Parallel Prefix

CSE 332 – Section 8

Slides by James Richie Sulaeman

Parallel Prefix

Parallel Prefix

Parallel prefix is a type of programming problem where:

- A given array of elements needs to be **processed in parallel**
- Each element is **combined with its predecessors** through some operation
    - i.e. in parallel prefix sum, each element is summed with its predecessors

Allows for efficient computation of certain operations on large datasets since it enables multiple threads to work simultaneously on different portions of the data.

Problem 0

Parallel Prefix Sum

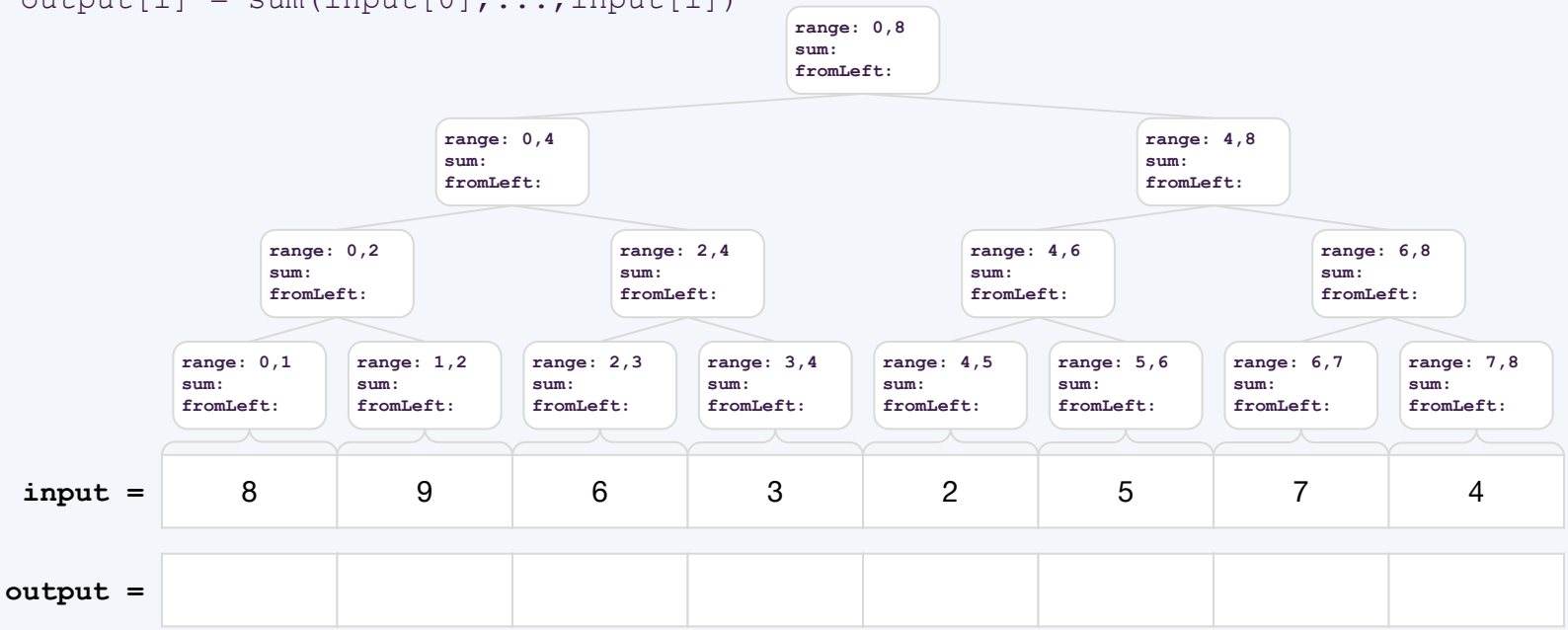
Problem 0

1. Divide problem into parallel pieces

   `cutoff = 1`

Given an array `input`, output an array such that

`output[i] = sum(input[0],...,input[i])`



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1. Divide problem into parallel pieces
   `cutoff = 1`
2. Find `sum` at cutoff
   `leaves[i].sum = input[i]`

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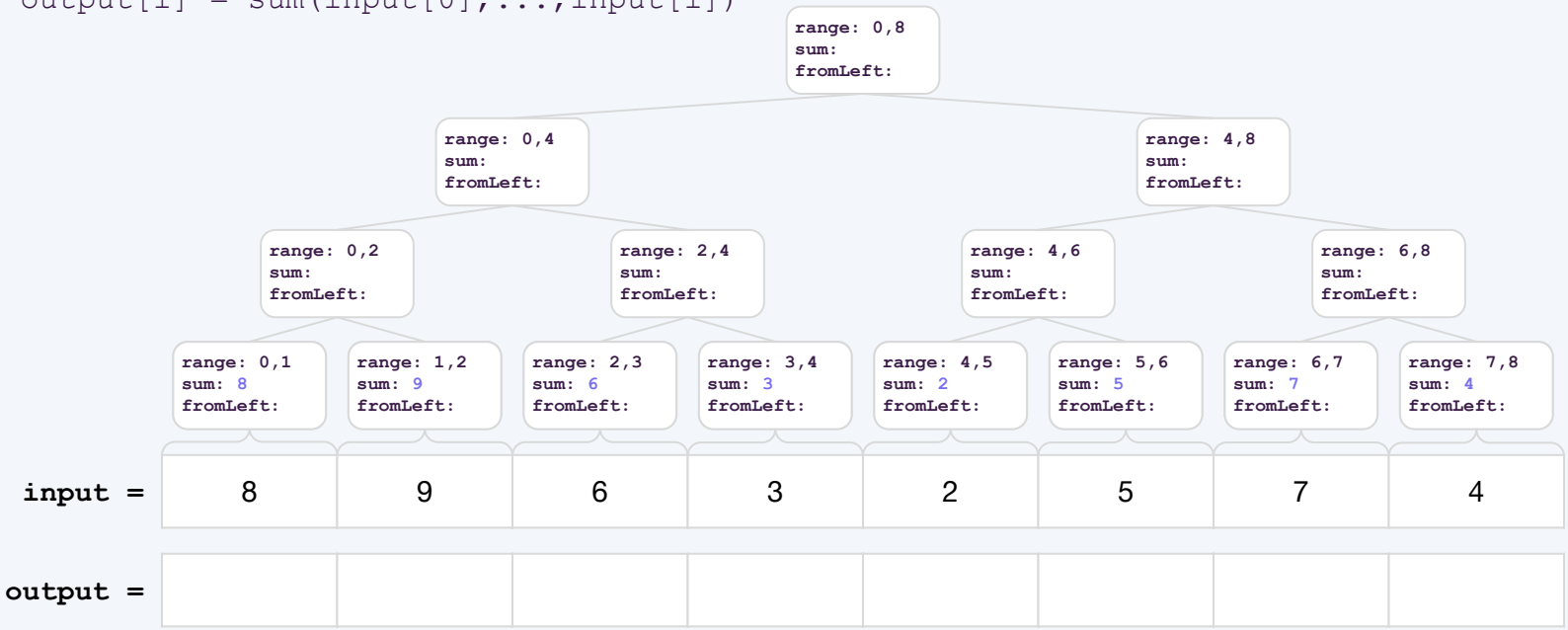

