CSE 332 Winter 2026
Lecture 11: hashing 2

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Dictionary (Map) ADT - Unordered

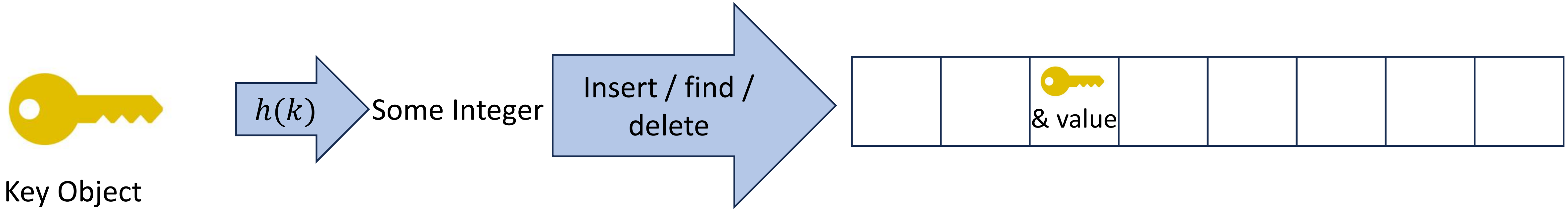
- Contents:
  - Sets of key+value pairs
  - ~~Keys must be comparable~~ Keys have a hash function
- Operations:
  - insert(key, value)
    - Adds the (key,value) pair into the dictionary
    - If the key already has a value, overwrite the old value
      - Consequence: Keys cannot be repeated
  - find(key)
    - Returns the value associated with the given key
  - delete(key)
    - Remove the key (and its associated value)

Next topic: Hash Tables

Data Structure	Time to insert	Time to find	Time to delete
Unsorted Array	$\Theta(n)$	$\Theta(n)$	$\Theta(n)$
Unsorted Linked List	$\Theta(n)$	$\Theta(n)$	$\Theta(n)$
Sorted Array	$\Theta(n)$	$\Theta(\log n)$	$\Theta(n)$
Sorted Linked List	$\Theta(n)$	$\Theta(n)$	$\Theta(n)$
Binary Search Tree	$\Theta(\text{height})$	$\Theta(\text{height})$	$\Theta(\text{height})$
AVL Tree	$\Theta(\log n)$	$\Theta(\log n)$	$\Theta(\log n)$
Hash Table (Worst case)	$\Theta(n)$	$\Theta(n)$	$\Theta(n)$
Hash Table (Average)	$\Theta(1)$	$\Theta(1)$	$\Theta(1)$

Hash Tables

- Idea:
  - Have a small array to store information
  - Use a **hash function** to convert the key into an index
    - Hash function should "scatter" the keys, behave as if it randomly assigned keys to indices
  - Store key at the index given by the hash function
  - Do something if two keys map to the same place (should be very rare)
    - Collision resolution



Properties of a "Good" Hash

- Definition: A hash function maps objects to integers
- Goal: A hash function should behave as if every key is assigned to a random index

- **Consistent**
  - Objects considered "equal" should hash to the same value
  - Deterministic: running the hash function on the same object twice should yield the same result
- **Uniform**
  - Should be able to use every index in a fixed-size array
  - Should use every index at roughly equal rates
- **Effective**
  - It should be difficult to find two objects which hash to the same value
  - Given on object, it should be hard to find a different object which hashes to the same value
  - "Avalanche effect": making a small change to the object yields big changes in the value it hashes to
- **Efficient**
  - Time to calculate the hash should be very small

Compare These Hash Functions (for strings)

- Let $s = s_0 s_1 s_2 \ldots s_{m-1}$ be a string of length m
  - Let $a(s_i)$ be the ascii encoding of the character s_i
- $h_1(s) = a(s_0)$
- $h_2(s) = \left(\sum_{i=0}^{m-1} a(s_i)\right)$
- $h_3(s) = \left(\sum_{i=0}^{m-1} a(s_i) \cdot 37^i\right)$
- $h_4(s) = \left(2 \cdot \sum_{i=0}^{m-1} a(s_i) \cdot 37^i\right)$

