

CSE 332: Data Structures & Parallelism

Lecture 4: Priority Queues

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Today

- Finish up Intro to Asymptotic Analysis
- New ADT! Priority Queues

Scenario

What is the difference between waiting for service at a pharmacy versus an ER?

Pharmacies usually follow the rule
First Come, First Served

Emergency Rooms assign priorities
based on each individual's need

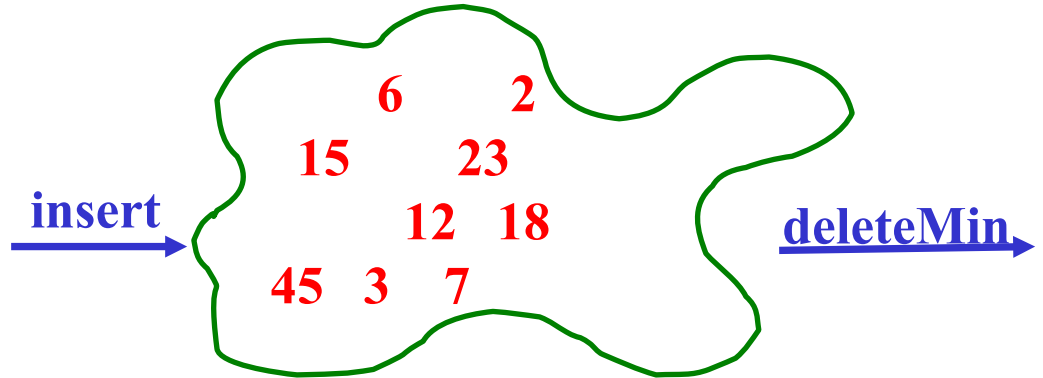
A new ADT: Priority Queue

- Textbook Chapter 6
 - We will go back to binary search trees (ch4) and hash tables (ch5) later
 - Nice to see a new and surprising data structure first
- A **priority queue** holds *compare-able data*
 - Unlike stacks and queues need to *compare items*
 - Given x and y , is x less than, equal to, or greater than y
 - What this means can depend on your data
 - Much of course will require comparable data: e.g. sorting
 - Integers are comparable, so will use them in examples
 - But the priority queue ADT is much more general
 - Typically two fields, the *priority* and the *data*

Priority Queue ADT

- Assume each item has a “priority”
 - The *lesser* item is the one with the *greater* priority
 - So “priority 1” is more important than “priority 4”
 - Just a convention, could also do a maximum priority

- Main Operations:
 - **insert**
 - **deleteMin**



- Key property: **deleteMin** returns and deletes from the queue the item with greatest priority (lowest priority value)
 - Can resolve ties arbitrarily

Aside: We will use ints as data and priority

For simplicity in lecture, we'll often suppose items are just `ints` and the `int` is also the priority

- So an operation sequence could be
 - `insert 6`
 - `insert 5`
 - `x = deleteMin // Now x = 5.`
- `int` priorities are common, but really just need comparable
- Not having “other data” is very rare
 - Example: print job has a priority *and* the file to print is the data

Priority Queue Example

To simplify our examples,
we will just use the priority
values from now on

`insert a` with priority `5`

`insert b` with priority `3`

`insert c` with priority `4`

`w = deleteMin`

`x = deleteMin`

`insert d` with priority `2`

`insert e` with priority `6`

`y = deleteMin`

`z = deleteMin`

after execution:

**Analogy: insert is like enqueue, deleteMin is like dequeue
But the whole point is to use priorities instead of FIFO**

Priority Queue Example (answer)

`insert a` with priority 5

`insert b` with priority 3

`insert c` with priority 4

`w = deleteMin`

`x = deleteMin`

`insert d` with priority 2

`insert e` with priority 6

`y = deleteMin`

`z = deleteMin`

after execution:

