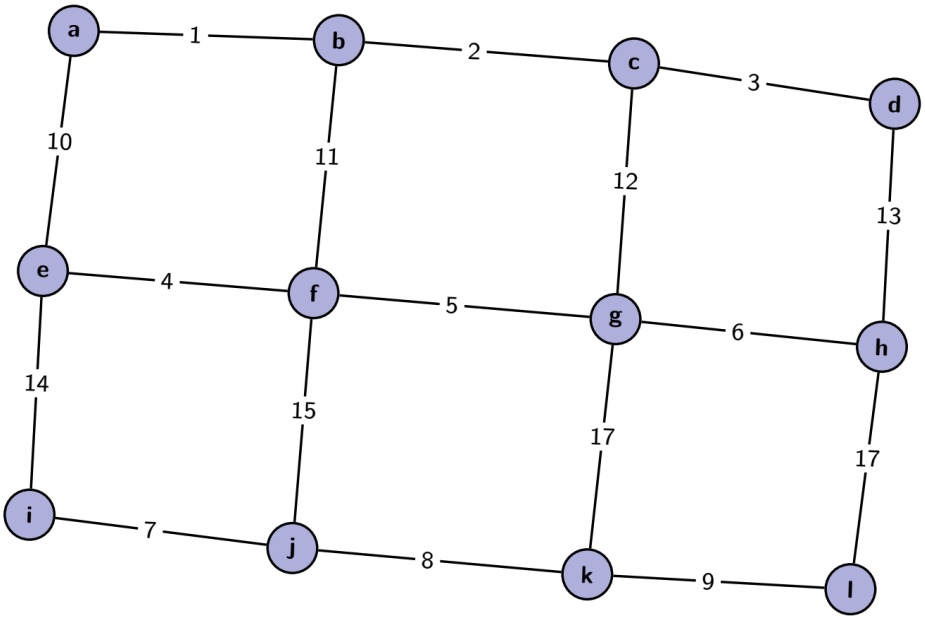
CSE 332: Data Structures and Parallelism

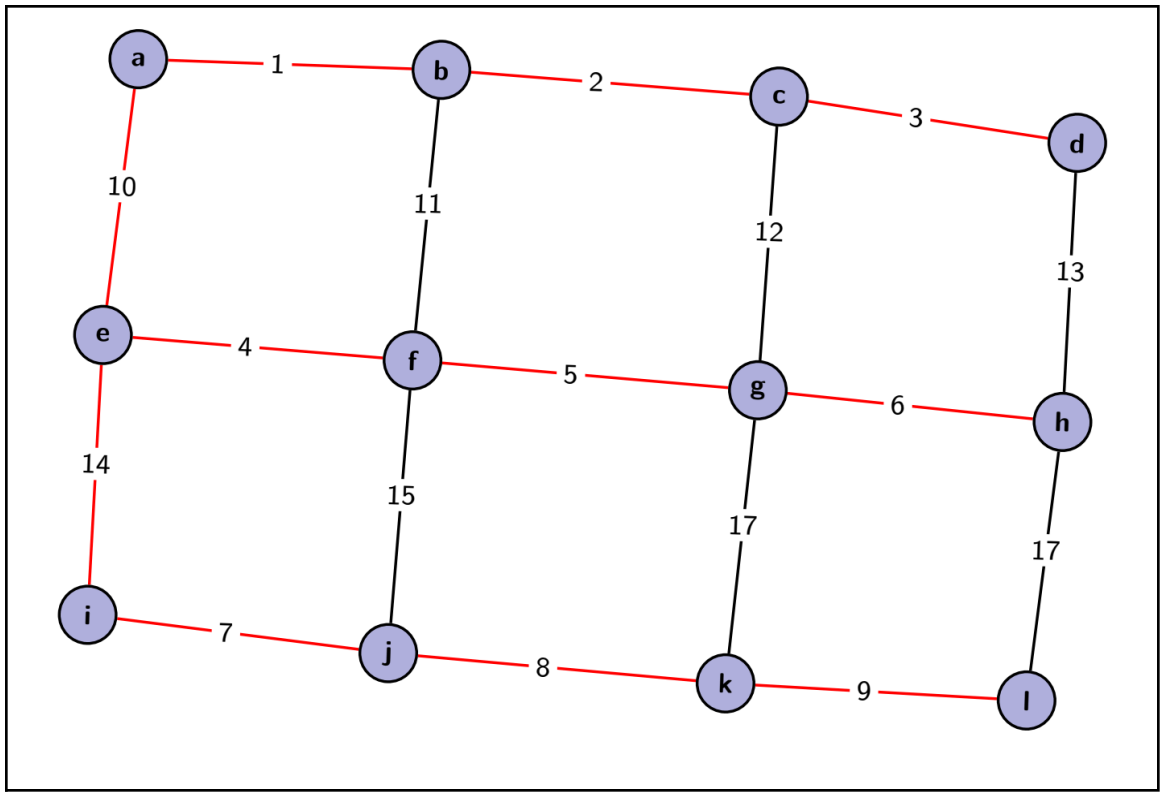
Section 9: Graphs Solutions

1. LMNST!

Consider the following graph:



a) Find an MST of this graph using both of the two algorithms we've discussed in lecture. Make sure you say which algorithm you're using and show your work.


a
b
c
d
e
f
g
h
i
j
k
l
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
17
17

Using Prim's algorithm:

Vertex	Known	Cost of Edge
a	True	0
b	True	~~∞~~ 01
c	True	~~∞~~ 02
d	True	~~∞~~ 03
e	True	~~∞~~ 10
f	True	~~∞~~ ~~11~~ 04
g	True	~~∞~~ ~~12~~ 05
h	True	~~∞~~ ~~13~~ 06
i	True	~~∞~~ ~~14~~
j	True	~~∞~~ ~~15~~ 07
k	True	~~∞~~ ~~17~~ 08
l	True	~~∞~~ ~~17~~ 09

Using Kruskal's algorithm:

Sorted Edges:
(a, b), (b, c), (c, d), (e, f), (f, g), (g, h), (i, j), (j, k), (k, l), (a, e), (b, f), (c, g), (d, h), (e, i), (f, j), (g, k), (h, l)

UF Forests:
{a}, {b}, {c}, {d}, {e}, {f}, {g}, {h}, {i}, {j}, {k}, {l}
{a, b, c, d}, {e, f, g, h}, {i, j, k, l}
{a, b, c, d, e, f, g, h}, {i, j, k, l}
{a, b, c, d, e, f, g, h, i, j, k, l}

b) Using just the graph, how can you determine if it's possible that there are multiple MSTs of the graph? Does this graph have multiple MSTs?

A graph can only have multiple MSTs if it has multiple edges of the same weight. This graph has two 17's, but neither of them are used in the MST. So, there's only one MST here.

c) What is the asymptotic runtime of the algorithms that you used to compute the MSTs?

Prim's Algorithm takes $\mathcal{O}(|V| \lg(|V|) + |E| \lg(|V|))$, and Kruskal's Algorithm takes $\mathcal{O}(|E| \lg(|E|))$.