Properties of Those Example Hash Functions

- Let $s = s_0 s_1 s_2 \dots s_{m-1}$ be a string of length m
  - Let $a(s_i)$ be the ascii encoding of the character s_i
- $h_1(s) = a(s_0)$
  - Is: consistent, efficient
- $h_2(s) = \left(\sum_{i=0}^{m-1} a(s_i)\right)$
  - Is: consistent, efficient, and possibly uniform
- $h_3(s) = \left(\sum_{i=0}^{m-1} a(s_i) \cdot 37^i\right)$
  - Is: Consistent, efficient, uniform, and effective
- $h_4(s) = \left(2 \cdot \sum_{i=0}^{m-1} a(s_i) \cdot 37^i\right)$
  - Is: Consistent, efficient, effective

Ideal Insert procedure

Supposing we have a “good” hash function:

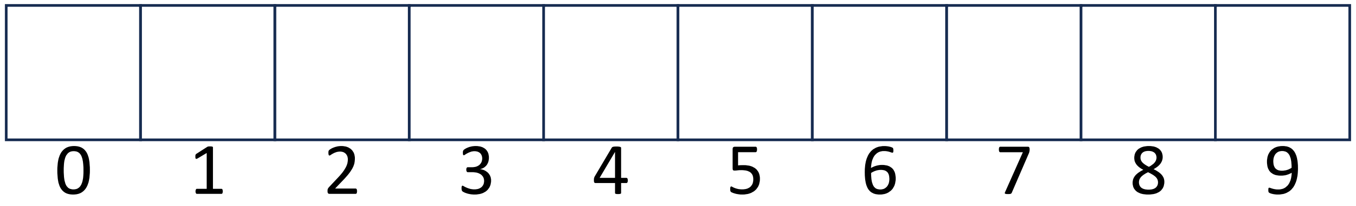
```
insert(key, value){
     h = key.hash();
     table[h % table.length] = value;
}
```

Problem: It’s possible that two different keys map to the same index!

This is called a “collision”

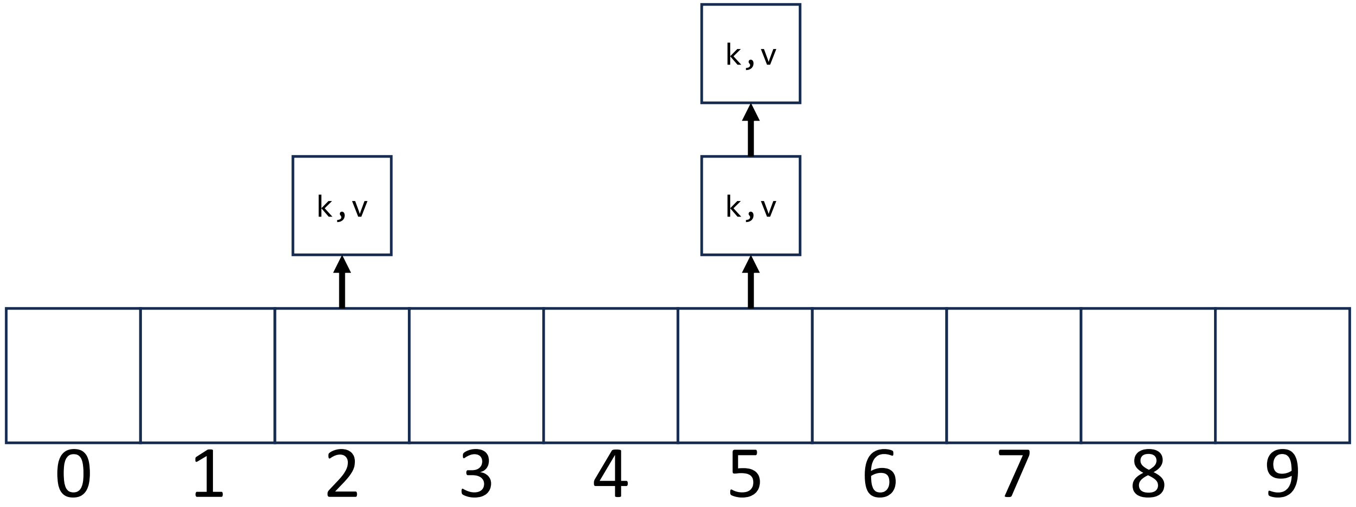
Collision Resolution

- A Collision occurs when we want to insert something into an already-occupied position in the hash table
- 2 main strategies:
  - Separate Chaining
    - Use a secondary data structure to contain the items
      - E.g. each index in the hash table is itself a linked list
  - Open Addressing
    - Use a different spot in the table instead
      - Linear Probing
      - Quadratic Probing
      - Double Hashing



