallelism have Map/Reduce built in.
 - Map and Reduce are two from **a trio, with Pack/Filter** being the third

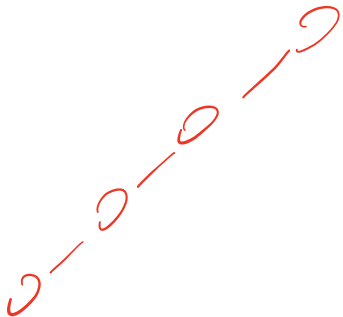
Which Data Structures are “Suitable” for Parallelism?

- For each data structure, can we write a parallel algorithm to sum all of its values that's *more efficient* than a sequential one?
 - Array ✓
 - Linked List ✗
 - Binary Tree (✓)

if you are performing a task on each element of linked list that requires $O(m)$ time,

then sequential: $O(n \cdot m)$

parallel: $O(n + m)$



Pack/Filter

- Given an array of values and a Boolean function, return a new array which contains only elements that were “true”

10	16	4	18	8	2
----	----	---	----	---	---



10	16	18
----	----	----

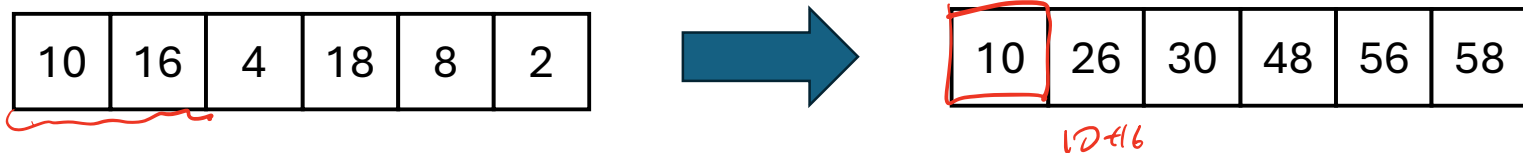
$$f(x) = x > 9$$

Question: Can we Pack/Filter in parallel?

Answer: yes, with help of parallel **Prefix Sum**

Prefix Sum

- Given an array, compute a new array where each index i is the sum of all values up to i



Later: will be useful for parallel packing/filtering!

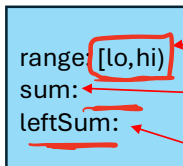
Parallel Prefix Sum

- Algorithm will have two major parallel steps
 - Called a “two pass” parallel algorithm
- First step:
 - Create a tree data structure
- Second Step:
 - Use the tree to fill in the output array



Richard Ladner
Allen School Faculty

Tree Node:



The “subproblem” this node represents
lower bound is inclusive, upper is exclusive

