CSE 332: Data Structures & Parallelism

Lecture 10:More Hashing

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Announcements

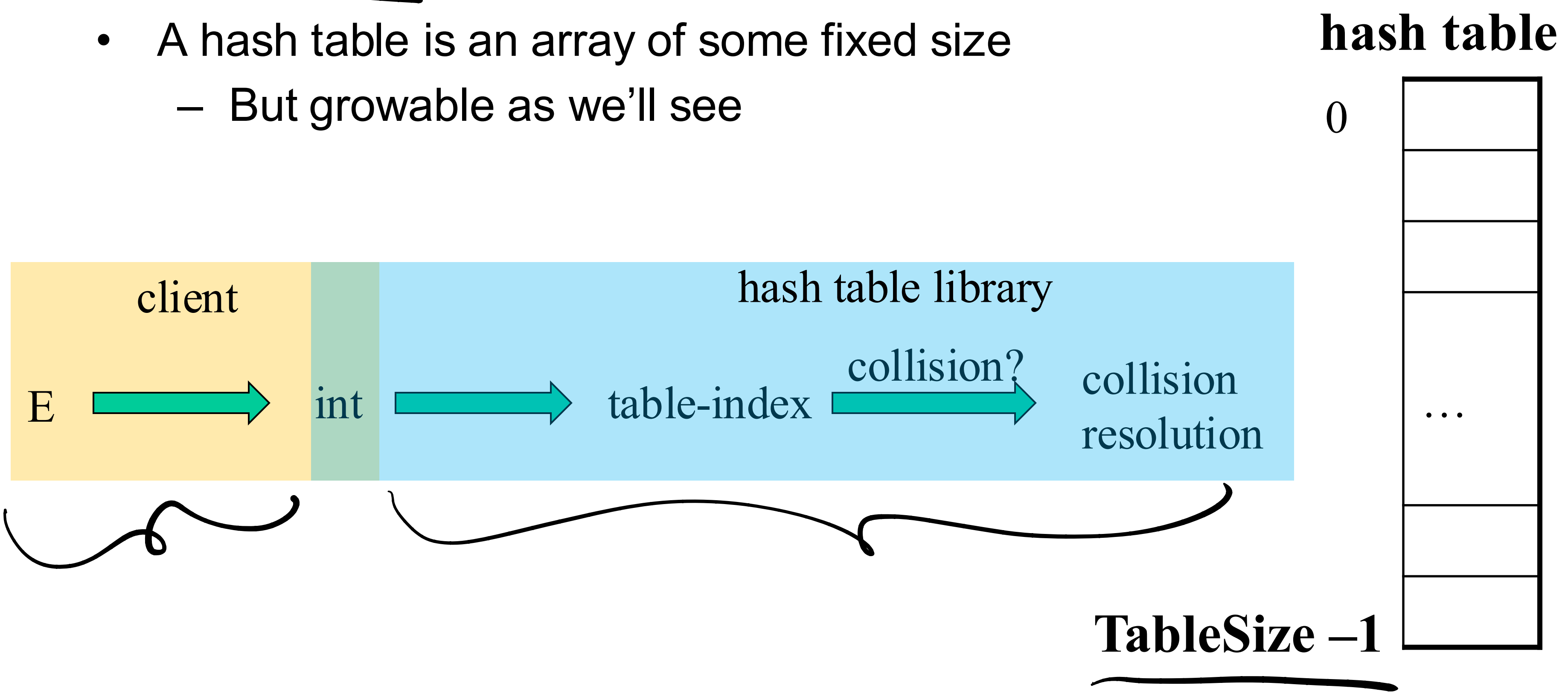
- EX04 Released
  - Start early!
- Exam 1 next Friday

Today

- Dictionaries
  - Finish Hashing

Hash Tables: Review

- Aim for constant-time (i.e., *O*(1)) **`find`**, **`insert`**, and **`delete`**
  - "On average" under some reasonable assumptions
- A hash table is an array of some fixed size
  - But growable as we'll see



Hashing Choices

1. Choose a Hash function • client
2. Choose TableSize • implementer
3. Choose a Collision Resolution Strategy from these: • implementer
   - Separate Chaining
   - Open Addressing
     - Linear Probing
     - Quadratic Probing
     - Double Hashing

- Other issues to consider:
  - Deletion? → Lazy deletion / actually remove
  - What to do when the hash table gets "too full"?

