

Midterm Review Questions

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1 Introduction

These are midterm review questions I made up to prep for exam 1.

Notes on exam logistics:

- There's not enough seats to have one empty space between each pair of students. So you'll be allowed to sit next to someone, but please leave space if you can. TAs will also be in the room to help proctor.

- The very front row will be reserved for students who come in late. Don't sit in the front row if you're early/on time.

2 Asymptotics

1. Select always, sometimes or never: $\underline{f(n)+g(n)}$ is \underline{S} $\mathcal{O}(\min\{f(n), g(n)\})$.
2. Select always, sometimes or never: $\underline{f(n)+g(n)}$ is \underline{A} $\Omega(\min\{f(n), g(n)\})$.

$$\underline{\min\{f(n), g(n)\}} = \begin{cases} \underline{f(n)} & , f(n) < g(n) \\ \underline{g(n)} & \text{o.t.h.} \end{cases}$$

$$\begin{array}{rcl} f(n) & \geq & \min\{f, g\} \\ + \quad g(n) & \geq & \min\{f, g\} \\ \hline \end{array}$$

$$\begin{array}{rcl} & 1 & \\ f(n) + g(n) & \geq & 2\min\{f, g\} \\ \therefore f(n) + g(n) & \in & \Omega(\downarrow) \end{array}$$

3 Code Analysis

2-3 min.

Describe the worst-case runtime of the code below, using a simplified big-oh bound:

```
void f(int n) {
    for (int i = 1; i <= n; i++) {
        int copy = i;
        while (copy > 0) {
            copy /= 2;
        }
    }
}
```

$\log(i)$

$\log(1) + \log(2) + \log(3) + \dots + \log(n) \in \Theta(n \log n)$

$O(\underline{n \log n})$

```
int g(int n) {
    int count = 0;  $n < 20$  cool fact :
```

```
    if (n * n < 20 * n) {
        for (int i = 0; i < n; i++) {
            for (int j = 0; j < n; j++) {
                for (int k = 0; k < n; k++) {
                    count++;
                }
            }
        }
    }
```

$$\sum_{i=0}^n f(i) \approx \int_0^n f(x) dx$$

```
    } else {  $n \geq 20$ 
        count--;
    }
    return count;
}
```

$O(\underline{1})$

$$n^2 < 20n$$

$$\implies n < 20$$

$$\forall n \geq n_0 \quad f(n) \leq c \cdot g(n)$$

$$\Rightarrow f(n) \in O(g(n))$$

$$n_0 = 21, \quad \forall n \geq 21. \quad T(n) \leq c \Rightarrow T(n) \in O(1)$$

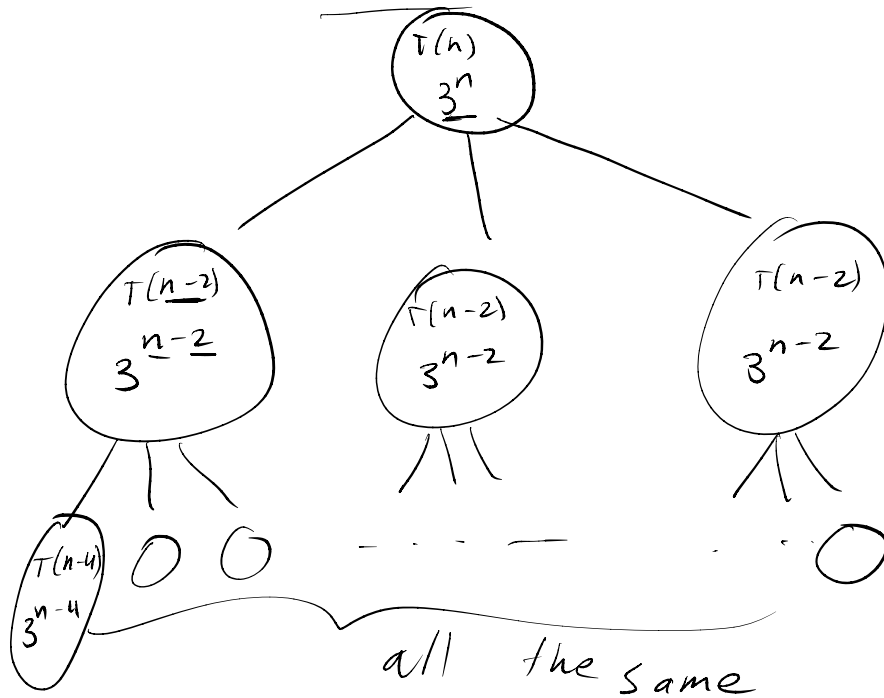
4 Solving a recurrence

5-7 min.

