Sorting

CSE 332

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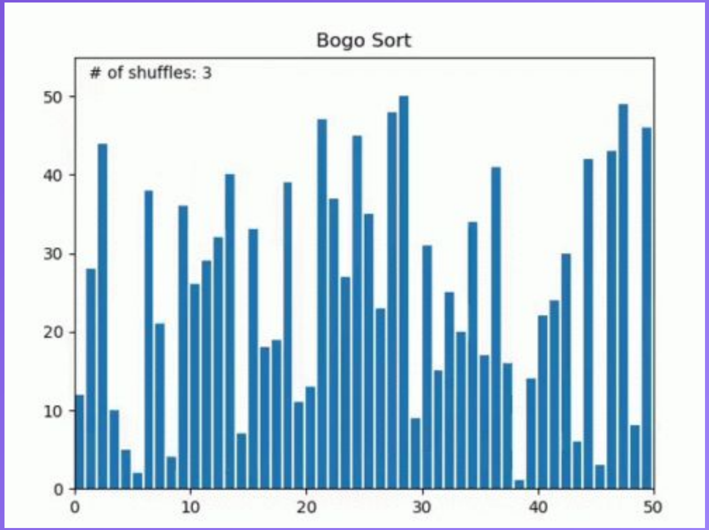
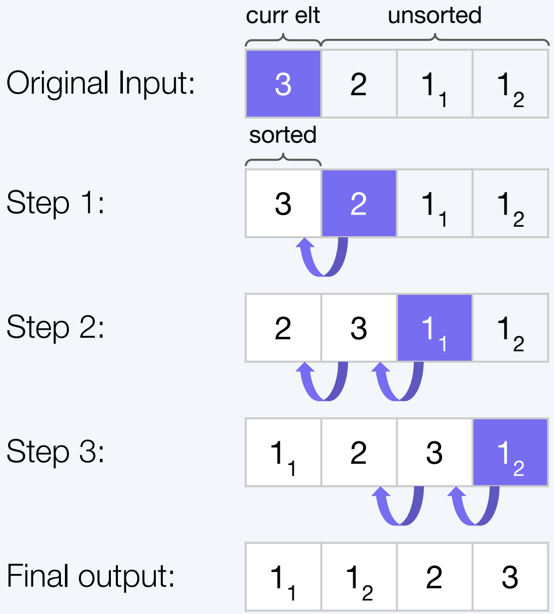


Comparison-based Sorting

Insertion Sort

Builds a sorted subarray at the front of the original array. Takes the first element of the unsorted subarray and inserts it into the sorted subarray.

- In-place and stable.
- Best-case runtime: $\mathcal{O}(n)$
- Worst-case runtime: $\mathcal{O}(n^2)$

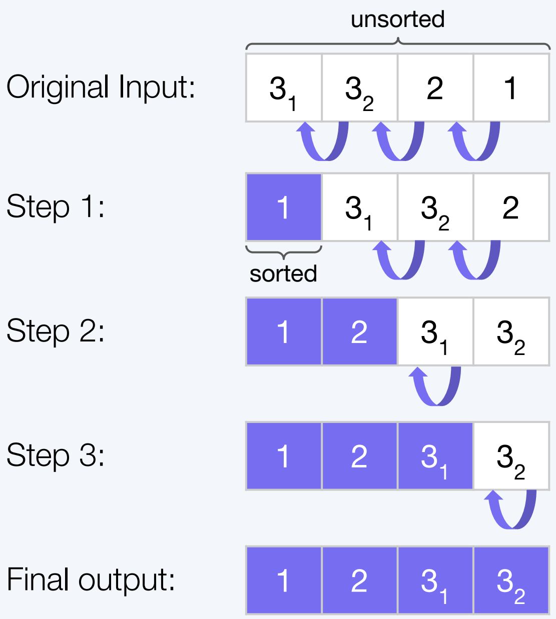


Selection Sort

Builds a sorted subarray at the front of the original array. Takes the smallest element of the unsorted subarray and **swaps it into the correct location** at the end of the sorted subarray.

- In-place and **not usually stable**
  - Can be made stable if you **shift** instead of swapping and break ties by selecting the left-most element (see image at right)
- Best-case runtime: $\mathcal{O}(n^2)$
- Worst-case runtime: $\mathcal{O}(n^2)$