Open Addressing: Linear Probing

- Why not use up the empty space in the table?
- Store directly in the array cell (no linked list)
- How to deal with collisions?
- If **`h(key)`** is already full,
  - try **`(h(key) + 1) % TableSize`**. If full,
  - try **`(h(key) + 2) % TableSize`**. If full,
  - try **`(h(key) + 3) % TableSize`**. If full…

- Example: insert 38, 19, 8, 109, 10

0	8
1	109
2	10
3	
4	
5	
6	
7	
8	38
9	19

Open Addressing: Linear Probing

- Another simple idea: If `h(key)` is already full,
  - try `(h(key) + 1) % TableSize`. If full,
  - try `(h(key) + 2) % TableSize`. If full,
  - try `(h(key) + 3) % TableSize`. If full…
- Example: insert 38, 19, 8, 109, 10

0	/
1	/
2	/
3	/
4	/
5	/
6	/
7	/
8	38
9	19

Open Addressing: Linear Probing

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1	109
2	10
3	/
4	/
5	/
6	/
7	/
8	38
9	19

Open addressing

Linear probing is *one example* of open addressing

In general, open addressing means resolving collisions by trying a sequence of other positions in the table.

Trying the *next* spot is called probing

- We just did linear probing:
  - i^{th} probe: `(h(key) + i) % TableSize`
- In general have some probe function `f` and :
  - i^{th} probe: `(h(key) + f(i)) % TableSize`

qudratic : $f(i) = i^2$

Open addressing does poorly with high load factor λ

- So want larger tables
- Too many probes means no more $O(1)$

Questions: Open Addressing: Linear Probing

How should **`find`** work? If value is in table? If not there?

Worst case scenario for **`find`**?

How should we implement **`delete`**?

How does **open addressing with linear probing** compare to **separate chaining**?

Open Addressing: Other Operations

`insert` finds an open table position using a probe function

What about **`find`**?

- Must use same probe function to "retrace the trail" for the data
- Unsuccessful search when reach empty position

What about **`delete`**?

- ***Must*** use "lazy" deletion. Why?
    - Marker indicates "no data here, but don't stop probing"

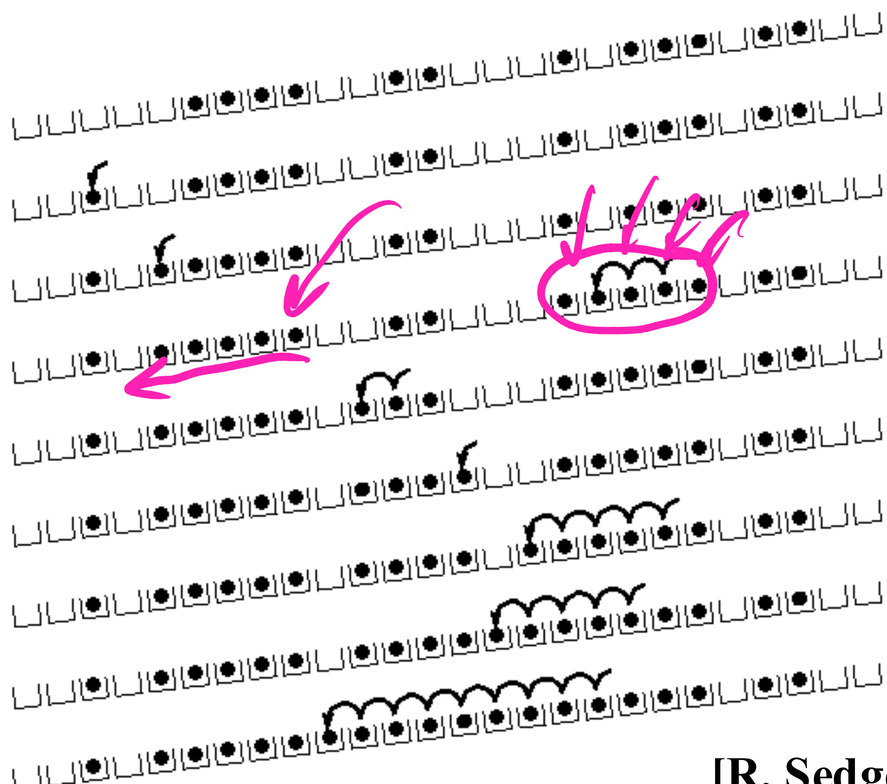
10	×	/	23	/	/	16	×	26

    - As with lazy deletion on other data structures, on insert, spots marked "deleted" can be filled in.
- Note: **`delete`** with chaining is just calling delete on the bucket (e.g. linked list)