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   `cutoff = 1`
2. Find `sum` at cutoff
   `leaves[i].sum = input[i]`
3. Propagate `sum` up
   `parent.sum = left.sum + right.sum`

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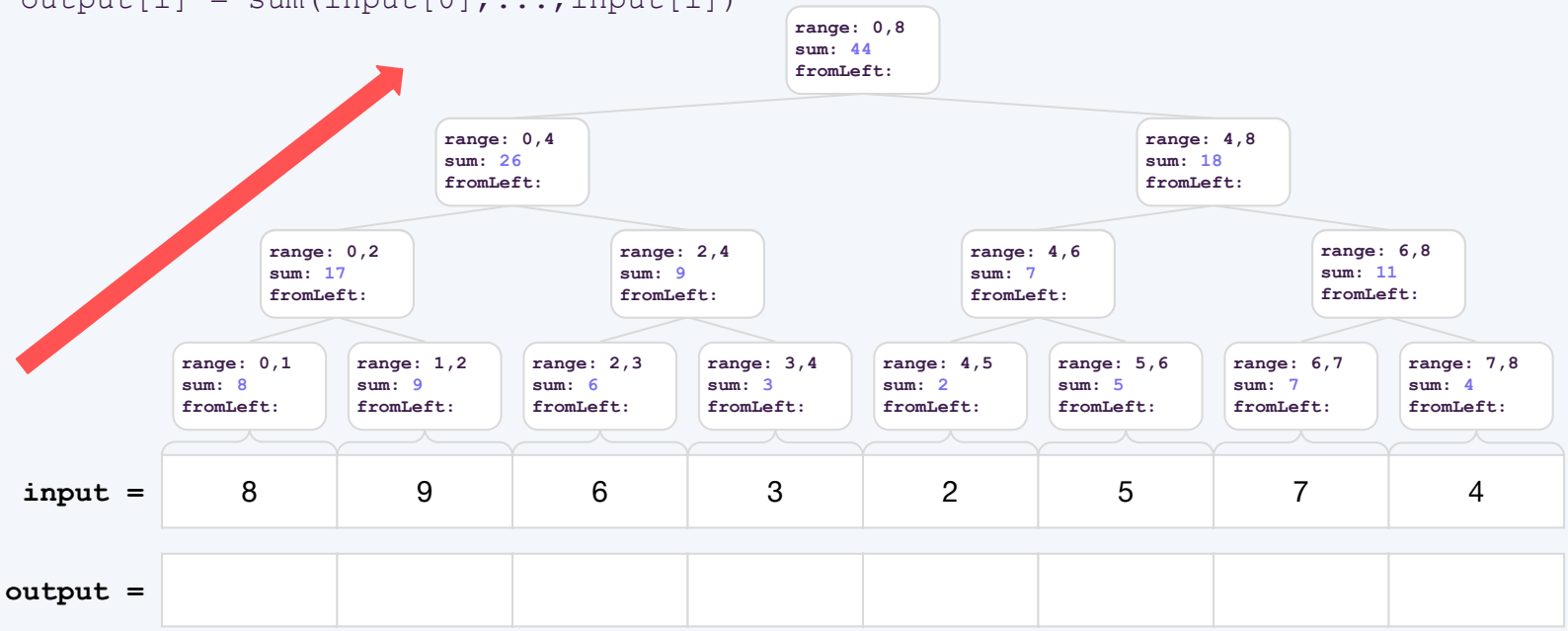

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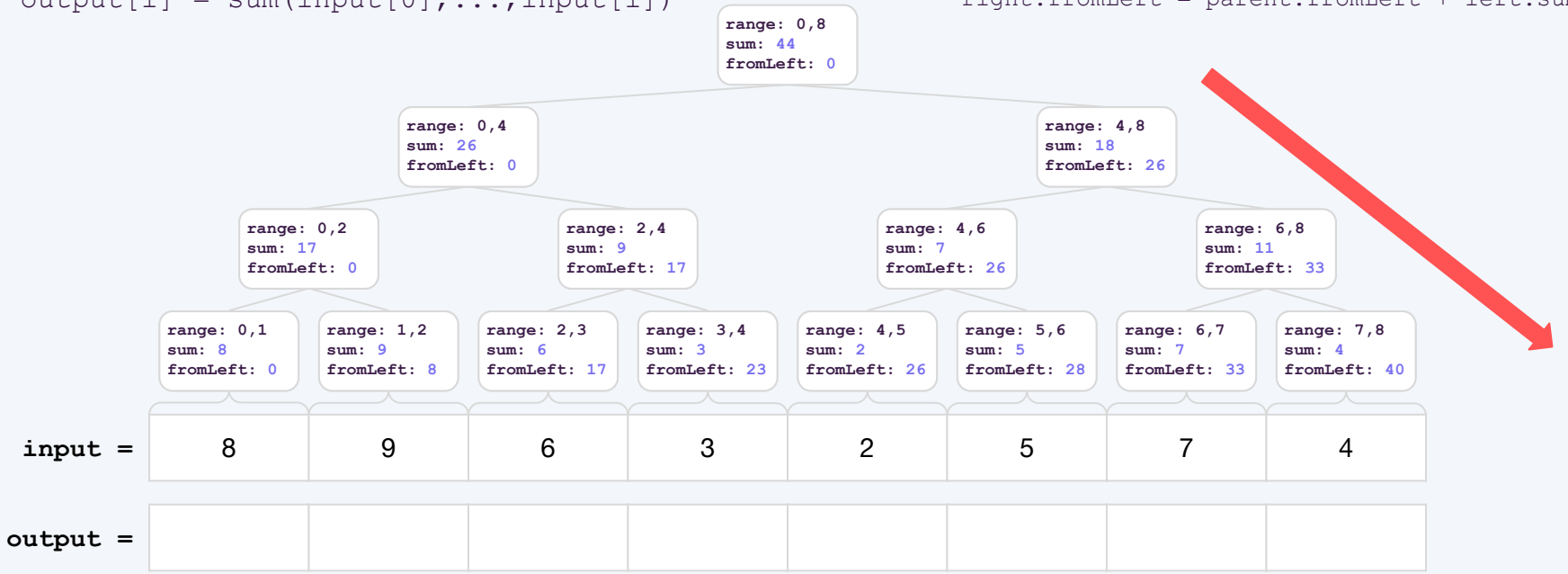

