Hashing

CSE 332 – Section 5

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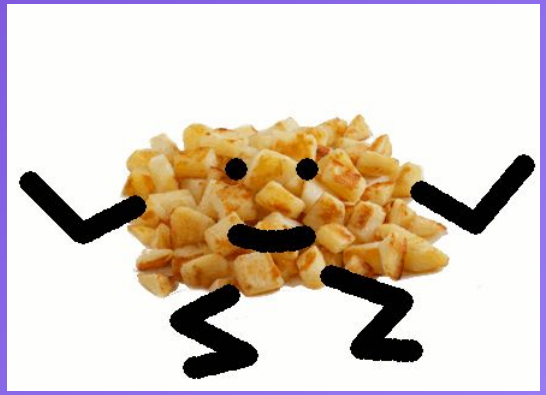


Announcements

- Midterm Next Week! Wednesday April 30th, 6:00-7:20 PM in BAG 154 and 131 (we will let you know which room to go to)
  - Covers everything up to and including Hash Tables
  - Check the bottom of the *Exams* section of the course website for past exams
  - Come to office hours if you have questions about **anything!**

Hashing

Collision Resolution

A collision occurs when two keys map onto the same location in a hash table.

- This is impossible to eliminate since the number of possible keys exceeds table size.

There are multiple ways to resolve conflicts:

- Separate Chaining
  - *All elements with keys that map to the same table location are kept in a linked list.*
- Open Addressing
  - *If the slot is full, we **probe** the next slot.*
  - *On the i^{th} probe, we check the slot with index* $(h(\text{key}) + f(i)) \,\%\, \text{TableSize}$.
    - Linear Probing: $f(i) = i$
    - Quadrating Probing: $f(i) = i^2$
    - Double Hashing: $f(i) = i \cdot g(\text{key})$

Problem 1a

Linear Probing

i^{th} probe: $(h(\text{key}) + i)\ \%\ \text{TableSize}$

Problem 1a

Insert 7, 9, 48, 8, 37, 57 into an empty table.

Linear Probing

i^{th} probe: $(h(\text{key}) + i)$ % TableSize

0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Problem 1a

Insert 7, 9, 48, 8, 37, 57 into an empty table.

- $(h(7) + 0) \% 10 = 7$

Linear Probing

i^{th} probe: $(h(\text{key}) + i) \% \text{TableSize}$

0	
1	
2	
3	
4	
5	
6	
7	7
8	
9	

Problem 1a

Insert 7, **9**, 48, 8, 37, 57 into an empty table.

- $(h(9) + 0)\ \%\ 10 = 9$

Linear Probing

i^{th} probe: $(h(\text{key}) + i)\ \%\ \text{TableSize}$

0	
1	
2	
3	
4	
5	
6	
7	7
8	
9	9

Problem 1a

Insert 7, 9, **48**, 8, 37, 57 into an empty table.

- $(h(48) + 0) \% 10 = 8$

Linear Probing

i^{th} probe: $(h(\text{key}) + i) \% \text{TableSize}$

0	
1	
2	
3	
4	
5	
6	
7	7
8	48
9	9

Problem 1a

Insert 7, 9, 48, **8**, 37, 57 into an empty table.

- $(h(8) + 0)\ \%\ 10 = 8$
- $(h(8) + 1)\ \%\ 10 = 9$
- $(h(8) + 2)\ \%\ 10 = 0$

Linear Probing

i^{th} probe: $(h(\text{key}) + i)\ \%\ \text{TableSize}$

0	8
1	
2	
3	
4	
5	
6	
7	7
8	48
9	9

Problem 1a

Insert 7, 9, 48, 8, **37**, 57 into an empty table.

- $(h(37) + 0)\ \%\ 10 = 7$
- $(h(37) + 1)\ \%\ 10 = 8$
- $(h(37) + 2)\ \%\ 10 = 9$
- $(h(37) + 3)\ \%\ 10 = 0$
- $(h(37) + 4)\ \%\ 10 = 1$

Linear Probing

i^{th} probe: $(h(\text{key}) + i)\ \%\ \text{TableSize}$

0	8
1	37
2	
3	
4	
5	
6	
7	7
8	48
9	9

Problem 1a

Insert 7, 9, 48, 8, 37, **57** into an empty table.

