

Section 10: P/NP

0. Definitions

a) What does P stand for?

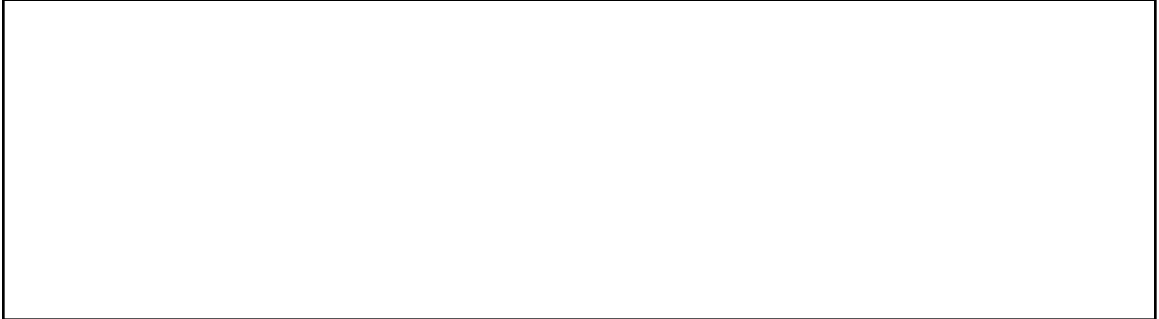
b) What is NP stand for?

c) What is the definition of P?

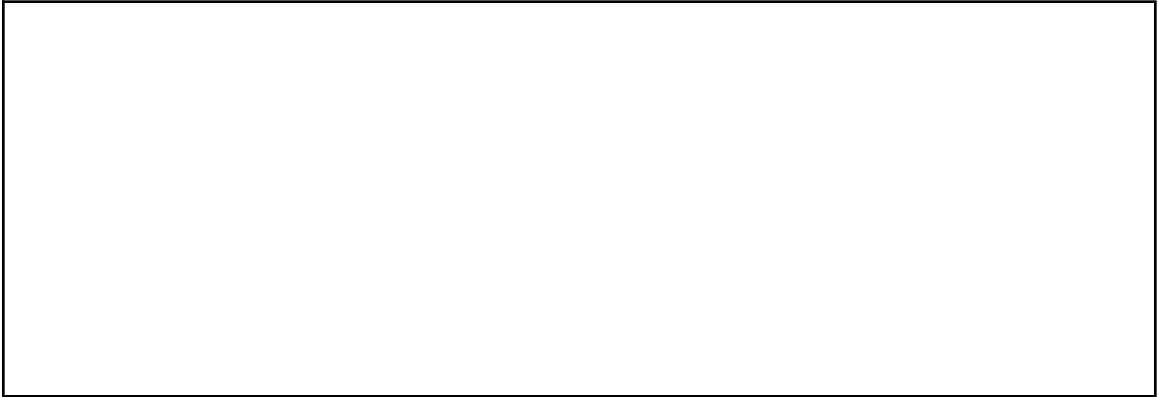
d) What is the definition of NP?

1. P & NP Membership

a) How would we show that a given problem belongs to the class P?

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b) How would we show that a given problem belongs to the class NP?

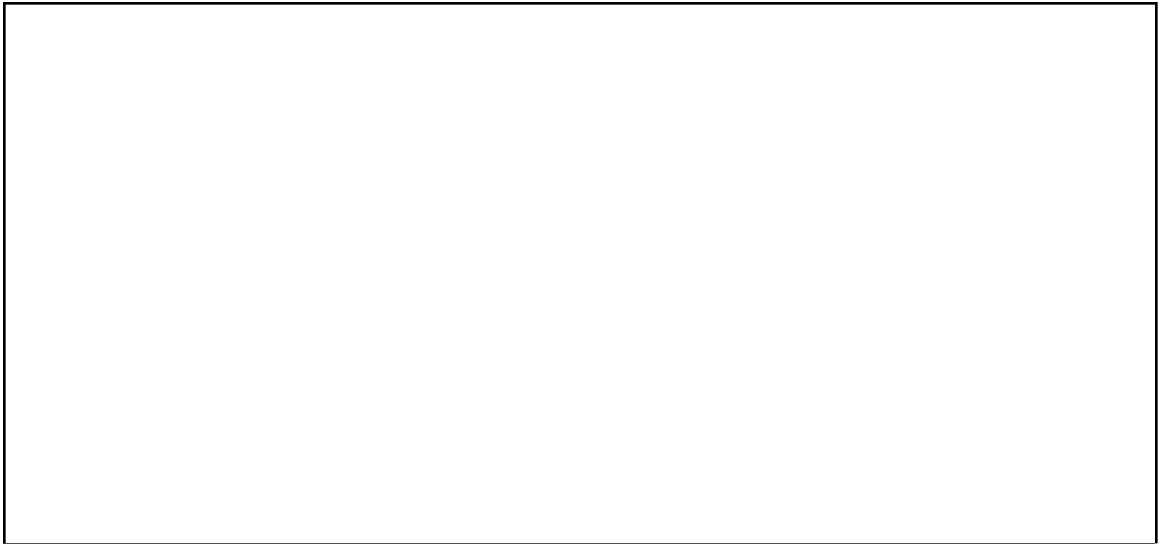
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2. P & NP Membership

For problems A and B below, show that they belong to both P and NP. Show that problem C belongs to NP.

- a) Problem A: Given a list of 2-dimensional points, return true or false to indicate whether some pair of points have a distance of less than 5.

Belongs to P



Belongs to NP



- b) Problem B: Given a list of integers, return true or false to indicate whether any items are duplicated (so return true if some value appears at least twice, false if all are distinct).

Belongs to P

Belongs to NP

- c) Problem C: Given a weighted graph, a pair of nodes X and Y , and a number k , return true or false to indicate whether there is a path from X to Y with a cost of at least k

Belongs to NP



3. NP-Hard and NP-Complete Definitions

a) What is the definition of NP-Hard?

b) What is the definition of NP-Complete?

4. NP-Hard and NP-Complete Membership

a) How do you show that a problem belongs to NP-Hard

b) How do you show that a problem belongs to NP-Complete

5. Practice

If A polynomial-time reduces to B and B is NP-Hard then A is NP-Hard.

True False

If B is NP-Hard and there exists a polynomial time algorithm for B, then $P=NP$.

True False

If B is NP-Hard and there does not exist a polynomial time algorithm for B, then P does not equal NP.

True False

If A reduces to B in polynomial time, and B reduces to C in polynomial time, and A is NP-Hard, then C is NP-Hard.

True False

If A reduces to B in polynomial time, and B reduces to C in polynomial time, and A is NP-Complete, then C is NP-Complete.

True False

If A reduces to B in polynomial time, and B reduces to C in polynomial time, and A is in EXP, then C is EXP.

True False

If A reduces to B in polynomial time, and B reduces to C in polynomial time, and A is in P, then C is P.

True False

If A reduces to B in polynomial time, and B reduces to C in polynomial time, and A is in NP, and C is in P, then $P=NP$.

True False

If A reduces to B in polynomial time, and B reduces to C in polynomial time, and A is in NP-Hard, and C is in P, then $P=NP$.

True False