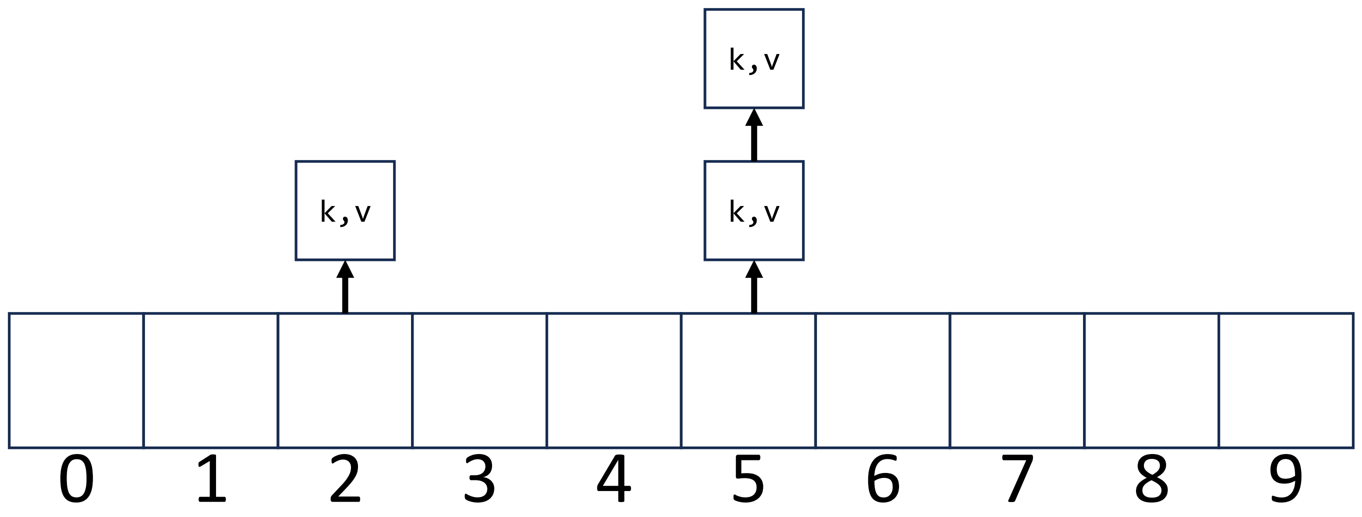
Separate Chaining Insert

- To insert `k, v`:
  - Compute the index using `i = h(k) % table.length`
  - Add the key-value pair to the data structure at `table[i]`



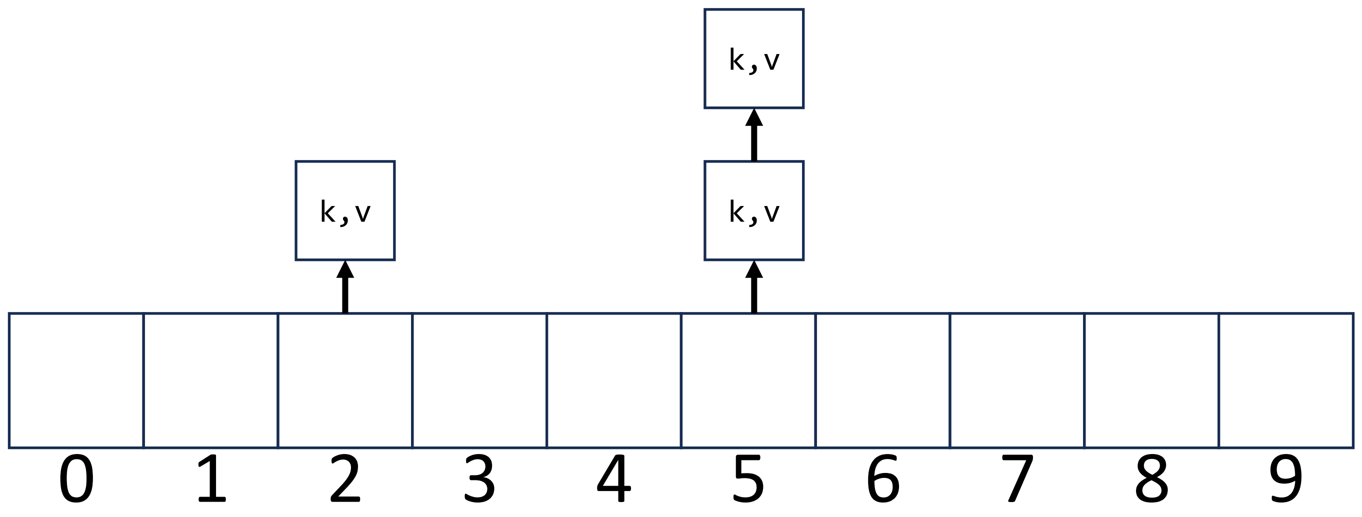
Separate Chaining Find

- To find k:
  - Compute the index using `i = h(k) % table.length`
  - Call find with the key on the data structure at `table[i]`