`w = b`

`x = c`

`y = d`

`z = a`

**To simplify our examples,
we will just use the priority
values from now on**

**Analogy: `insert` is like `enqueue`, `deleteMin` is like `dequeue`
But the whole point is to use priorities instead of FIFO**

Applications

Like all good ADTs, the priority queue arises often

- Sometimes “directly”, sometimes less obvious
- Run multiple programs in the operating system
 - “critical” before “interactive” before “compute-intensive”
 - Maybe let users set priority level
- Treat hospital patients in order of severity (or triage)
- Select print jobs in order of decreasing length?
- Forward network packets in order of urgency
- Select most frequent symbols for data compression (cf. CSE143)
- Sort: **insert** all, then repeatedly **deleteMin**

More applications

- “Greedy” algorithms
 - Select the ‘best-looking’ choice at the moment
 - Will see an example when we study graphs in a few weeks
- Discrete event simulation (system modeling, virtual worlds, ...)
 - Simulate how state changes when events fire
 - Each event e happens at some time t and generates new events e_1, \dots, e_n at times $t+t_1, \dots, t+t_n$
 - Naïve approach: advance “clock” by 1 unit at a time and process any events that happen then
 - Better:
 - *Pending events* in a priority queue (priority = time happens)
 - Repeatedly: **deleteMin** and then **insert** new events
 - Effectively, “set clock ahead to next event”

Preliminary Implementations of Priority Queue ADT

	insert	deleteMin
Unsorted Array		
Unsorted Linked-List		
Sorted Circular Array		
Sorted Linked-List		
Binary Search Tree (BST)		

Aside: More on possibilities

- Note: If priorities are inserted in random order, binary search tree will likely do better than $O(n)$
 - $O(\log n)$ **insert** and $O(\log n)$ **deleteMin** on average
 - Could get same performance from a *balanced* binary search tree (e.g. AVL tree we will study later)
- One more idea: if priorities are $0, 1, \dots, k$ can use array of lists
 - **insert**: add to front of list at **arr[priority]**, $O(1)$
 - **deleteMin**: remove from lowest non-empty list $O(k)$

Our Data Structure: The Binary Min Heap

The Heap:

- Worst case: $O(\log n)$ for insert
- Worst case: $O(\log n)$ for deleteMin
- If items arrive in random order, then the average-case of insert is $O(1)$
- Very good constant factors

Key idea: Only pay for functionality needed

- We need something better than scanning unsorted items
- But we do not need to maintain a full sorted list

- We will *visualize* our heap as a tree, so we need to review some tree terminology

Q: Reviewing Some Tree Terminology

root(T):

leaves(T):

children(B):

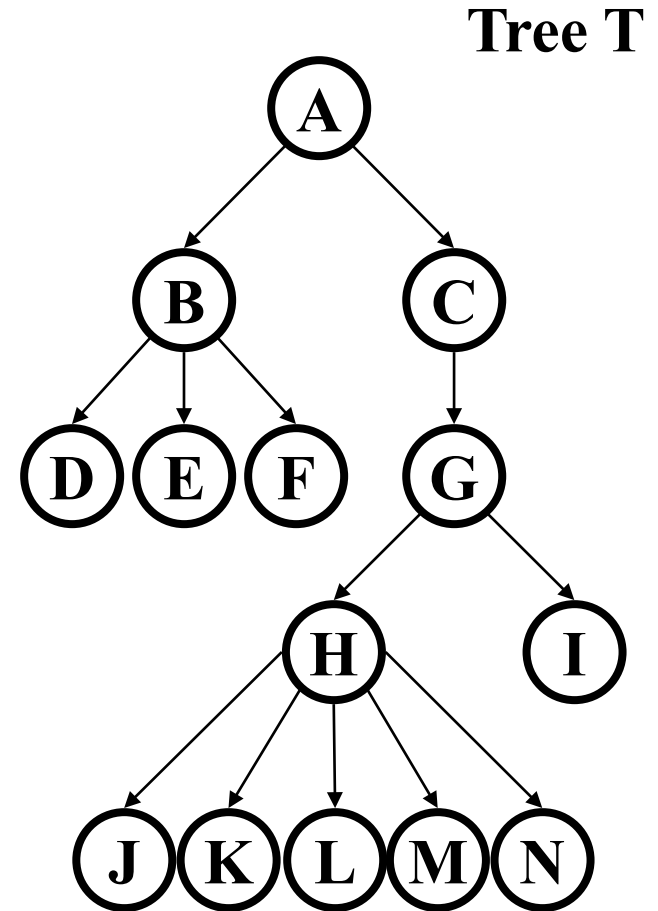
parent(H):