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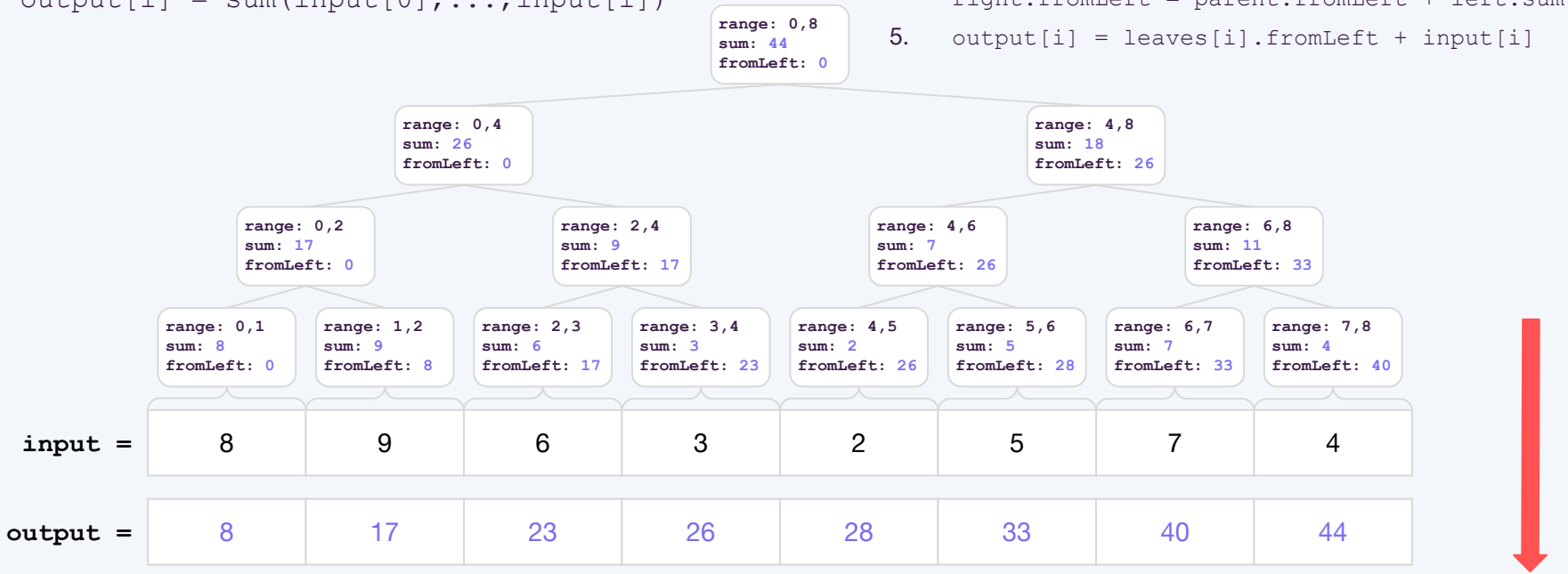
5. `output[i] = leaves[i].fromLeft + input[i]`