Separate Chaining Delete

- To delete k:
  - Compute the index using `i = h(k) % table.length`
  - Call delete with the key on the data structure at `table[i]`



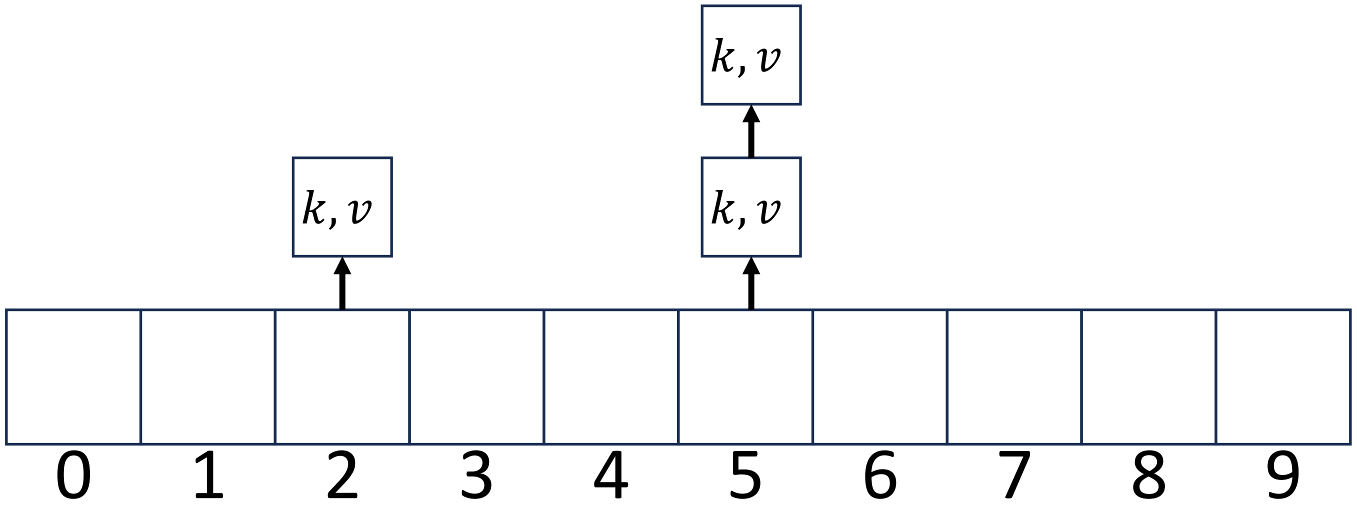
Formal Running Time Analysis

- The **load factor** of a hash table represents the average number of items per “bucket”
  - $\lambda = \frac{n}{length}$
- Assume we have a hash table that uses a linked-list for separate chaining
  - What is the expected number of comparisons needed in an unsuccessful find?
  - What is the expected number of comparisons needed in a successful find?
- How can we make the expected running time $\Theta(1)$?