siblings(E):

ancestors(F):

descendants(G):

subtree(G):



Q: Some More Tree Terminology

depth(B):

height(G):

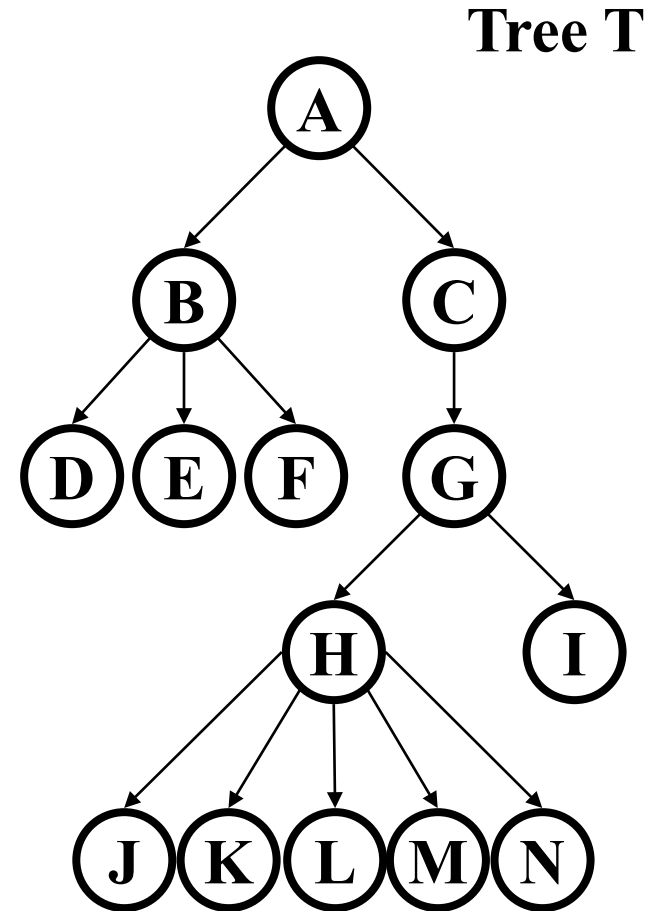
height(T):

degree(B):

branching factor(T):

height – number of edges in path from node to deepest descendent

depth – number of edges in path from node to root



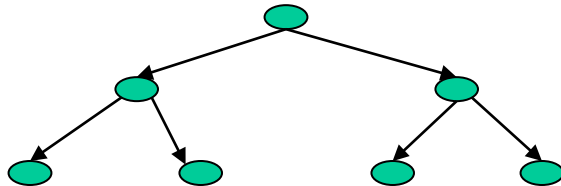
Types of Trees

Binary tree: Every node has ≤ 2 children

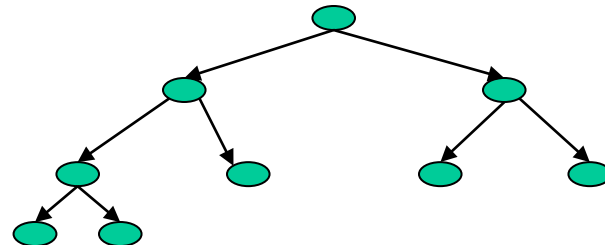
n-ary tree: Every node has $\leq n$ children