Problem 1

Parallel Prefix FindMin

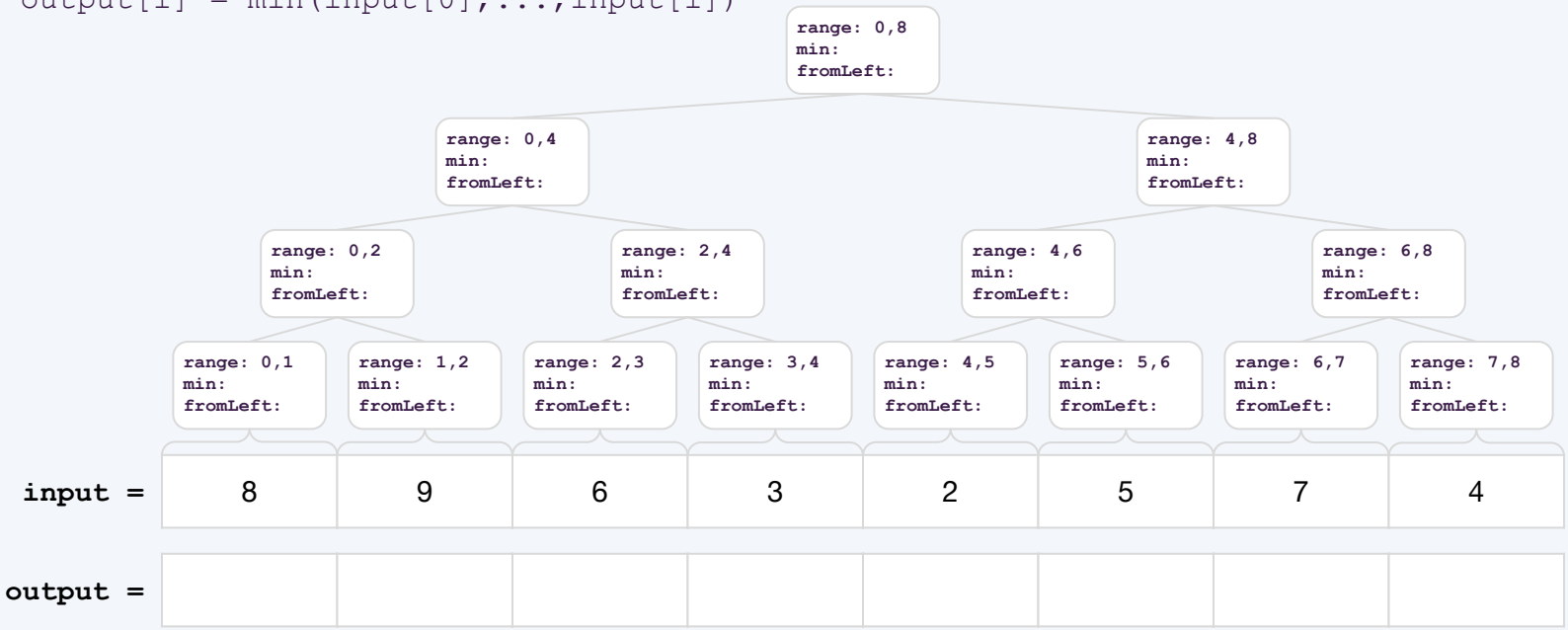
Problem 1

1. Divide problem into parallel pieces

   `cutoff = 1`

Given an array `input`, output an array such that

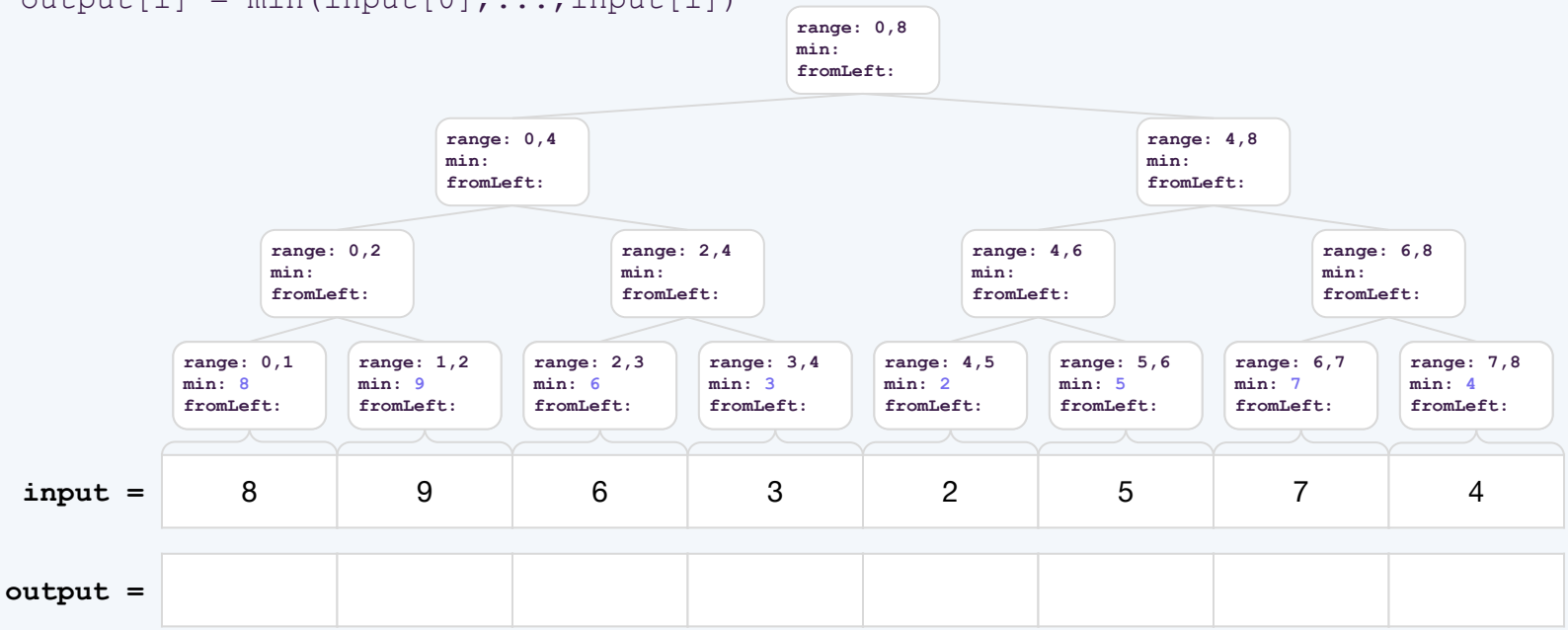
`output[i] = min(input[0],...,input[i])`



Problem 1

1. Divide problem into parallel pieces
   `cutoff = 1`
2. Find `min` at cutoff
   `leaves[i].min = input[i]`

Given an array `input`, output an array such that
`output[i] = min(input[0],...,input[i])`



Problem 1

1. Divide problem into parallel pieces
   `cutoff = 1`
2. Find `min` at cutoff
   `leaves[i].min = input[i]`
3. Propagate `min` up
   `parent.min = min(left.min, right.min)`

Given an array `input`, output an array such that
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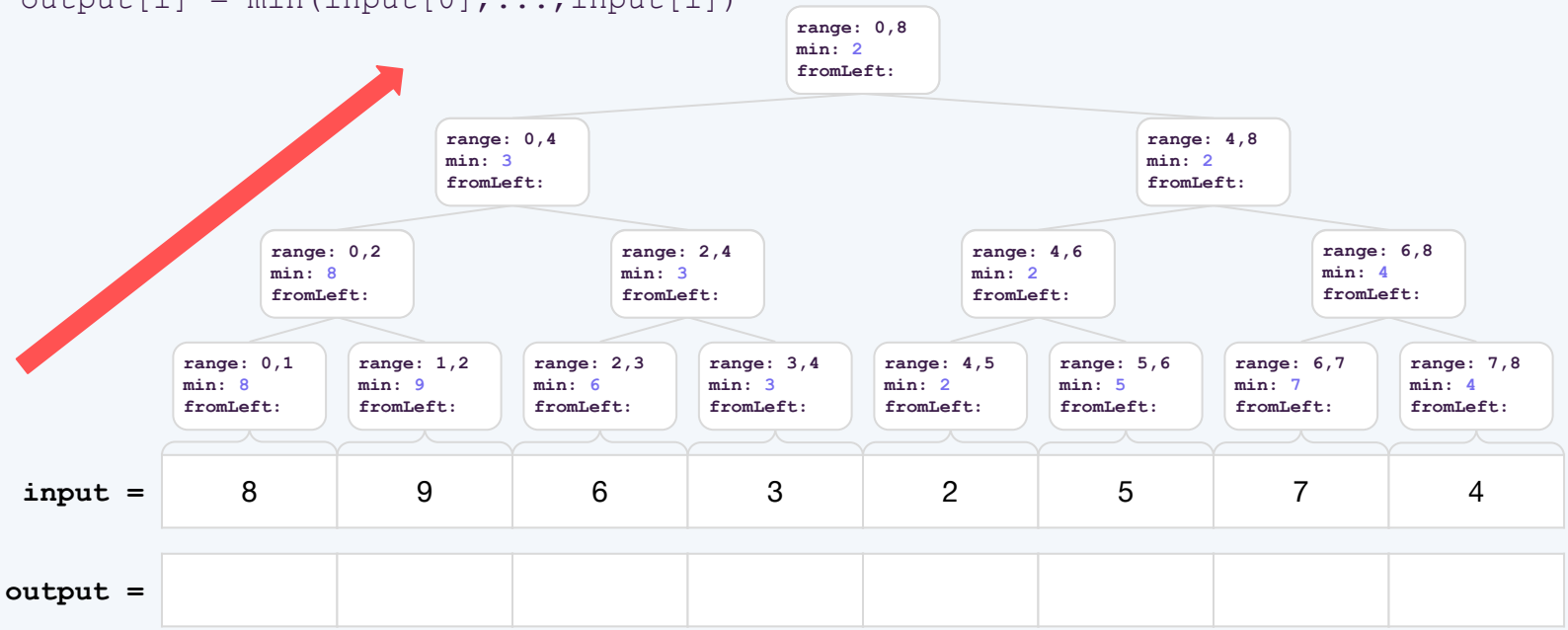

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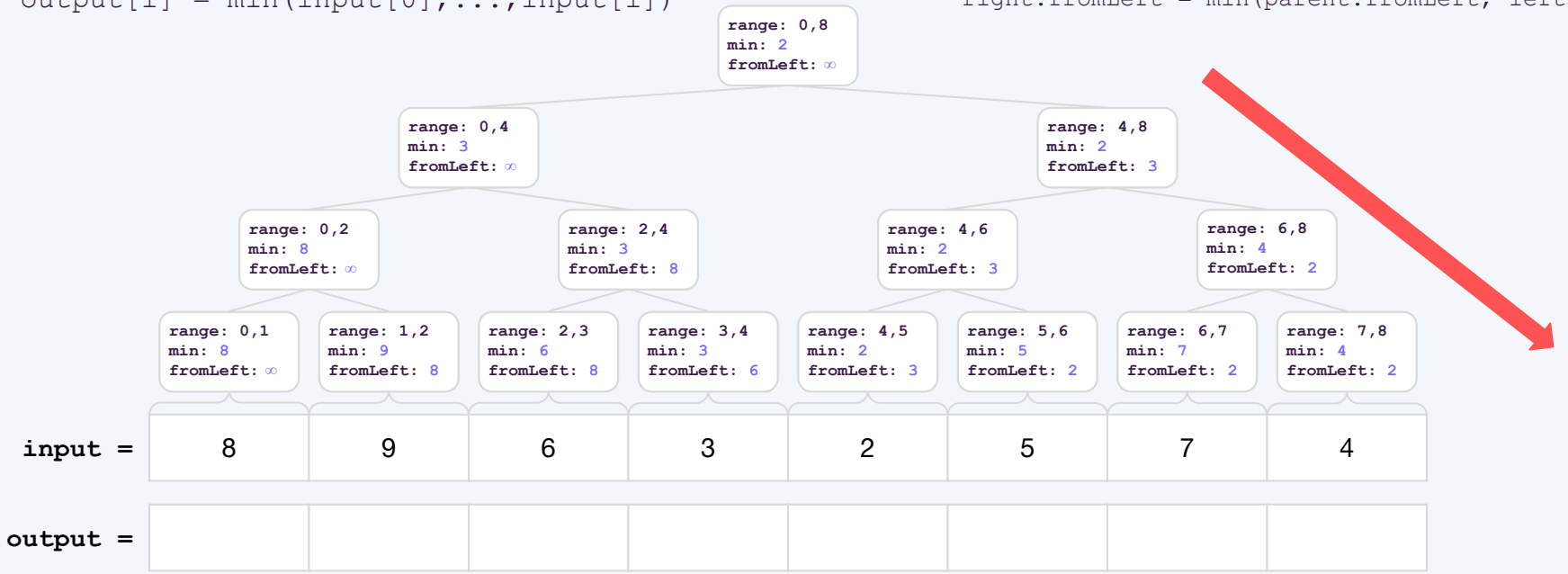