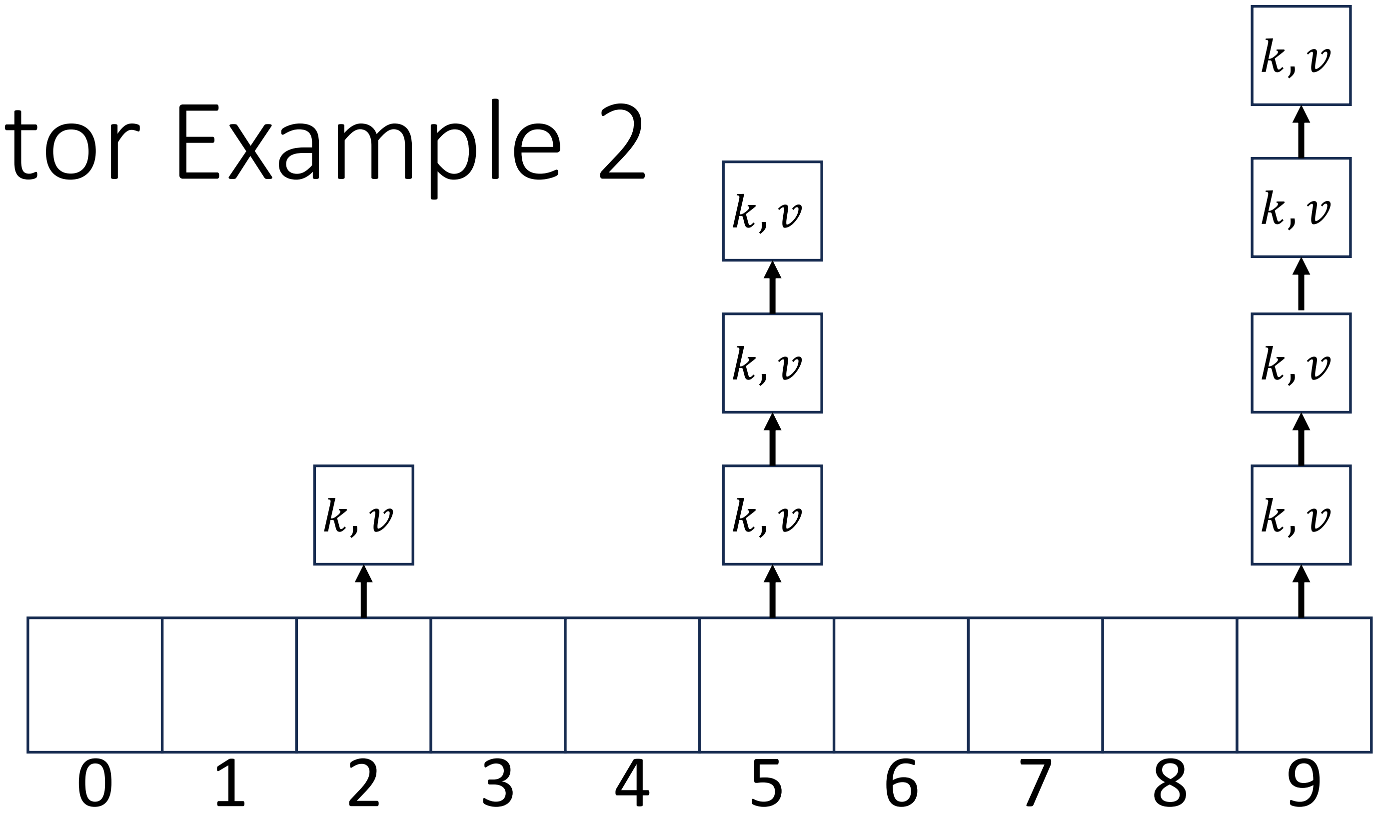
Formal Running Time Analysis (Answers)

- The **load factor** of a hash table represents the average number of items per "bucket"
  - $\lambda = \frac{n}{length}$
- Assume we have a hash table that uses a linked-list for separate chaining
  - What is the expected number of comparisons needed in an unsuccessful find?
    - λ
  - What is the expected number of comparisons needed in a successful find?
    - $\frac{\lambda}{2}$
- How can we make the expected running time $\Theta(1)$?
  - Pick a constant value, resize the array whenever λ exceeds that constant
    - We'll talk about which constant we should pick later