Primary Clustering

It turns out linear probing is a *bad idea*, even though the probe function is quick to compute (a good thing)

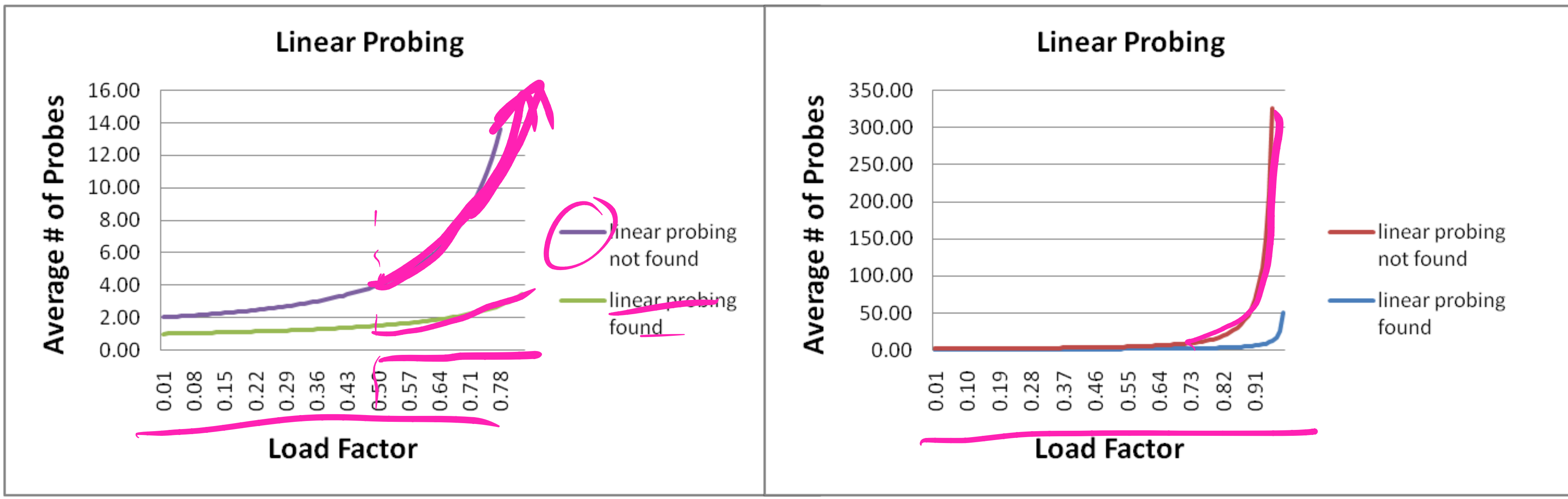
- Tends to produce *clusters*, which lead to long probe sequences
- Called primary clustering
- Saw the start of a cluster in our linear probing example



[R. Sedgewick]

Analysis in chart form

- Linear-probing performance degrades rapidly as table gets full
  - (Formula assumes "large table" but point remains)



- By comparison, separate chaining performance is linear in λ and has no trouble with λ**>1**

$\lambda = 3$

Open Addressing: Linear probing

`(h(key) + f(i)) % TableSize`

- For linear probing:

  `f(i) = i`

- So probe sequence is:
  - 0th probe: `h(key) % TableSize`
  - 1st probe: `(h(key) + 1) % TableSize`
  - 2nd probe: `(h(key) + 2) % TableSize`
  - 3rd probe: `(h(key) + 3) % TableSize`
  - …
  - ith probe: `(h(key) + i) % TableSize`

Open Addressing: Quadratic probing

- We can avoid primary clustering by changing the probe function…

  `(h(key) + f(i)) % TableSize`

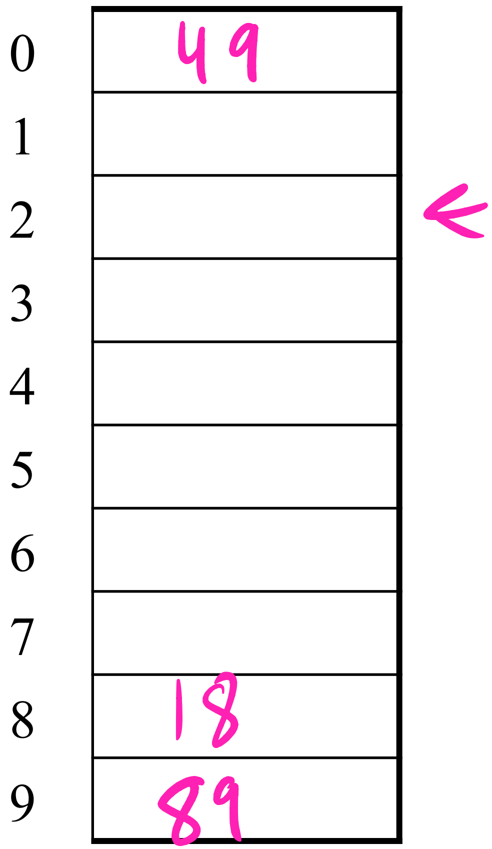
  - For quadratic probing:

    `f(i) = i²`

  - So probe sequence is:
    - 0^{th} probe: `h(key) % TableSize`
    - 1^{st} probe: `(h(key) + 1) % TableSize`
    - 2^{nd} probe: `(h(key) + 4) % TableSize`
    - 3^{rd} probe: `(h(key) + 9) % TableSize`
    - …
    - i^{th} probe: `(h(key) + i²) % TableSize`

- Intuition: Probes quickly “leave the neighborhood” ith probe: (h(key) + i^2) % TableSize

Quadratic Probing Example



TableSize=10

Insert:

89

18

49

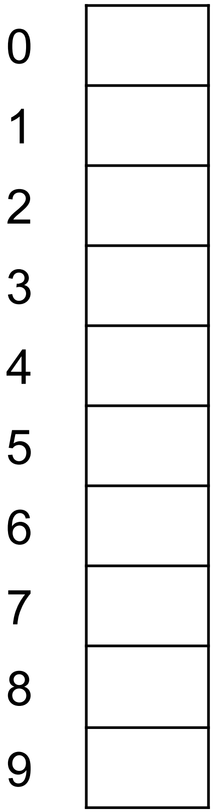
58

79

Quadratic Probing Example

TableSize = 10

insert(89)



Quadratic Probing Example

0	
1	
2	
3	
4	
5	
6	
7	
8	
9	89

TableSize = 10

insert(89)

insert(18)

Quadratic Probing Example

0	
1	
2	
3	
4	
5	
6	
7	
8	**18**
9	89

TableSize = 10

insert(89)

insert(18)

insert(49)

Quadratic Probing Example

0	49
1	
2	
3	
4	
5	
6	
7	
8	18
9	89

TableSize = 10

insert(89)

insert(18)

insert(49)

  49 % 10 = 9 collision!

  (49 + 1) % 10 = 0

insert(58)

Quadratic Probing Example

0	49
1	
2	58
3	
4	
5	
6	
7	
8	18
9	89

TableSize = 10

insert(89)

insert(18)

insert(49)

insert(58)

- **58 % 10 = 8 collision!**
- **(58 + 1) % 10 = 9 collision!**
- **(58 + 4) % 10 = 2**

insert(79)

Quadratic Probing Example

0	49
1	
2	58
3	79
4	
5	
6	
7	
8	18
9	89

TableSize = 10

insert(89)

insert(18)

insert(49)

insert(58)

insert(79)

  79 % 10 = 9 collision!

  (79 + 1) % 10 = 0 collision!

  (79 + 4) % 10 = 3

ith probe: $(h(key) + i^2) \% TableSize$

Another Quadratic Probing Example

0	48
1	
2	5
3	55
4	
5	40
6	76

TableSize = 7

Insert:

76	(76 % 7 = 6)
40	(40 % 7 = 5)
48	(48 % 7 = 6)
5	(5 % 7 = 5)
55	(55 % 7 = 6)
47	(47 % 7 = 5)

Another Quadratic Probing Example

0	
1	
2	
3	
4	
5	
6	76

TableSize = 7

Insert:

76	**(76 % 7 = 6)**
40	**(40 % 7 = 5)**
48	**(48 % 7 = 6)**
5	**(5 % 7 = 5)**
55	**(55 % 7 = 6)**
47	**(47 % 7 = 5)**

ith probe: (`h(key) + i`^{2}`) % TableSize`

Another Quadratic Probing Example

0	
1	
2	
3	
4	
5	40
6	76

TableSize = 7

Insert:

76	**(76 % 7 = 6)**
40	**(40 % 7 = 5)**
48	**(48 % 7 = 6)**
5	**(5 % 7 = 5)**
55	**(55 % 7 = 6)**
47	**(47 % 7 = 5)**

ith probe: (h(key) + i^2) % TableSize

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47	**(47 % 7 = 5)**

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Insert:

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ith probe: (`h(key) + i`^{2}`) % TableSize`