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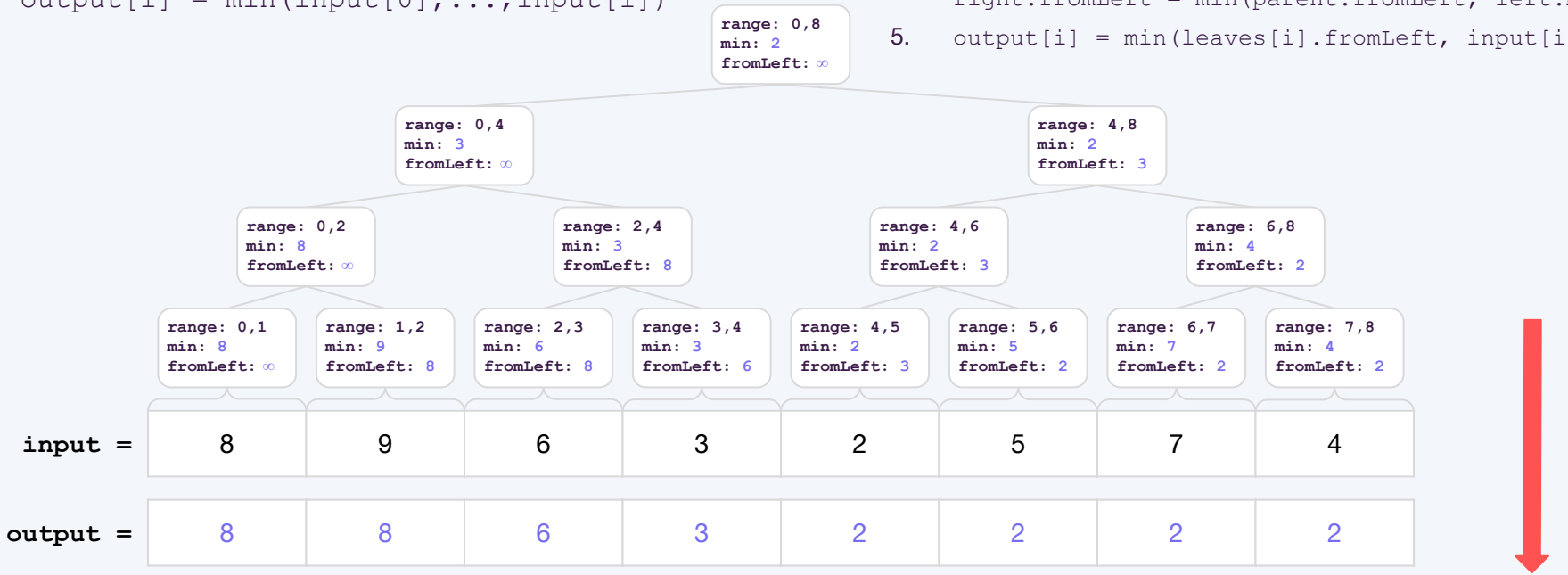
5. `output[i] = min(leaves[i].fromLeft, input[i])`



Problem 2

Parallel Pack

Problem 2 Overview

Given an array `input`, output an array that contains only the elements that are less than 10