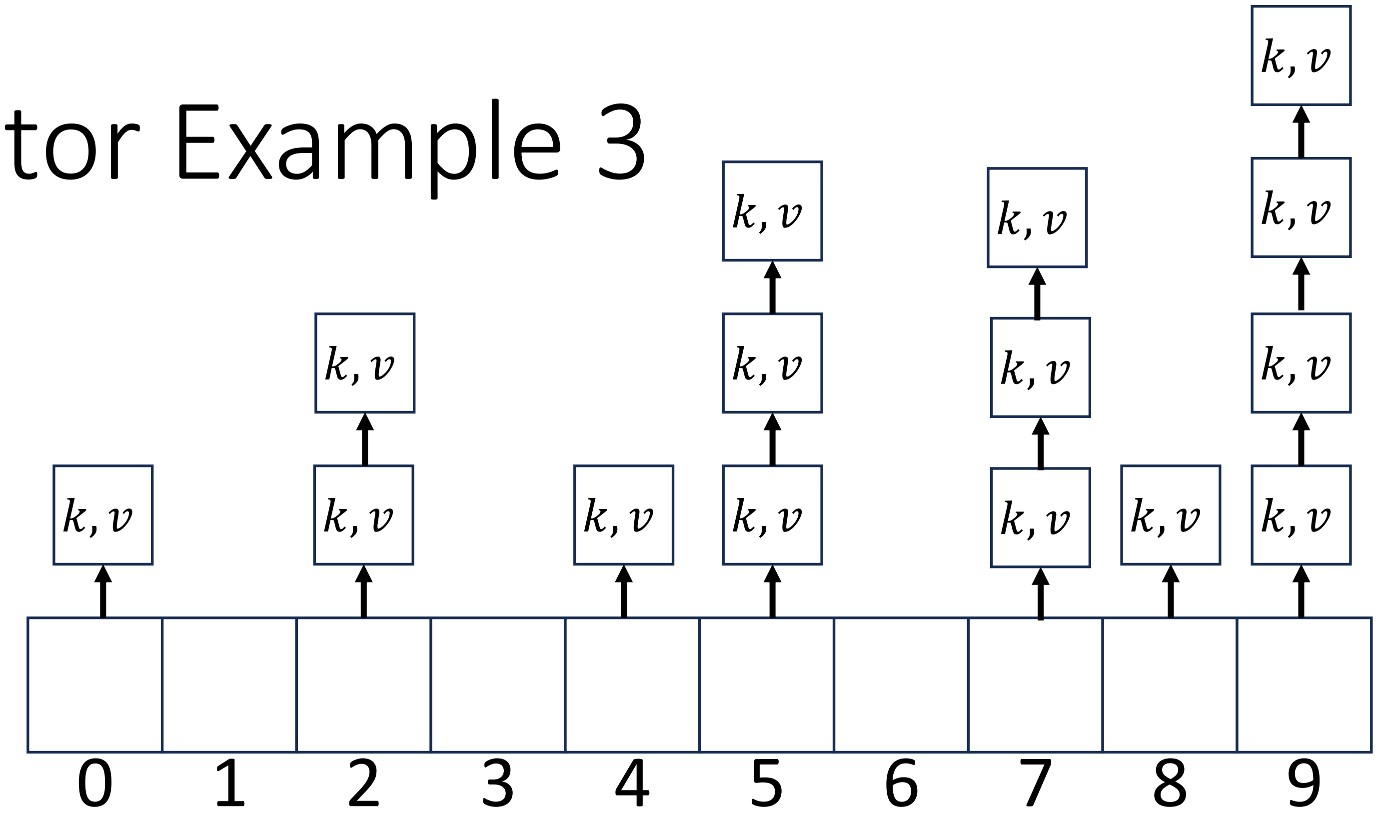
Load Factor Example 1



Load Factor Example 2

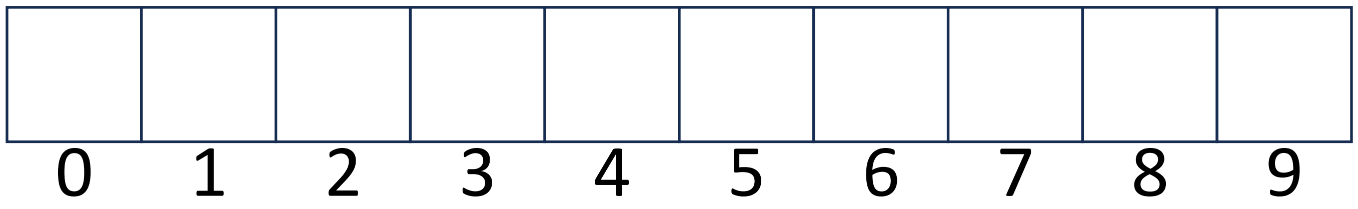


Load Factor Example 3



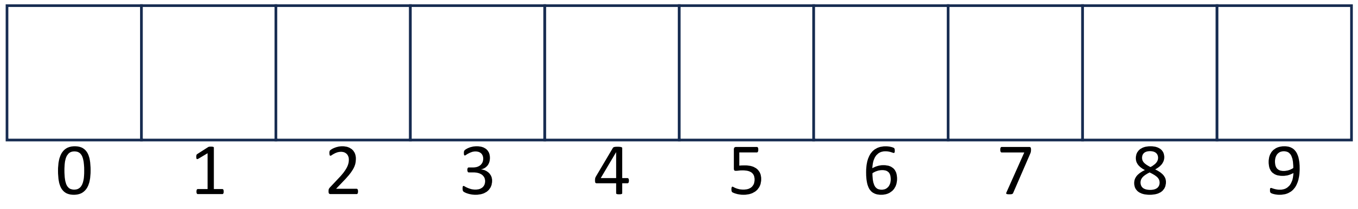
Collision Resolution: Linear Probing

- When there’s a collision, use the next open space in the table



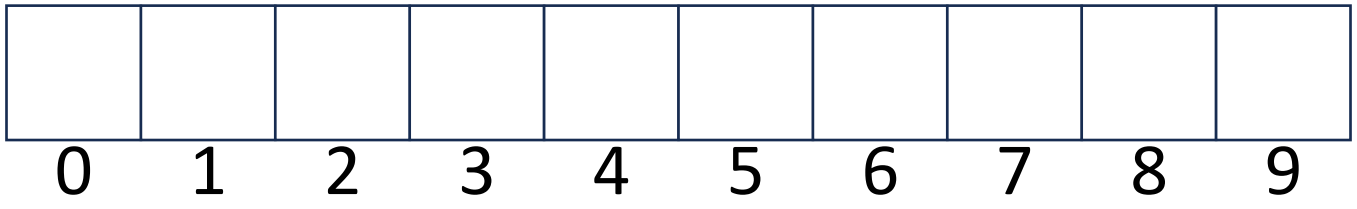
Linear Probing: Insert Procedure

- To insert `k,v`
  - Calculate `i = h(k) % table.length`
  - If `table[i]` is occupied then try index `(i+1) % table.length`
  - If that is occupied try index `(i+2) % table.length`
  - If that is occupied try index `(i+3) % table.length`
  - …



Linear Probing: How to find?

- What do you think?



Linear Probing: Find

- To find key k
  - Calculate `i = h(k) % table.length`
  - If `table[i]` is occupied but doesn't have k, check `(i+1) % table.length`
  - If that is occupied and doesn't contain k, check `(i+2) % table.length`
  - If that is occupied and doesn't contain k, check `(i+3) % table.length`
  - Repeat until you either find k or else you reach an empty cell in the table

0	1	2	3	4	5	6	7	8	9

Linear Probing: Delete is Hard (Example 1)

- Suppose we insert A, B, C, D, and E in that order, where all map to 3
- How should we delete B?

Before:

0	1	2	3	4	5	6	7	8	9
			A	B	C	D	E		

We need future probing to work correctly, so we must fill in the hole.

After:

0	1	2	3	4	5	6	7	8	9
			A	C	D	E			

Linear Probing: Delete is Hard (Example 2)

- Suppose we insert A, B, C, D, and E in that order, where A,B,E all map to 3 and C,D map to 5.
- How should we delete B?

Before:

			A	B	C	D	E		
0	1	2	3	4	5	6	7	8	9