Solve this recurrence:

$$T(n) = 3T(n-2) + 3^n$$

$$T(0) = 1$$



work per level: $3^i (3^{n-2i})$

$$= 3^{n-i} = \frac{3^n}{3^i}$$

base case occurs
at level $i = n/2$.

$$T(n) = \sum_{i=0}^{n/2-1} \frac{3^n}{3^i} + 3^{n/2} (1)$$

$$= \sqrt{3}^n + 3^n \sum_{i=0}^{n/2-1} \left(\frac{1}{3}\right)^i \leq \underline{\underline{3^n \cdot c}}$$

$$\geq 3^n$$

4 Solving a recurrence

Solve this recurrence:

$$T(n) = 3T(n-2) + 3^n$$

✓ geom. series
that converges

$$T(n) = \sqrt{3}^n + \underline{3^n \sum_{i=0}^{n/2-1} \left(\frac{1}{3}\right)^i} \leq \underline{3^n \cdot C}$$

$$\underline{3^n} \leq \underline{T(n)} \leq 3^n \cdot C$$

$$\boxed{T(n) \in \Theta(3^n)}$$

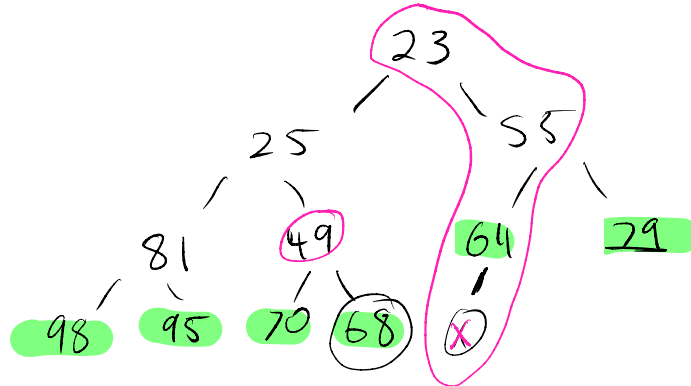
5 Heaps

5-7 minutes

Here's a 0-indexed binary min heap, named `heap`:

[23, 25, 55, 81, 49, 64, 79, 98, 95, 70, 68]

1. Draw the visual representation of heap.



2. Suppose we execute the code: `heap.insert(x)`, where `x` is some integer (`x` is both the value and the priority, just like in lecture). For each assertion below, give a value of `x` such that the assertion is true after the code is executed, or explain why no such `x` exists (1-2 sentences max). Consider each assertion independently; use the same initial heap for each one.

- (a) No leaf node has a value of "68"

Not possible. Inserting `x` cannot change the position of the 68 above.

- (b) Some node with value "64" has a child with value "71".

71

- (c) No node with value ~~79~~ has a parent with value "55".

24

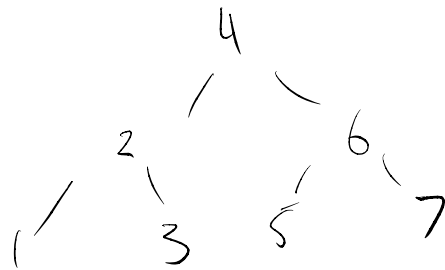
- (d) Every node with value "49" has exactly one child

Not possible. Inserting `x` cannot change the positions of 49, 70, 68 in the original heap.

3 min

6 AVL

Insert these elements into an AVL tree: 4, 7, 2, 1, 6, 3, 5



AVERAGE CASE

for BST insert

$O(\log n)$, so asking

Chat GPT for random

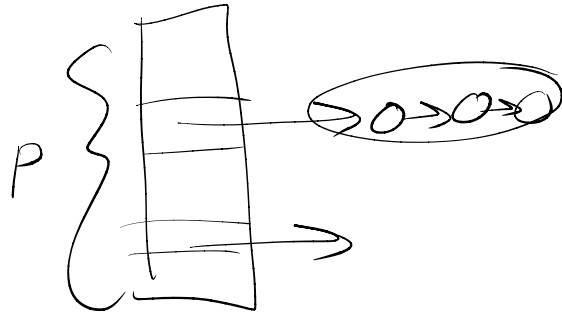
numbers was not a good idea!

2-3 min

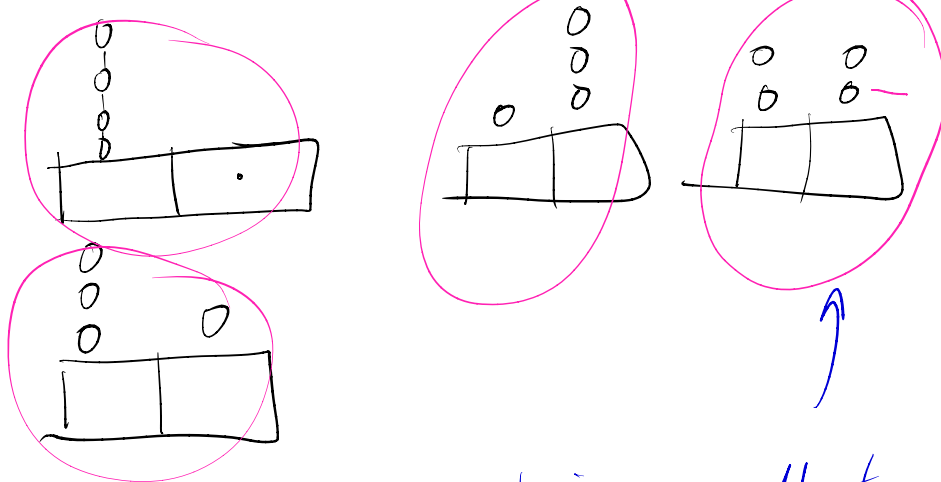
7 Hashing

1. If you have a separate chaining hash table with n elements and table size p , what is the minimum possible length of the longest chain? You may assume n is a multiple of p .

$$n/p$$



$$n = 4, \quad p = 2$$



longest chain smallest when
evenly distributed