1. **Parallel map** to compute `bits` array where `bits[i] = (input[i] < 10)`

input =	12	5	-8	34	6	10	2	7
bits =	0	1	1	0	1	0	1	1

2. **Parallel prefix sum** on `bits` array to compute `bitsum` array

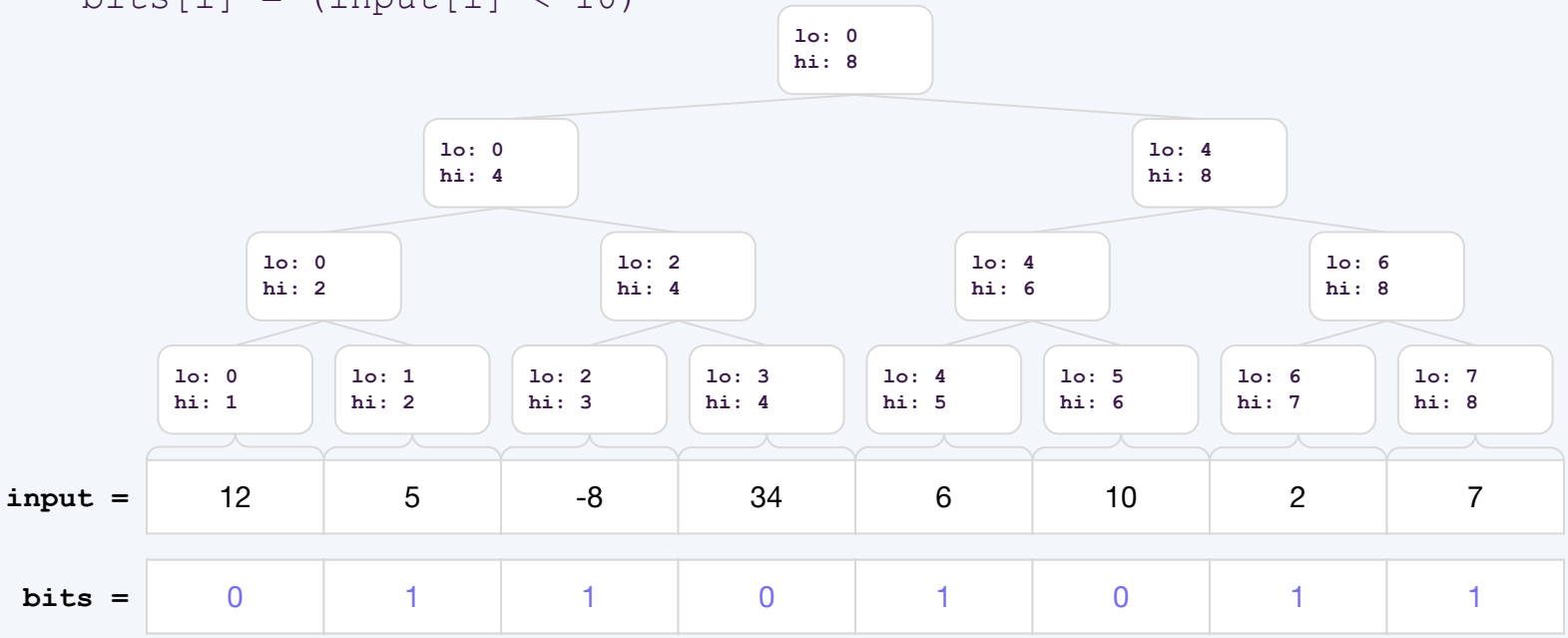
bitsum =	0	1	2	2	3	3	4	5

3. **Parallel map** to produce `output` array where `output[bitsum[i] - 1] = input[i]` if `bits[i] == 1`

output =	5	-8	6	2	7

Problem 2

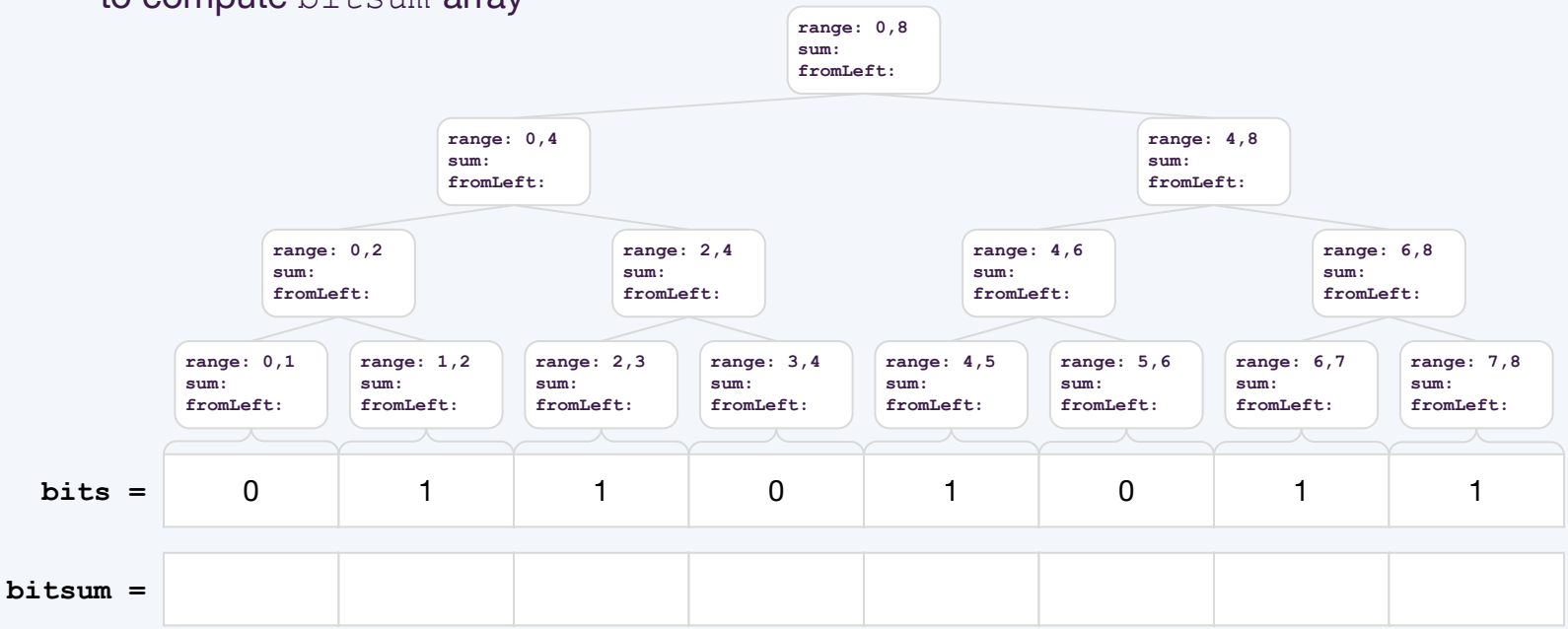
1. **Parallel map** to compute `bits` array where
   `bits[i] = (input[i] < 10)`



Problem 2

1. Divide problem into parallel pieces
   `cutoff = 1`

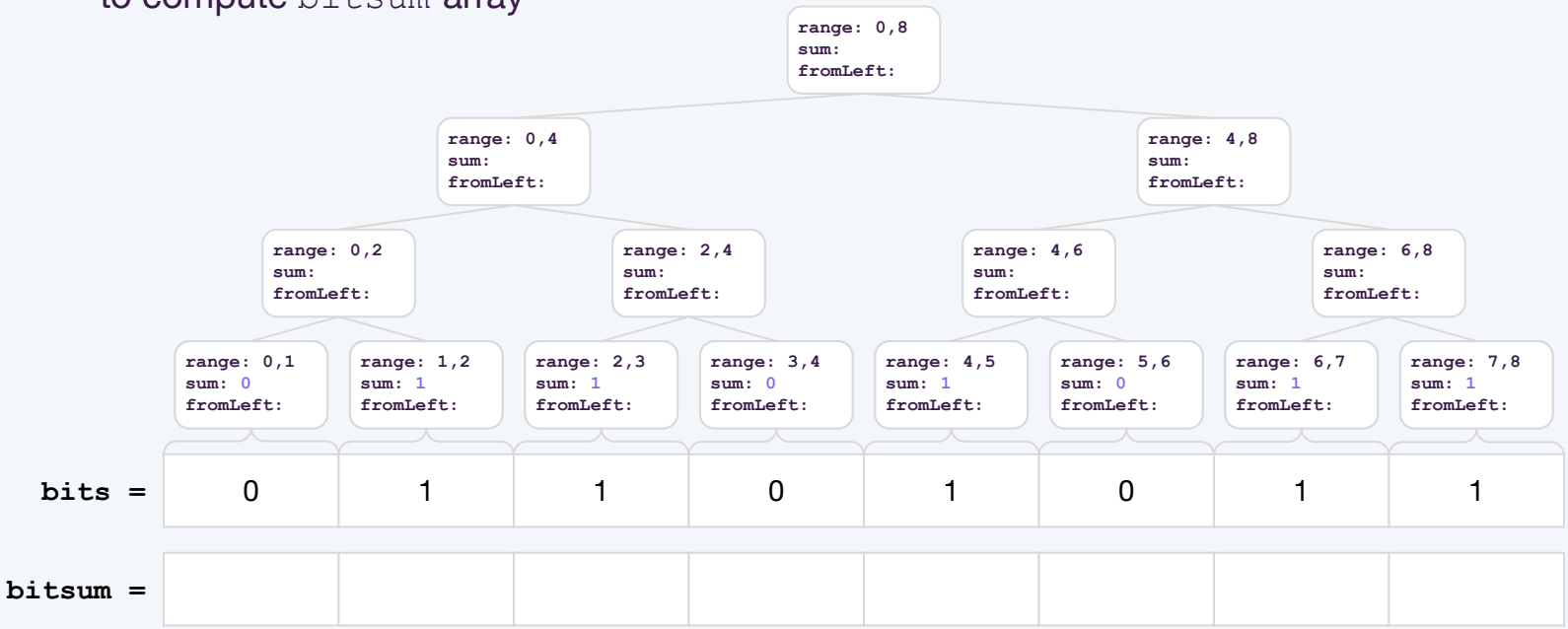
2. **Parallel prefix sum** on `bits` array to compute `bitsum` array