Perfect tree: Every row is completely full

Complete tree: All rows except possibly the bottom are completely full, and it is filled from left to right



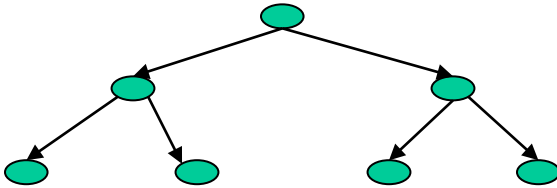
Perfect Tree



Complete Tree

More on Perfect Trees

Perfect tree: Every row is completely full



Perfect Tree

Some Basic Tree Properties

Nodes in a perfect binary tree of height h ?

Leaf nodes in a perfect binary tree of height h ?

Height of a perfect binary tree with n nodes?

Height of a complete binary tree with n nodes?

Properties of a Binary Min-Heap

More commonly known as a **binary heap** or simply a **heap**

- **Structure Property:**

A complete [binary] tree

- **Heap Property:**

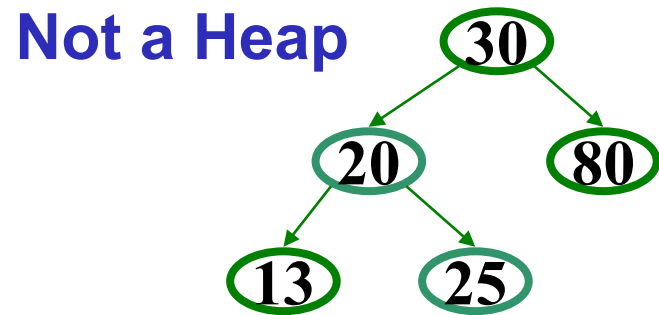
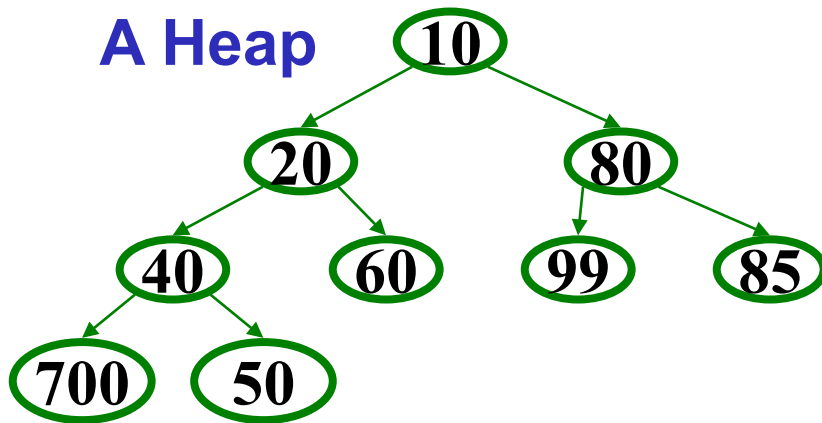
Every non-root node has a priority value larger than (or possibly equal to) the priority of its parent

How is this different from a binary search tree?

Properties of a Binary Min-Heap (example)

More commonly known as a **binary heap** or simply a **heap**

- **Structure Property:**
A complete [binary] tree
- **Heap Order Property:**
Every non-root node has a priority value larger than (or possibly equal to) the priority of its parent



More Properties of a Binary Min-Heap

- Where is the minimum priority item?
- What is the height of a heap with n items?

More Properties of a Binary Min-Heap (answer)

- Where is the minimum priority item?