The sum of all values in the range

The sum of all values to the left of the range
i.e. in the range $[0, lo)$

Step 1: Using D&C

Create a Tree, Fill in sum

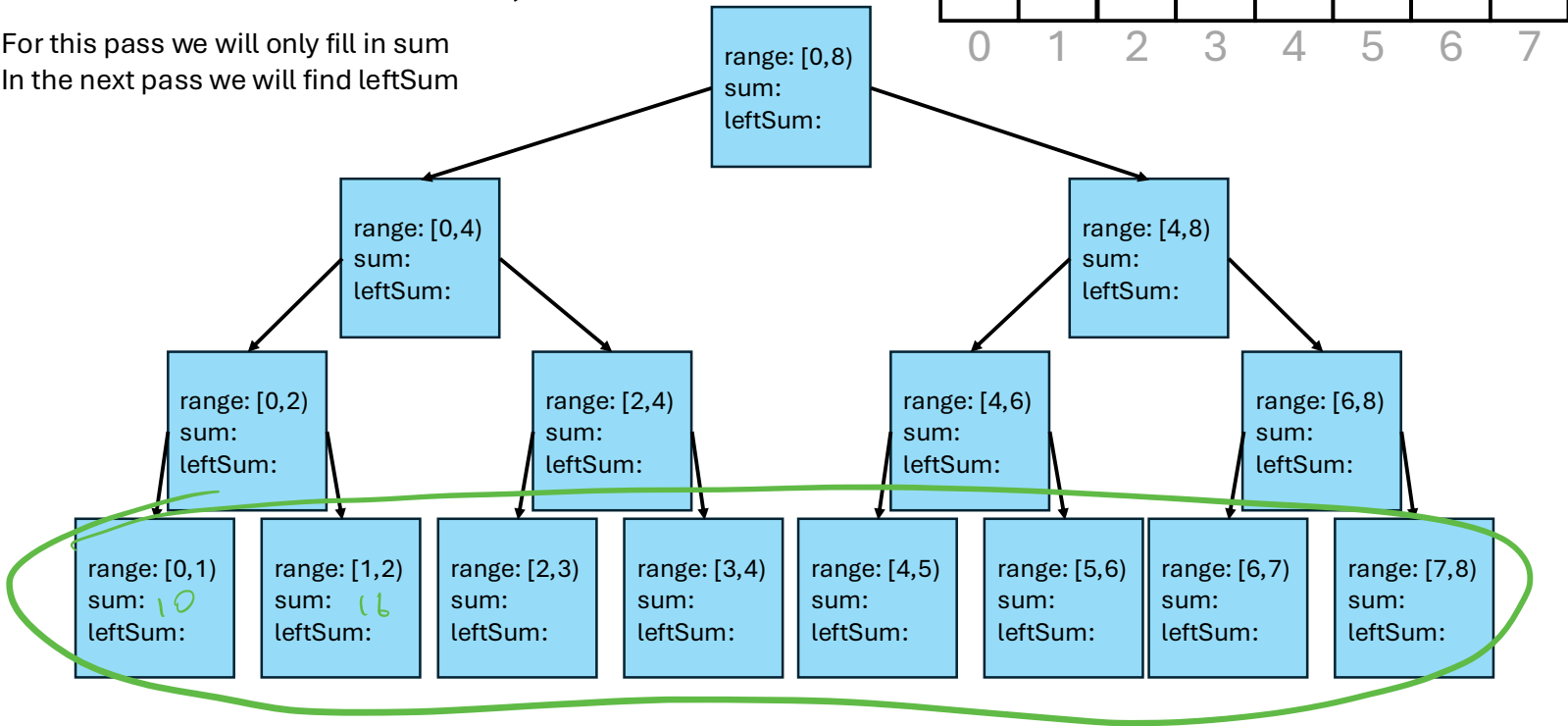
For this pass we will only fill in sum
In the next pass we will find leftSum

Input:

10	16	4	18	8	2	14	9
----	----	---	----	---	---	----	---

Output:

0	1	2	3	4	5	6	7



Step 1: Create a Tree, Fill in sum

10	16	4	18	8	2	14	9
----	----	---	----	---	---	----	---

4

range: [2,3)
sum: 4
leftSum:

8	2	1	9
		4	

1	1	4	1
0	6		8

range: [0,4)
sum: 48
leftSum:

range: [4,8)
sum: 33
leftSum:

range: [0,8)
sum: 81
leftSum:

range: [0,4)
sum: 48
leftSum:

range: [4,8)
sum: 33
leftSum:

- **Base Case:**
 - If the range is smaller than the Sequential Cutoff, create a node for that range and find the sum sequentially
- **Divide:**
 - Split the list into two “sublists” of (roughly) equal length, create a thread for each sublist.
- **Conquer:**
 - Compute the left and right subtrees in parallel
- **Combine:**
 - Create parent node, connect to children, fill in sum

Step 1 pseudocode (create tree, compute sum)

BuildTree(arr)

1. **If** $\text{len}(\text{arr}) < \ell$:

1. Create new TreeNode **Leaf**

2. Set **Leaf**.sum = sum of values in arr and return **Leaf**

2. **Else:**

1. Divide arr in half into arr1 and arr2

2. Conquer: TreeNode **leftChild** = **BuildTree**(arr1) in new thread,
TreeNode **rightChild** = **BuildTree**(arr2) in this thread

3. Wait for parallel computations to finish

4. Combine: Create TreeNode **parent**, set **parent**.sum =
leftChild.sum + **rightChild**.sum, **parent**.left = **leftChild**,
parent.right = **rightChild**

5. Return **parent**

After Step 1

Input:

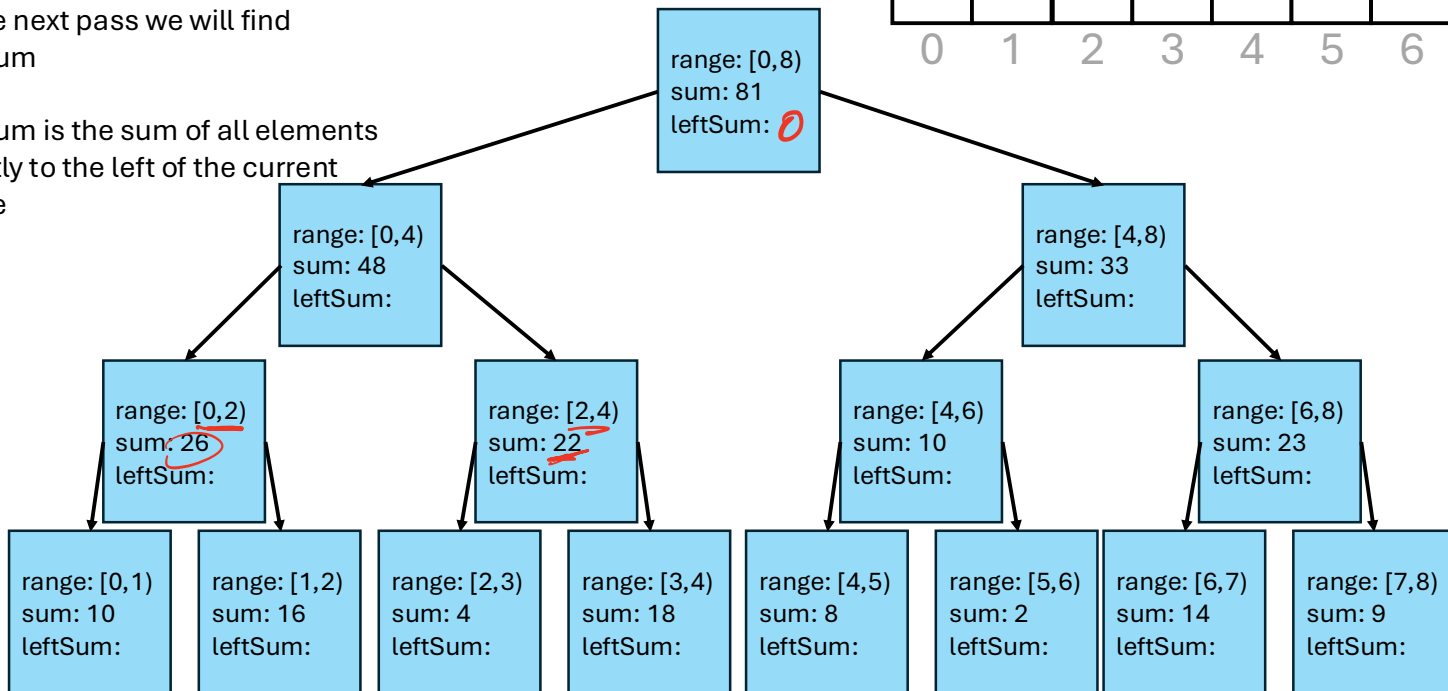
10	16	4	18	8	2	14	9
----	----	---	----	---	---	----	---

Output:

0	1	2	3	4	5	6	7

All sums filled in per node
In the next pass we will find
leftSum

leftSum is the sum of all elements
strictly to the left of the current
range



Step 2: fill in leftSum and Output

Input:

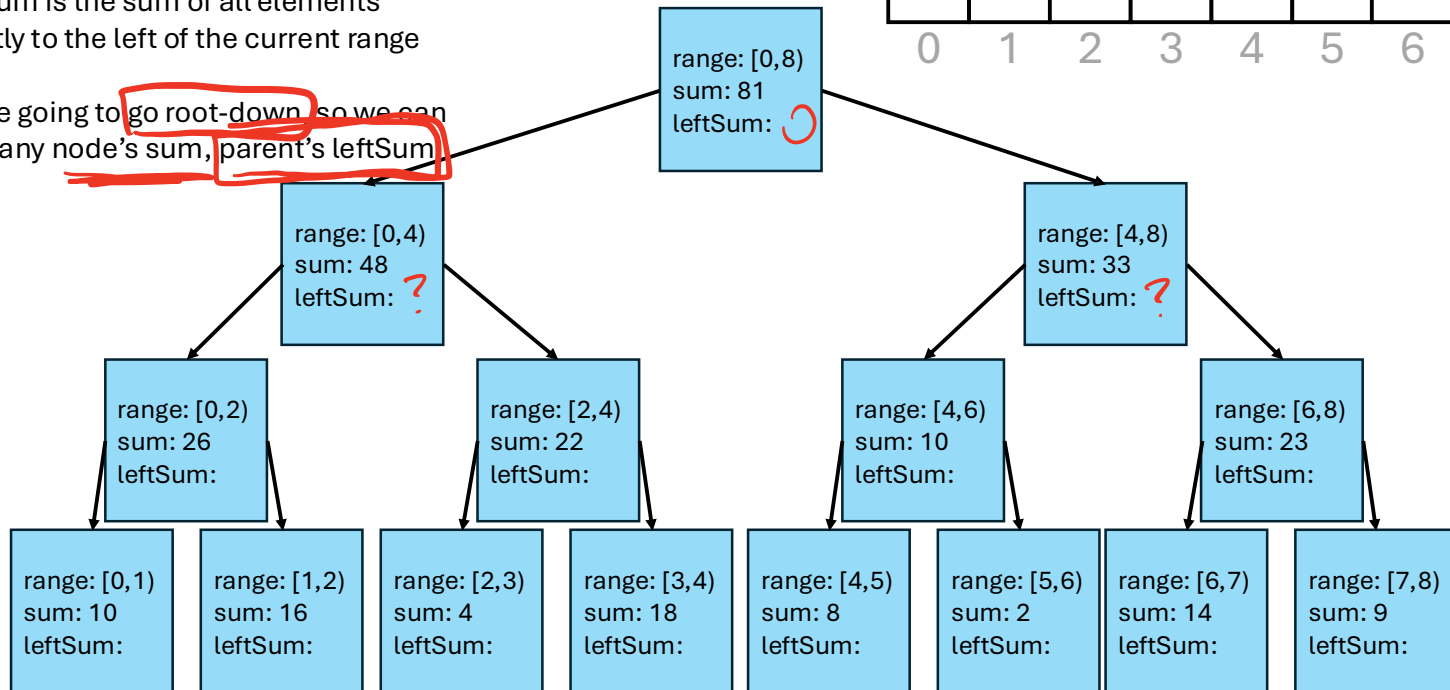
10	16	4	18	8	2	14	9
----	----	---	----	---	---	----	---

Output:

0	1	2	3	4	5	6	7

leftSum is the sum of all elements
strictly to the left of the current range

We're going to go root-down so we can
use: any node's sum, parent's leftSum



Step 2: fill in leftSum and Output

Input:

10	16	4	18	8	2	14	9
----	----	---	----	---	---	----	---

Output:

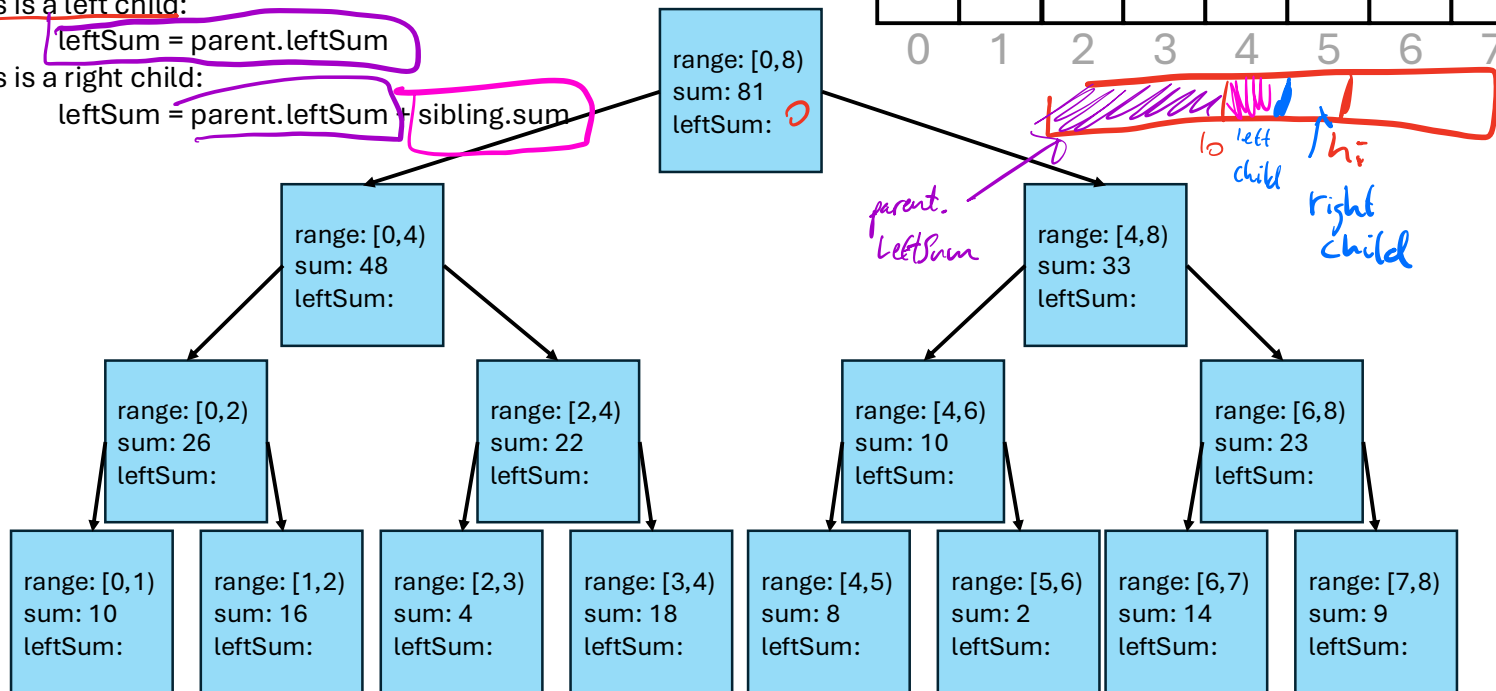
0	1	2	3	4	5	6	7

If this is a left child:

$\text{leftSum} = \text{parent.leftSum}$

If this is a right child:

$\text{leftSum} = \text{parent.leftSum} + \text{sibling.sum}$



Step 2: fill in leftSum and Output

If this is a left child:

$\text{leftSum} = \text{parent.leftSum}$

If this is a right child:

$\text{leftSum} = \text{parent.leftSum} + \text{sibling.sum}$

For the leaves:

use leftSum+sum

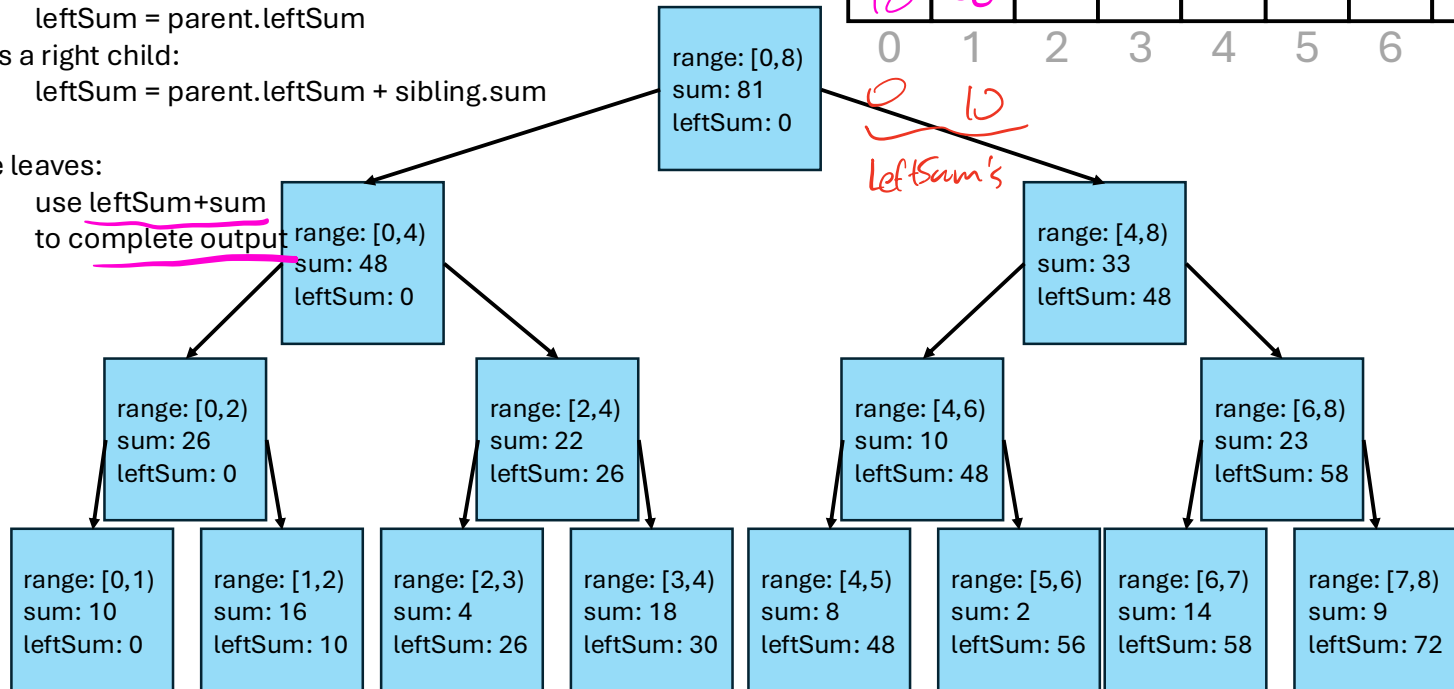
to complete output

Input:

10	16	4	18	8	2	14	9
----	----	---	----	---	---	----	---

Output:

10	26						
0	1	2	3	4	5	6	7



Step 2: fill in leftSum and Output

If this is a left child:

$\text{leftSum} = \text{parent.leftSum}$

If this is a right child:

$\text{leftSum} = \text{parent.leftSum} + \text{sibling.sum}$

For the leaves:

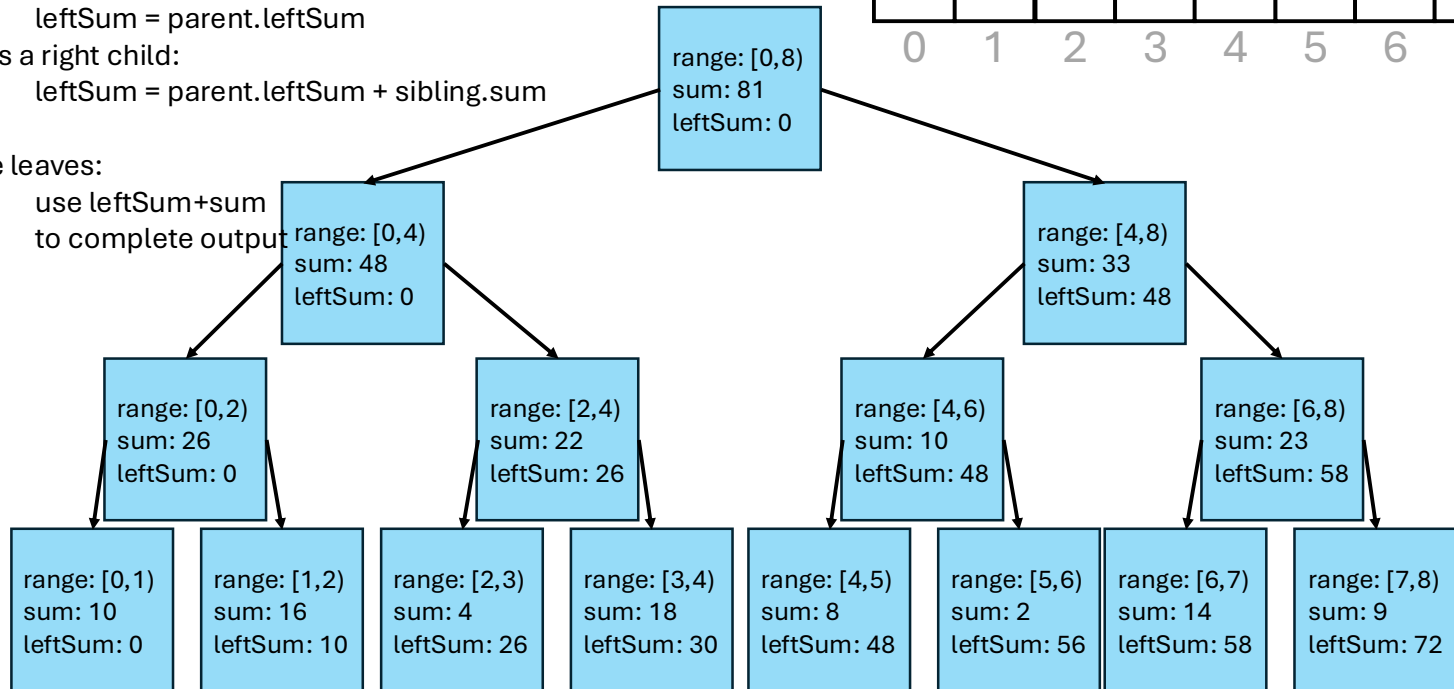
use $\text{leftSum} + \text{sum}$
to complete output

Input:

10	16	4	18	8	2	14	9
----	----	---	----	---	---	----	---

Output:

10	26	30	48	56	58	72	81
0	1	2	3	4	5	6	7



Step 2 pseudocode (Compute Leftsum)

CompleteTree(Node curr)

1. **If** curr is a left child of curr.parent:

1. Set curr.LeftSum = curr.parent.LeftSum

2. **Else:**

1. Set curr.LeftSum = curr.parent.LeftSum + curr.sibling.Sum

3. **If** curr is not a leaf:

1. Divide and conquer: call **CompleteTree**(curr.left) and **CompleteTree**(curr.right) in parallel threads

2. Wait for parallel computations to finish

4. **Else (curr is a leaf):**

1. Set output[curr.lo] = curr.LeftSum + input[curr.lo]

2. for i = curr.lo + 1 to curr.hi:

1. Set output[i] = output[i-1] + input[i]

naïve sequential alg.

Base
Case