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TableSize = 7

Insert:

76	**(76 % 7 = 6)**
40	**(40 % 7 = 5)**
48	**(48 % 7 = 6)**
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55	**(55 % 7 = 6)**
47	**(47 % 7 = 5)**

ith probe: `(h(key) + i²) % TableSize`

Another Quadratic Probing Example

0	48
1	
2	5
3	55
4	
5	40
6	76

TableSize = 7

Insert:

76	**(76 % 7 = 6)**
40	**(40 % 7 = 5)**
48	**(48 % 7 = 6)**
5	**(5 % 7 = 5)**
55	**(55 % 7 = 6)**
47	**(47 % 7 = 5)**

(47 + 1) % 7 = 6 collision!

(47 + 4) % 7 = 2 collision!

(47 + 9) % 7 = 0 collision!

(47 + 16) % 7 = 0 collision!

(47 + 25) % 7 = 2 collision!

Will we ever get a 1 or 4?!?

Another Quadratic Probing Example

0	48
1	
2	5
3	55
4	
5	40
6	76

insert(47) will always fail here. Why?

For all *i*, $(5 + i^2) \% 7$ is 0, 2, 5, or 6

Proof uses induction and

$$(5 + i^2) \% 7 = (5 + (i - 7)^2) \% 7$$

In fact, for all *c* and *k*,

$$(c + i^2) \% k = (c + (i - k)^2) \% k$$

Risk: you might not be able to insert an element!

From bad news to good news

Bad News:

- After `TableSize` quadratic probes, we cycle through the same indices

Good News:

- If `TableSize` is *prime* and λ < ½, then quadratic probing will find an empty slot in at most `TableSize/2` probes
- So: If you keep λ < ½ and `TableSize` is *prime*, no need to detect cycles
- Proof posted in `lecture10.txt` (slightly less detailed proof in textbook)

For prime `TableSize` and $0 \le i, j \le$ `TableSize/2` where $i \ne j$,

$$(h(key) + i^2) \,\%\, TableSize \ne (h(key) + j^2) \,\%\, TableSize$$

That is, if `TableSize` is prime, the first `TableSize`/2 quadratic probes map to different locations (and one of those will be empty if the table is < half full).

Clustering reconsidered

- Quadratic probing does not suffer from primary clustering: As we resolve collisions we are not merely growing "big blobs" by adding one more item to the end of a cluster, we are looking i^2 locations away, for the next possible spot.

- But quadratic probing does not help resolve collisions between keys that initially hash *to the same **index***
  - Any 2 keys that initially hash to the same index **will have the same series of moves after that** looking for any empty spot
  - Called secondary clustering

- Can avoid secondary clustering with *a probe function that depends on the key*: double hashing…

Open Addressing: Double hashing

Idea: Given two good hash functions *h* and *g*, and two different keys *k1* and *k2*, it is very unlikely that: `h(k1)==h(k2)` and `g(k1)==g(k2)`

`(h(key) + f(i)) % TableSize`

- For double hashing:

  `f(i) = i*g(key)`

- So probe sequence is:
  - 0th probe: `h(key) % TableSize`
  - 1st probe: `(h(key) + g(key)) % TableSize`
  - 2nd probe: `(h(key) + 2*g(key)) % TableSize`
  - 3rd probe: `(h(key) + 3*g(key)) % TableSize`
  - …
  - ith probe: `(h(key) + i*g(key)) % TableSize`

- Detail: Make sure `g(key)` can't be `0`

Open Addressing: Double Hashing

0	
1	
2	
3	13
4	
5	
6	
7	33
8	28
9	149

T = 10 (TableSize)

<u>Hash Functions</u>:

h(key) = key mod T

g(key) = 1 + ((key/T) mod (T-1))

Insert these values into the hash table in this order. Resolve any collisions with double hashing:

13

28

33

147

43

ith probe: `(h(key) + i*g(key)) % TableSize`

Double Hashing

0	
1	
2	
3	13
4	
5	
6	
7	
8	
9	

T = 10 (TableSize)

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13

28

33

147

43

ith probe: **`(h(key) + i*g(key)) % TableSize`**

Double Hashing

0	
1	
2	
3	13
4	
5	
6	
7	
8	28
9	

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13

28

33

147

43

ith probe: `(h(key) + i*g(key)) % TableSize`

Double Hashing

0	
1	
2	
3	13
4	
5	
6	
7	33
8	28
9	

T = 10 (TableSize)

Hash Functions:

h(key) = key mod T

g(key) = 1 + ((key/T) mod (T-1))