We need future probing to work correctly, so we must fill in the hole.

We cannot move C or D over because they map to an index later in the probe path

After:

			A	E	C	D			
0	1	2	3	4	5	6	7	8	9

Linear Probing: Delete is Hard (Example 3)

- Suppose we insert A, B, C, D, and E in that order, where A,B,E map to 8 and C,D map to 0.
- How should we delete B?

Before:

C	D	E						A	B
0	1	2	3	4	5	6	7	8	9

We cannot move C or D over because they map to an index later in the probe path, even though the index number happens to be smaller.
E moves to a larger index because it's earlier in its probe path

After:

C	D							A	E
0	1	2	3	4	5	6	7	8	9

Linear Probing: Delete

- **Option 1 (harder)**: Plug the hole with other items in a way that makes probes behave correctly
  - Something like: to delete at index i, starting from i and going until we find an empty index, if probe path of the key at index j has index i before j then move the key-value pair at j to index i, then repeat on index j.
- **Option 2 (easier)**: "Tombstone" deletion. Leave a special object that indicates an something was deleted from there
  - The tombstone does not act as an open space when finding (so keep looking after its reached)
  - When inserting you can replace a tombstone with a new item

k, v		k, v	k, v		k, v				
0	1	2	3	4	5	6	7	8	9

Linear Probing + Tombstone: Find

- To find key k
  - Calculate `i = h(k) % table.length`
  - While `table[i]` has a key other than k, set `i = (i+1) % table.length`
  - If you come across k return `table[i]`
  - If you come across an empty index, the find was unsuccessful
    - Tombstones do not count as empty!