A stable version of Selection Sort

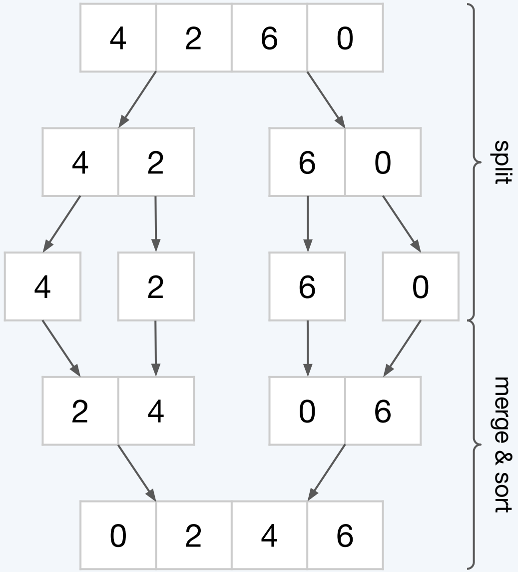


Merge Sort

```
mergeSort(input) -> sorted input:
1. sortedLeft = mergeSort(left half of input)
2. sortedRight = mergeSort(right half of input)
3. return (merged 'sortedLeft' and 'sortedRight')
```

Recursively splits an array into two halves, sorts them, and then merges them back together to obtain a sorted array.

- Not in-place but stable.
- Best-case runtime: $\mathcal{O}(n \log n)$
- Worst-case runtime: $\mathcal{O}(n \log n)$

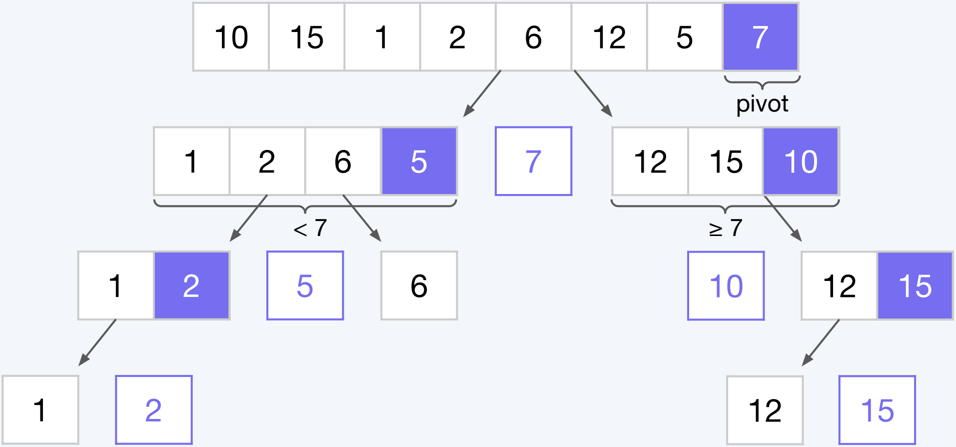


Quick Sort

```
quickSort(input) -> void:
1. pick pivot
2. partition input into 'lessThanPivot' and
   'greaterThanPivot' parts
3. quickSort(lessThanPivot)
4. quickSort(greaterThanPivot)
```

Recursively partitions an array based on a pivot element, sorts the subarrays on either side of the pivot, and merges them back together to obtain a sorted array.

- In-place but not stable.
- Best-case runtime: $\mathcal{O}(n \log n)$
- Worst-case runtime: $\mathcal{O}(n^2)$



Problem 0

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Suppose we sort an array of numbers, but it turns out every element of the array is the same (e.g. `[17, 17, 17, ..., 17]`).

What is the asymptotic runtime of the following sorting algorithms?

Sorting Algorithm	Asymptotic Runtime	Explanation
Insertion Sort		
Selection Sort		
Merge Sort		
Quick Sort		

Problem 0

Suppose we sort an array of numbers, but it turns out every element of the array is the same (e.g. `[17, 17, 17, ..., 17]`).

What is the asymptotic runtime of the following sorting algorithms?

Sorting Algorithm	Asymptotic Runtime	Explanation
Insertion Sort	$\mathcal{O}(n)$	Insertion Sort will traverse the array, but since it is already 'sorted', no extra computation is necessary
Selection Sort	$\mathcal{O}(n^2)$	Selection Sort always has $\mathcal{O}(n^2)$ runtime since it has to find the smallest item in the unsorted subarray n times
Merge Sort	$\mathcal{O}(n \log n)$	Merge Sort always has $\mathcal{O}(n \log n)$ runtime. You can prove this using recurrences!
Quick Sort	$\mathcal{O}(n^2)$	This is the worst case for Quick Sort. Since all the elements are the same, all items will end up on the same side of each partition.

Problem 1

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Given an array of integers as such: {11, 13, 55, 67, 79, 10, 8, 6, 4, 2}. Please answer the following questions (assume all sorts to be done in ascending order):

What is the asymptotic runtime of the following sorting algorithms?

(assuming that we choose the leftmost element as the pivot each time for quick sort)

Sorting Algorithm	Asymptotic Runtime	Explanation
Insertion Sort		
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Problem 1

Given an array of integers as such: {11, 13, 55, 67, 79, 10, 8, 6, 4, 2}. Please answer the following questions (assume all sorts to be done in ascending order):
What is the asymptotic runtime of the following sorting algorithms?