Insert these values into the hash table in this order. Resolve any collisions with double hashing:

13

28

33 → g(33) = 1 + 3 mod 9 = 4

147

43

ith probe: `(h(key) + i*g(key)) % TableSize`

Double Hashing

0	
1	
2	
3	13
4	
5	
6	
7	33
8	28
9	147

T = 10 (TableSize)

Hash Functions:

h(key) = key mod T

g(key) = 1 + ((key/T) mod (T-1))

Insert these values into the hash table in this order. Resolve any collisions with double hashing:

13

28

33

147 → g(147) = 1 + 14 mod 9 = 6

43

ith probe: `(h(key) + i*g(key)) % TableSize`

Double Hashing

0	
1	
2	
3	13
4	
5	
6	
7	33
8	28
9	147

T = 10 (TableSize)

<u>Hash Functions</u>:

h(key) = key mod T

g(key) = 1 + ((key/T) mod (T-1))

Insert these values into the hash table in this order. Resolve any collisions with double hashing:

13

28

33

147 → g(147) = 1 + 14 mod 9 = 6

43 → g(43) = 1 + 4 mod 9 = 5

We have a problem:

$3 + 0 = 3$ $\quad 3 + \mathbf{5} = \mathbf{8}$ $\quad \mathbf{3} + 10 = \mathbf{13}$

$3 + \mathbf{15} = \mathbf{18}$ $\quad 3 + 20 = 23$

Double-hashing analysis

Intuition: Since each probe is "jumping" by **`g(key)`** each time, we "leave the neighborhood" *and* "go different places from other initial collisions"

But, as in quadratic probing, we could still have a problem where we are not "safe" due to an infinite loop despite room in table:

- No guarantee that i*g(key) will let us try all/most indices
- It is known that this cannot happen in at least one case:

  For primes p and q such that $2 < q < p$

  h(key) = key % p

  g(key) = q – (key % q)

Yet another reason to use a prime TableSize

- So, for double hashing

  i^{th} probe: (**`h(key) + i*g(key))% TableSize`**
- Say g(key) divides Tablesize
  - That is, there is some integer x such that x*g(key)=Tablesize
  - After x probes, we'll be back to trying the same indices as before
- Ex:
  - Tablesize=50
  - g(key)=25
  - Probing sequence:
    - h(key)
    - h(key)+25
    - h(key)+50=h(key)
    - h(key)+75=h(key)+25
- Only 1 & itself divide a prime

Where are we?

- Separate Chaining is easy
  - `find, insert, delete` proportional to load factor on average if using unsorted linked list nodes
  - If using another data structure for buckets (e.g. AVL tree) , runtime is proportional to runtime for that structure.
- Open addressing uses probing, has clustering issues as table fills
  Why use it:
  - Less memory allocation?
    - Some run-time overhead for allocating linked list (or whatever) nodes; open addressing could be faster
  - Easier data representation?
- Now:
  - Growing the table when it gets too full (aka "rehashing")

Rehashing

- As with array-based stacks/queues/lists, if table gets too full, create a bigger table and copy everything over
- With **separate chaining,** we get to decide what "too full" means
  - Keep load factor reasonable (e.g., < 1)?
  - Consider average or max size of non-empty chains?
- For **open addressing**, half-full is a good rule of thumb
- New table size
  - Twice-as-big is a good idea, except, uhm, that won't be prime!
  - So go *about* twice-as-big
  - Can have a list of prime numbers in your code since you probably won't grow more than 20-30 times, and then calculate after that

A Generally Good hashCode()


```
int result = 17; // start at a prime
foreach field f
    int fieldHashcode =
        boolean: (f ? 1: 0)
        byte, char, short, int: (int) f
        long: (int) (f ^ (f >>> 32))
        float: Float.floatToIntBits(f)
        double: Double.doubleToLongBits(f), then above
        Object: object.hashCode( )
        result = 31 * result + fieldHashcode;
return result;
```

Final word on hashing

- The hash table is one of the most important data structures
  - Efficient find, insert, and delete
  - Operations based on sorted order are not so efficient!
  - Useful in many, many real-world applications
  - Popular topic for job interview questions
- Important to use a good hash function
  - Good distribution, Uses enough of key's components
  - Not overly expensive to calculate (bit shifts good!)
- Important to keep hash table at a good size
  - Prime #
  - Preferable λ depends on type of table
- Side-comment: hash functions have uses beyond hash tables
  - Examples: Cryptography, check-sums