k_1, v_1		k_2, v_2	k_3, v_3		k_4, v_4				
0	1	2	3	4	5	6	7	8	9

Linear Probing + Tombstone: Insert

- To insert `k,v`
  - Calculate `i = h(k) % table.length`
  - While `table[i]` has a key other than k, set `i = (i+1) % table.length`
    - If `table[i]` has a tombstone, set `dest = i`
      - That is where we will insert if the find is unsuccessful
    - If you come across k, set `table[i] = k,v`
    - If you come across an empty index, the find was unsuccessful
      - Set `table[x] = k,v` if we saw a tombstone
    - Set `table[x] = k,v` otherwise

k_1, v_1		k_2, v_2	k_3, v_3		k_4, v_4				
0	1	2	3	4	5	6	7	8	9

Downsides of Linear Probing

- What happens when λ approaches 1?
  - Get longer and longer contiguous blocks
  - A collision is guaranteed to grow a block
    - Larger blocks experience more collisions
    - Feedback loop!
- What happens when λ exceeds 1?
  - Impossible!
  - You can't insert more stuff

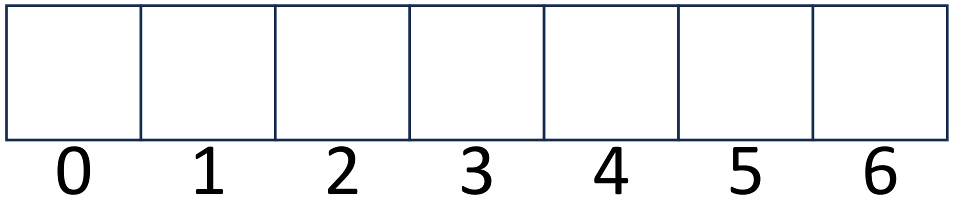
Quadratic Probing: Insert Procedure

- To insert `k,v`
  - Calculate `i = h(k) % table.length`
  - If `table[i]` is occupied then try $(\texttt{i}+1^2)$ `% table.length`
  - If that is occupied try $(\texttt{i}+2^2)$ `% table.length`
  - If that is occupied try $(\texttt{i}+3^2)$ `% table.length`
  - If that is occupied try $(\texttt{i}+4^2)$ `% table.length`
  - …

0	1	2	3	4	5	6	7	8	9

Quadratic Probing: Example

- Insert:
  - 76
  - 40
  - 48
  - 5
  - 55
  - 47



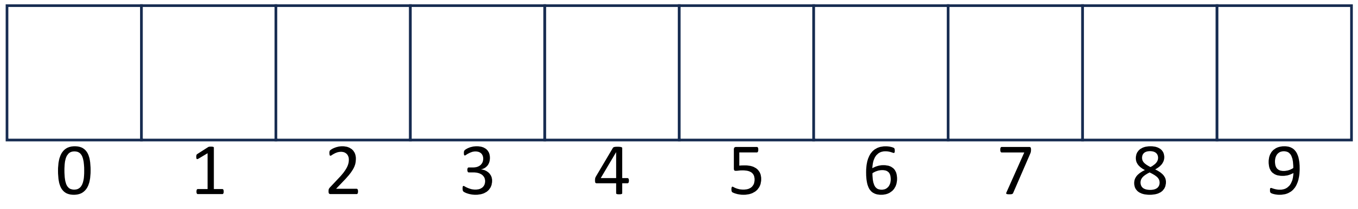
Using Quadratic Probing

- If you probe `table.length` times, you start repeating indices
- If `table.length` is prime and $\lambda < \frac{1}{2}$ then you're guaranteed to find an open spot in at most `table.length`/2 probes

- Helps with the clustering problem of linear probing, but does not help if many things hash to the same value

Double Hashing: Insert Procedure

- Given h and g are both good hash functions
- To insert `k,v`
  - Calculate `i = h(k) % table.length`
  - If `table[i]` is occupied then try `(i+g(k)) % table.length`
  - If that is occupied try `(i+2*g(k)) % table.length`
  - If that is occupied try `(i+3*g(k)) % table.length`
  - If that is occupied try `(i+4*g(k)) % table.length`
  - …



Rehashing

- If your load factor λ gets too large, copy everything over to a larger hash table
  - To do this: make a new, larger array
  - Re-insert all items into the new hash table by reapplying the hash function
    - We need to reapply the hash function because items should map to a different index
  - New array should be "roughly" double the length (but probably still want it to be prime)
- What does "too large" mean?
  - For separate chaining, typically we want $\lambda < 2$
  - For open addressing, typically we want $\lambda < \frac{1}{2}$