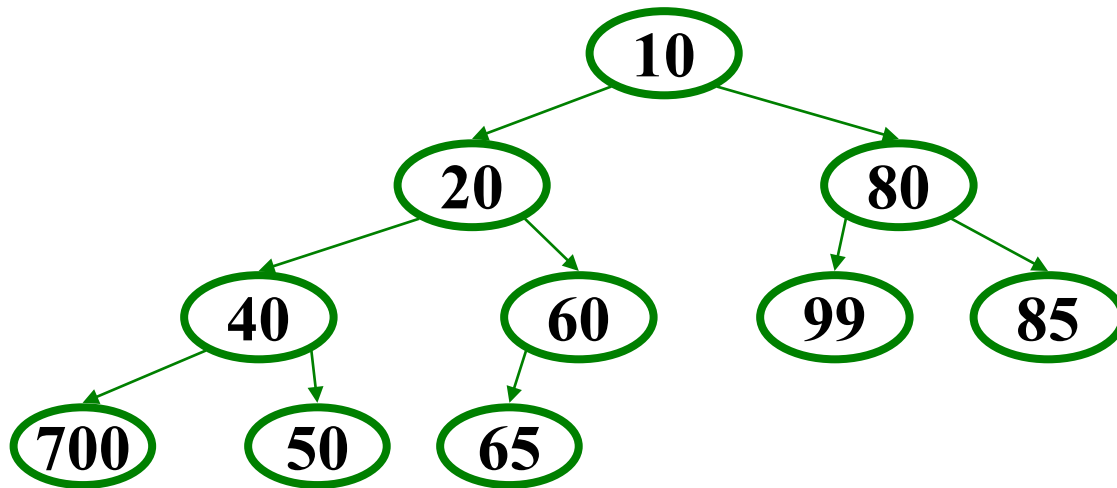
At the root

- What is the height of a heap with n items?

$\lceil \log_2 n \rceil$

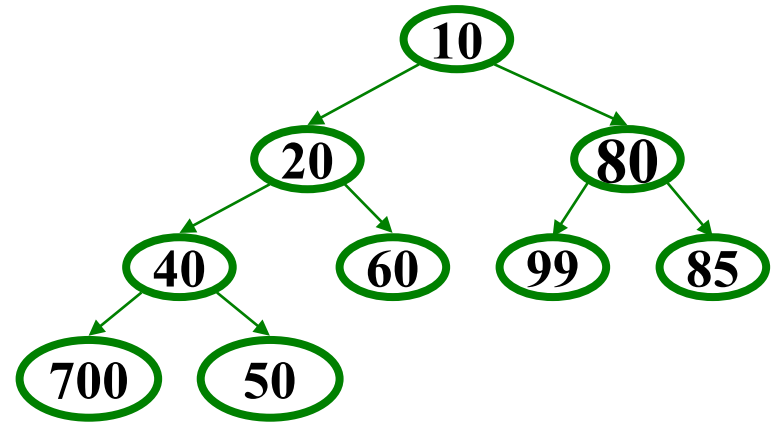
Heap Operations

- findMin:
- deleteMin: percolate down.
- insert(val): percolate up.



Operations: basic idea

- **findMin:**
return `root.data`
- **deleteMin:**
 1. `answer = root.data`
 2. Move right-most node in last row to root to restore structure property
 3. “Percolate down” to restore heap order property
- **insert:**
 1. Put new node in next position on bottom row to restore structure property
 2. “Percolate up” to restore heap order property

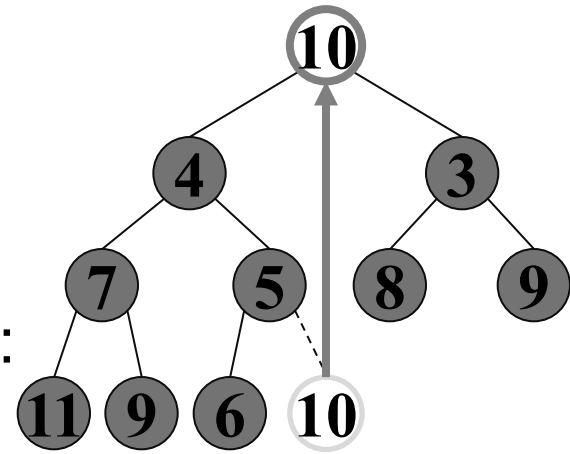
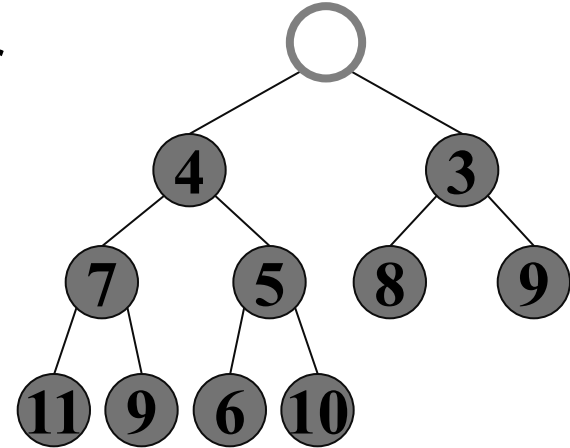