(assuming that we choose the leftmost element as the pivot each time for quick sort)

Sorting Algorithm	Asymptotic Runtime	Explanation
Insertion Sort	$\mathcal{O}(n^2)$	The array is not already sorted. At least half of the elements need to iterate through half of the array. So this follow the running time of a worst case.
Selection Sort	$\mathcal{O}(n^2)$	Selection Sort always has $\mathcal{O}(n^2)$ runtime, regardless of the nature of data
Merge Sort	$\mathcal{O}(n*log(n))$	Merge Sort always has $\mathcal{O}(n \log n)$ runtime, regardless of the nature of data
Quick Sort	$\mathcal{O}(n^2)$	After finish the sort of first pivot (11), we end up sort the left subarray in a worst case running time ({10, 8, 6, 4, 2} {11} {13 55 67 79})

Non-Comparison Sorting

Bucket Sort

```
bucketSort(input) -> sorted input:
1. create array of size B
2. put each element into their corresponding bucket
3. generate sorted array by iterating through the
   buckets in order
```

Distributes elements into their corresponding buckets. Buckets have an inherent ordering and are merged together in this ordering to produce the sorted array.

- Not in-place but stable.
- Runtime: $\mathcal{O}(n + B)$
  - Need to iterate over n elements and B buckets.
  - Good when $B \ll n$ or $B \approx n$.
  - Bad when $B \gg n$.

B: 5

Original Input:

$[5_1,\ 1_1,\ 3_1,\ 4_1,\ 3_2,\ 2,\ 1_2,\ 1_3,\ 5_2,\ 4_2,\ 5_3]$

Bucket Array	
1	$1_1,\ 1_2,\ 1_3$
2	2
3	$3_1,\ 3_2$
4	$4_1,\ 4_2$
5	$5_1,\ 5_2,\ 5_3$

Final output:

$[1_1,\ 1_2,\ 1_3,\ 2,\ 3_1,\ 3_2,\ 4_1,\ 4_2,\ 5_1,\ 5_2,\ 5_3]$

Radix Sort

```
radixSort(input) -> sorted input:
1. create array of size b
2. for each significant digit:
   3. run bucket sort on the elements using their
      corresponding significant digit values
```

Repeatedly runs bucket sort on the elements for each significant digit, from least significant to most significant.

Original Input: `[478, 537, 9, 721, 3, 38, 143, 67]` *b*: 10

Bucket Sort on 1's Digit

0	1	2	3	4	5	6	7	8	9
	721		3, 143				537, 67	478, 38	9

Bucket Sort on 10's Digit

0	1	2	3	4	5	6	7	8	9
3, 9		721	537, 38	143		67	478		

Radix Sort

```
radixSort(input) -> sorted input:
1. create array of size b
2. for each significant digit:
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```

Repeatedly runs bucket sort on the elements for each significant digit, from least significant to most significant.

Original Input: `[478, 537, 9, 721, 3, 38, 143, 67]` *b*: 10

Bucket Sort on 10's Digit

0	1	2	3	4	5	6	7	8	9
3, 9		721	537, 38	143		67	478		

- Notice how elements are now sorted with respect to their last two digits.
- By running bucket sort from the least to the most significant digit, the order of the more significant digits take precedence over the less significant digits.

Radix Sort

```
radixSort(input) -> sorted input:
1. create array of size b
2. for each significant digit:
   3. run bucket sort on the elements using their
      corresponding significant digit values
```

Repeatedly runs bucket sort on the elements for each significant digit, from least significant to most significant.

Original Input: `[478, 537, 9, 721, 3, 38, 143, 67]` *b*: 10

Bucket Sort on 10's Digit

0	1	2	3	4	5	6	7	8	9
3, 9		721	537, 38	143		67	478		

Bucket Sort on 100's Digit

0	1	2	3	4	5	6	7	8	9
3, 9, 38, 67	143			478	537		721		

Radix Sort

```
radixSort(input) -> sorted input:
1. create array of size b
2. for each significant digit:
   3. run bucket sort on the elements using their
      corresponding significant digit values
```

Repeatedly runs bucket sort on the elements for each significant digit, from least significant to most significant.

Original Input: `[478, 537, 9, 721, 3, 38, 143, 67]` b: 10

Bucket Sort on 100's Digit

0	1	2	3	4	5	6	7	8	9
3, 9, 38, 67	143			478	537		721		

Output: `[3, 9, 38, 67, 143, 478, 537, 721]`

- Not in-place but stable.
- Runtime: $\mathcal{O}(p(n+b))$
  - Bucket sort has $\mathcal{O}(n+b)$ runtime, and we're running it $p = \log_b(\max(n))$ times.

Thank You!