Problem 2

1. Divide problem into parallel pieces
   `cutoff = 1`
2. Find `sum` at cutoff
   `leaves[i].sum = bits[i]`

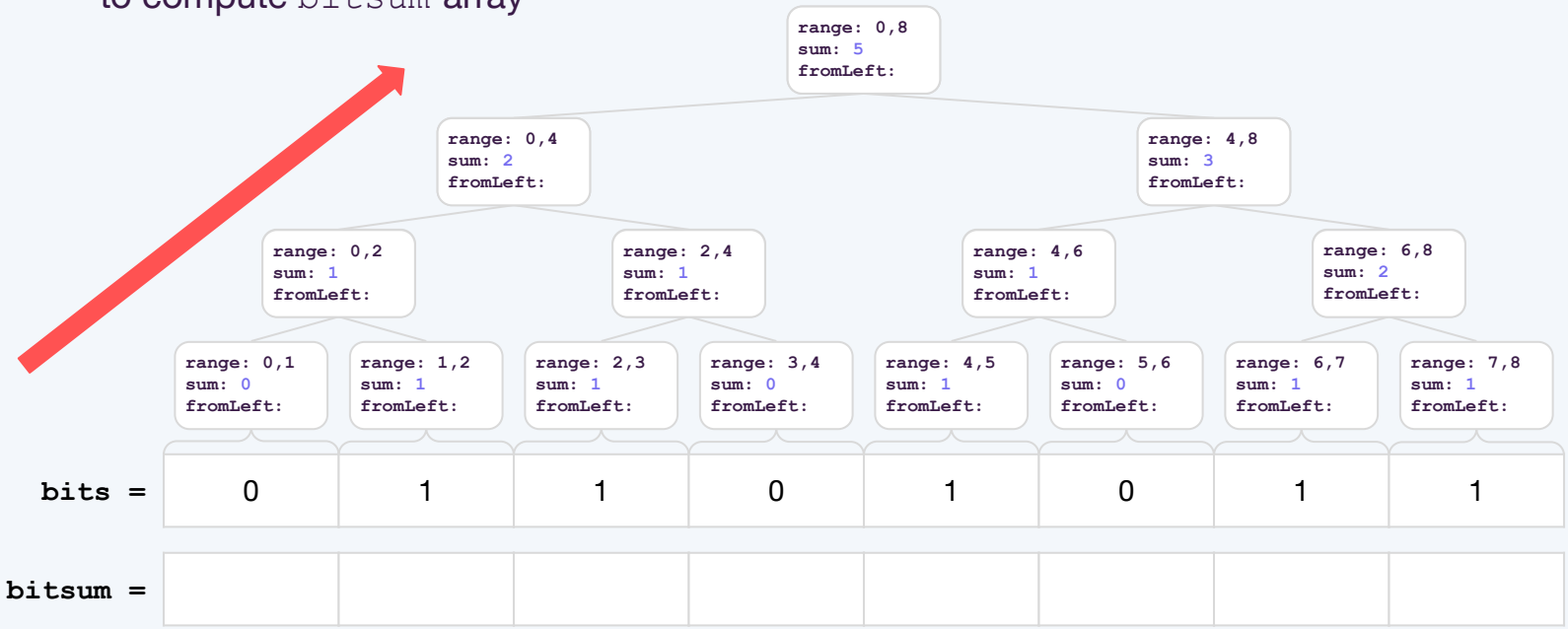
2. **Parallel prefix sum** on `bits` array to compute `bitsum` array



Problem 2

1. Divide problem into parallel pieces
   `cutoff = 1`
2. Find `sum` at cutoff
   `leaves[i].sum = bits[i]`
3. Propagate `sum` up
   `parent.sum = left.sum + right.sum`

2. **Parallel prefix sum** on `bits` array to compute `bitsum` array



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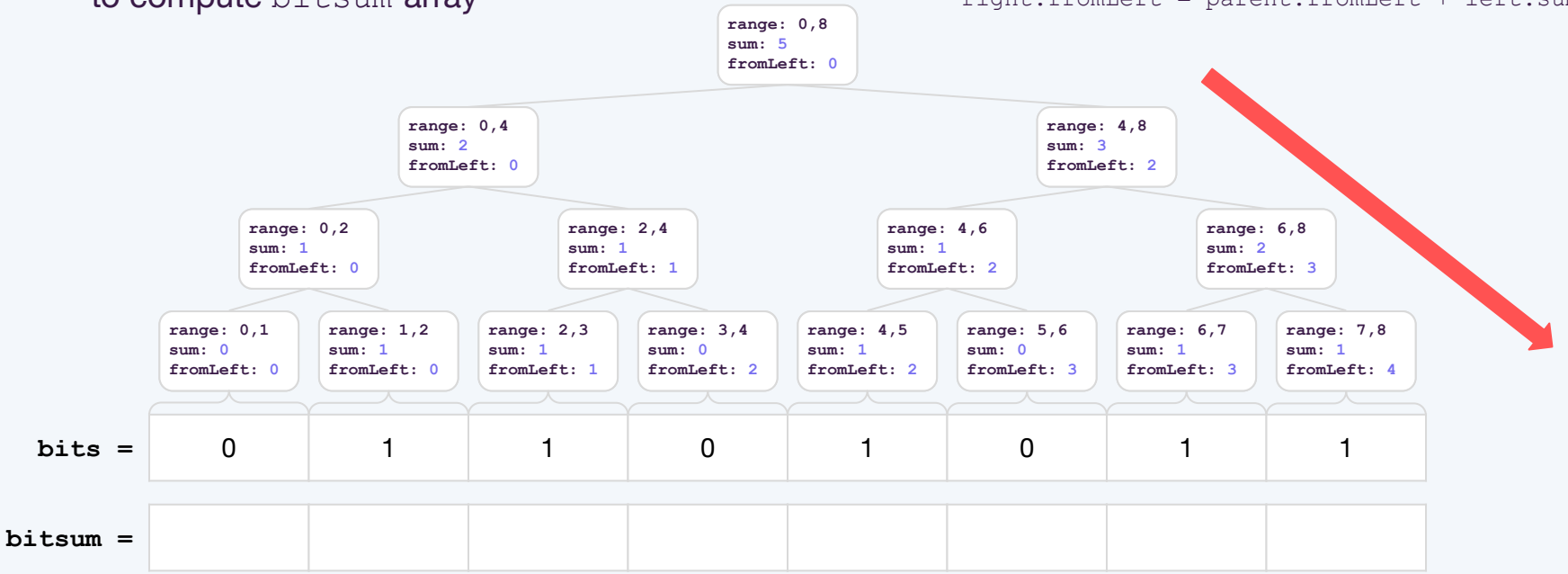