Overall strategy:

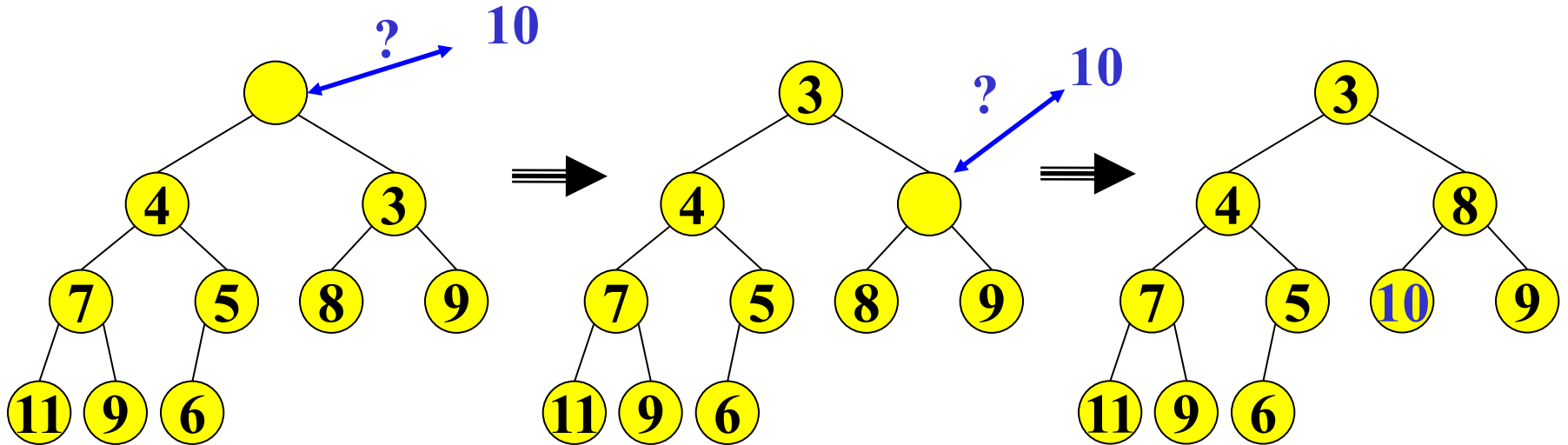
- *Preserve complete tree structure property*
- *This may break heap order property*
- *Percolate to restore heap order property*

DeleteMin Implementation

1. Delete value at root node (and store it for later return)
2. There is now a "hole" at the root. We must "fill" the hole with another value, must have a tree with one less node, and it must still be a complete tree
3. The "last" node is the obvious choice, but now the heap order property is violated
4. We **percolate down** to fix the heap order:
While greater than either child
 Swap with smaller child



Percolate Down



Percolate down:

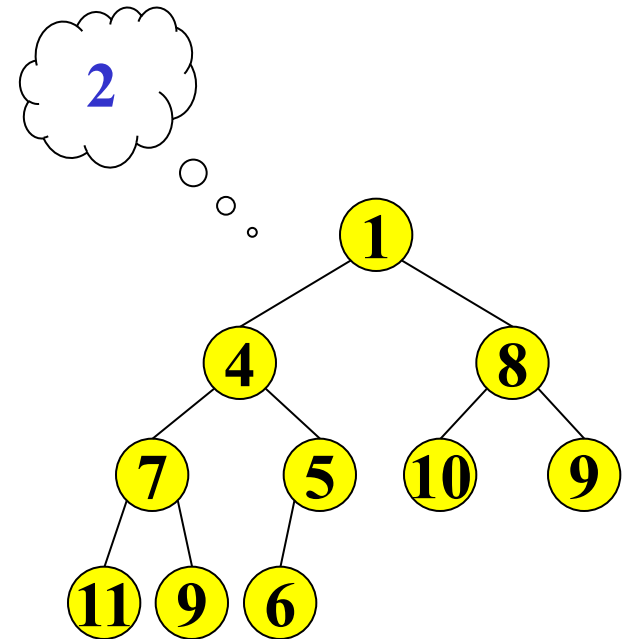
- Keep comparing with both children
- Move smaller child up and go down one level
- Done if both children are \geq item or reached a leaf node
- Why does this work? What is the run time?

DeleteMin: Run Time Analysis

- Run time is $O(\text{height of heap})$
- A heap is a complete binary tree
- Height of a complete binary tree of n nodes?
 - height = $\lfloor \log_2(n) \rfloor$
- Run time of **deleteMin** is $O(\log n)$

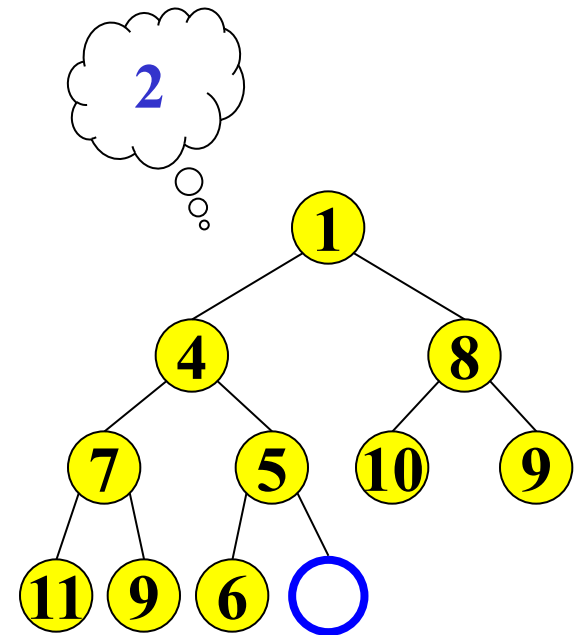
Insert

- Add a value to the tree
- Structure and heap order properties must still be correct afterwards

