

CSE 332 - Section 3 Worksheet

0. Recurrence Relations

a) Find a recurrence $T(n)$ modeling the worst-case runtime complexity of $f(n)$

```
1  f(n) {  
2    if (n <= 0) {  
3      return 1  
4    }  
5    return 2 * f(n - 1) + 1  
6  }
```

b) Find a recurrence $T(n)$ modeling the worst-case runtime complexity of $g(n)$

```
1  f(n) {  
2    if (n <= 1) {  
3      return 1000  
4    }  
5    if (g(n/3) > 5) {  
6      for (int i = 0; i < n; i++) {  
7        println("Yay")  
8      }  
9      return 5 * g(n/3)  
10   } else {  
11     for (int i = 0; i < n * n; i++) {  
12       println("Yay")  
13     }  
14     return 4 * g(n/3)  
15   }
```

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1. Tree Method

For each of the following recurrence relations, use the tree method to convert it to closed form:

$$T(n) = \begin{cases} 1 & \text{if } n \leq 1 \\ 2T\left(\frac{n}{2}\right) + n & \text{otherwise} \end{cases}$$

a)

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$$T(n) = \begin{cases} 4 & \text{if } n \leq 4 \\ 2T\left(\frac{n}{2}\right) + n & \text{otherwise} \end{cases}$$

b)

$$T(n) = \begin{cases} 100 & \text{if } n \leq 1 \\ 2T\left(\frac{n}{2}\right) + 100n & \text{otherwise} \end{cases}$$

c)

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$$T(n) = \begin{cases} 1 & \text{if } n \leq 1 \\ 3T\left(\frac{n}{2}\right) + n & \text{otherwise} \end{cases}$$

d)

$$T(n) = \begin{cases} 1 & \text{if } n \leq 1 \\ 2T\left(\frac{n}{3}\right) + n & \text{otherwise} \end{cases}$$

e)

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f)

$$T(n) = \begin{cases} 1 & n \leq 1 \\ 2T(n-2) + 1 & \text{otherwise} \end{cases}$$

2. Putting It All Together

Consider the function $f(n)$. Find a recurrence modeling the worst-case runtime of this function and then find a Big-Oh bound for this recurrence.

```
1  f(n) {  
2      if (n <= 1) {  
3          return 0  
4      }  
5      int result = f(n/2)  
6      for (int i = 0; i < n; i++) {  
7          result *= 4  
8      }  
9      return result + f(n/2)  
10 }
```

a) Find a recurrence $T(n)$ modeling the *worst-case runtime complexity* of $f(n)$

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- b) Use your answer in part (a) to find a closed form for $T(n)$