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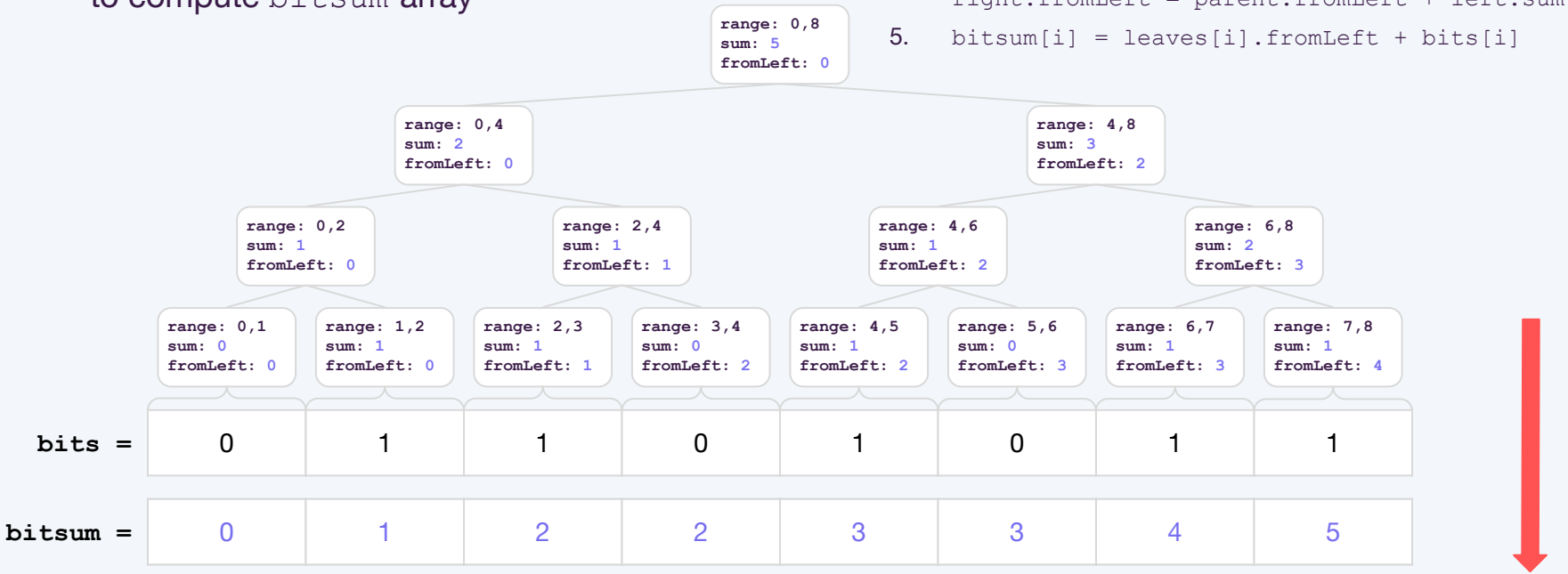
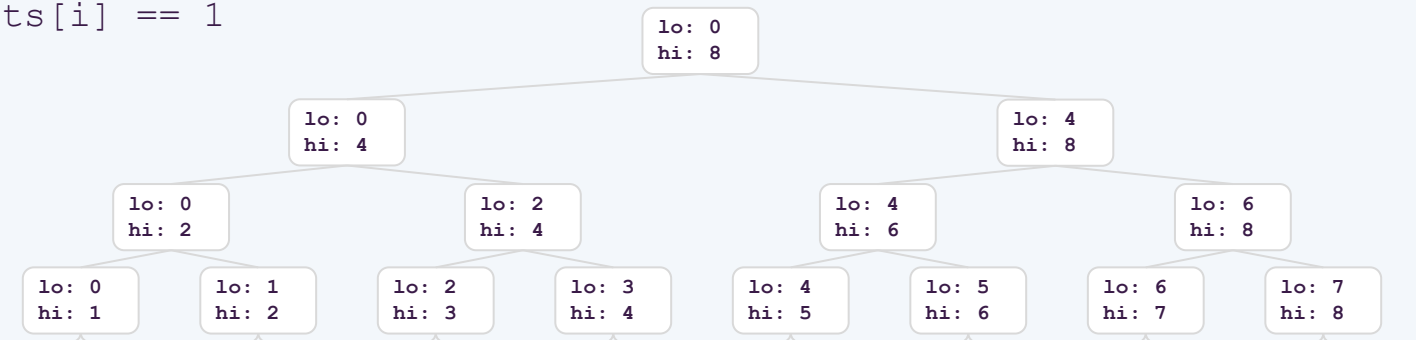


Problem 2

3. **Parallel map** to produce `output` array where `output[bitsum[i] - 1] = input[i]` if `bits[i] == 1`



input =	12	5	-8	34	6	10	2	7
bits =	0	1	1	0	1	0	1	1
bitsum =	0	1	2	2	3	3	4	5

output =	5	-8	6	2	7

Problem 3

Problem 3

a) Define work and span.

Work is the runtime of a program on **one** processor.

Span is the runtime of a program on **infinitely** many processors.

b) How do we calculate work and span?

Work is the sum of all the work done by all processors

Span is the sum of all the work done by the longest dependence chain / parallel ‘tree’ branch

c) Does adding more processors affect the work or span?

Neither. Work is defined for one processor and span is defined for infinitely many processors.

Adding more processors will not affect either work or span.

Problem 3

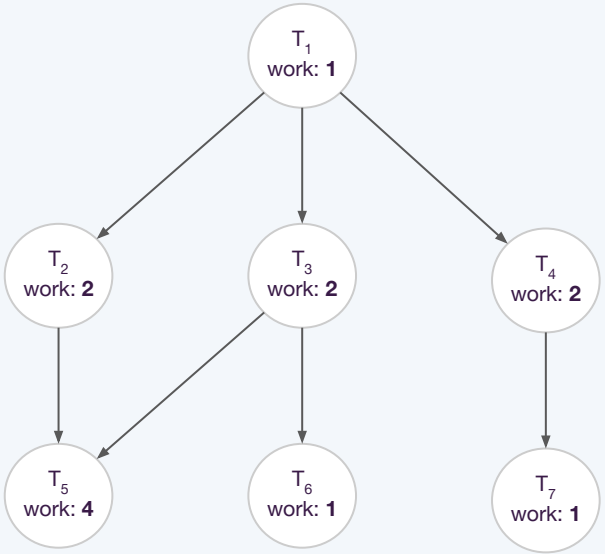
d) What is the work and span of the graph?

Work = 1 + 2 + 2 + 2 + 4 + 1 + 1

= 13

Span ($T_1 \rightarrow T_2 \rightarrow T_5$) = 1 + 2 + 4

= 7



Thank You!