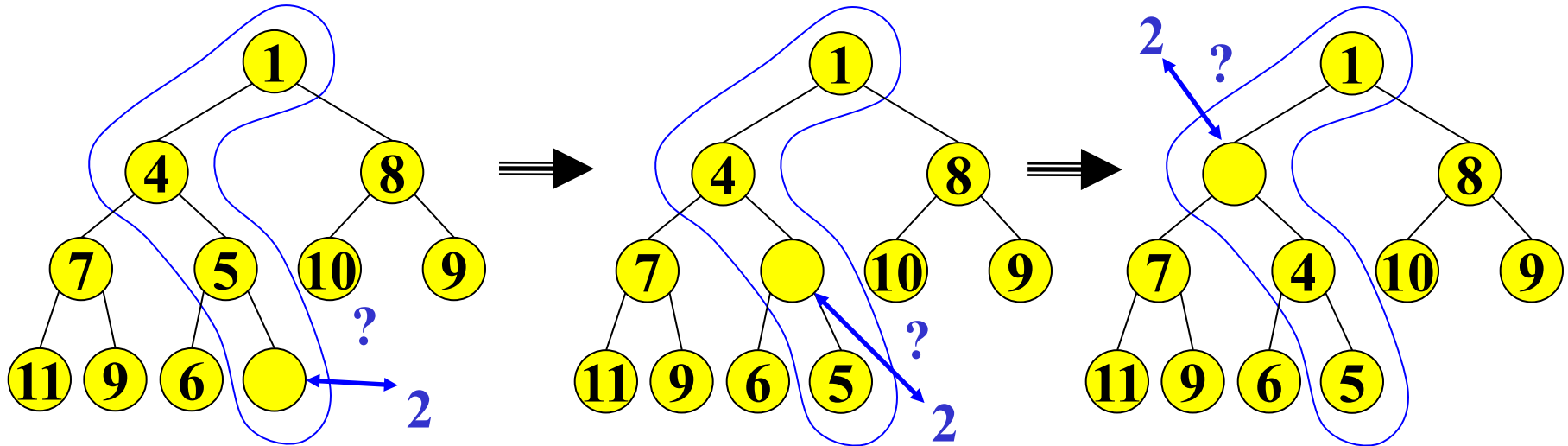


Insert: Maintain the Structure Property

- There is only **one** valid tree shape after we add one more node!
- So put our new data there and then focus on restoring the heap order property



Maintain the heap order property



Percolate up:

- Put new data in new location
- If parent larger, swap with parent, and continue
- Done if parent \leq item or reached root
- Why does this work? What is the run time?

A Clever Trick for Storing the Heap...

Clearly, insert and deleteMin are worst-case $O(\log n)$

- But we promised average-case $O(1)$ insert (how??)

Insert requires access to the “next to use” position in the tree

- Walking the tree from root to leaf requires $O(\log n)$ steps
- Insert and Deletemin would have to update the “next to use” reference each time: $O(\log n)$

We should only pay for the functionality we need!!

- Why have we insisted the tree be complete? 😊

All complete trees of size n contain the same edges

- So why are we even representing the edges?

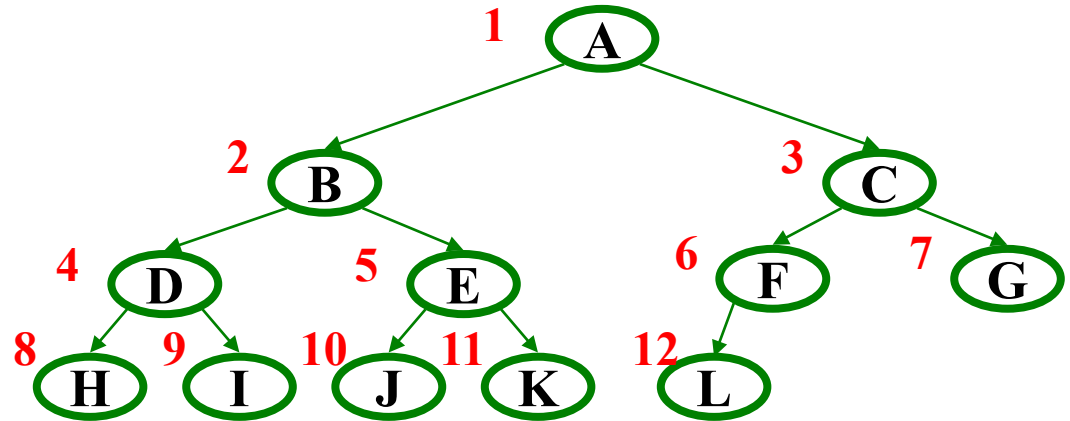
Here comes the really clever bit about implementing heaps!!!

Important: On our Exercises start counting from 0

Array Representation of a Binary Heap

From node i :

- left child:
- right child:
- parent:



	A	B	C	D	E	F	G	H	I	J	K	L	
0	1	2	3	4	5	6	7	8	9	10	11	12	13

- Here we skip index 0 to make the math simpler
- On your exercises, you will start counting at 0