- $(h(57) + 0) \% 10 = 7$
- $(h(57) + 1) \% 10 = 8$
- $(h(57) + 2) \% 10 = 9$
- $(h(57) + 3) \% 10 = 0$
- $(h(57) + 4) \% 10 = 1$
- $(h(57) + 5) \% 10 = 2$

Linear Probing

i^{th} probe: $(h(\text{key}) + i) \% \text{TableSize}$

0	8
1	37
2	57
3	
4	
5	
6	
7	7
8	48
9	9

Problem 1a

Delete 37, 7, 57 from the table.

Linear Probing

i^{th} probe: $(h(\text{key}) + i)$ % TableSize

0	8
1	37
2	57
3	
4	
5	
6	
7	7
8	48
9	9

Problem 1a

Delete **37**, 7, 57 from the table.

- $(h(37) + 0) \% 10 = 7$
- $(h(37) + 1) \% 10 = 8$
- $(h(37) + 2) \% 10 = 9$
- $(h(37) + 3) \% 10 = 0$
- $(h(37) + 4) \% 10 = 1$

Linear Probing

i^{th} probe: $(h(\text{key}) + i) \% \text{TableSize}$

0	8
1	DELETED
2	57
3	
4	
5	
6	
7	7
8	48
9	9

Problem 1a

Delete 37, **7**, 57 from the table.

- $(h(7) + 0)\ \%\ 10 = 7$

Linear Probing

i^{th} probe: $(h(\text{key}) + i)\ \%$ TableSize

0	8
1	DELETED
2	57
3	
4	
5	
6	
7	DELETED
8	48
9	9

Problem 1a

Delete 37, 7, **57** from the table.

- $(h(57) + 0)\ \%\ 10 = 7$
- $(h(57) + 1)\ \%\ 10 = 8$
- $(h(57) + 2)\ \%\ 10 = 9$
- $(h(57) + 3)\ \%\ 10 = 0$
- $(h(57) + 4)\ \%\ 10 = 1$
- $(h(57) + 5)\ \%\ 10 = 2$

Linear Probing

i^{th} probe: $(h(\text{key}) + i)\ \%\ \text{TableSize}$

0	8
1	DELETED
2	DELETED
3	
4	
5	
6	
7	DELETED
8	48
9	9

Experiment

What happens if we now try to remove a non-existent element (e.g. 17) from the table?

Experiment

Delete 17 from the table.

- $(h(17) + 0) \% 10 = 7$
- $(h(17) + 1) \% 10 = 8$
- $(h(17) + 2) \% 10 = 9$
- $(h(17) + 3) \% 10 = 0$
- $(h(17) + 4) \% 10 = 1$
- $(h(17) + 5) \% 10 = 2$
- $(h(17) + 6) \% 10 = 3$

We have reached an empty slot, but have not encountered 17. Therefore, it must not exist.

Linear Probing

i^{th} probe: $(h(\text{key}) + i) \% \text{TableSize}$

0	8
1	DELETED
2	DELETED
3	
4	
5	
6	
7	DELETED
8	48
9	9

Problem 1b

Quadratic Probing

i^{th} probe: $(h(\text{key}) + i^2)$ % TableSize

Problem 1b

Insert 7, 9, 48, 8, 37, 57 into an empty table.

Quadratic Probing

i^{th} probe: $(h(\text{key}) + i^2)$ % TableSize

0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Problem 1b

Insert 7, 9, 48, 8, 37, 57 into an empty table.

- $(h(7) + 0^2)\ \%\ 10 = 7$

Quadratic Probing

i^{th} probe: $(h(\text{key}) + i^2)\ \%\ \text{TableSize}$

0	
1	
2	
3	
4	
5	
6	
7	7
8	
9	

Problem 1b

Insert 7, 9, 48, 8, 37, 57 into an empty table.

- $(h(9) + 0^2)\ \%\ 10 = 9$

Quadratic Probing

i^{th} probe: $(h(\text{key}) + i^2)\ \%\ \text{TableSize}$

0	
1	
2	
3	
4	
5	
6	
7	7
8	
9	9

Problem 1b

Insert 7, 9, **48**, 8, 37, 57 into an empty table.

- $(h(48) + 0^2)\ \%\ 10 = 8$

Quadratic Probing

i^{th} probe: $(h(\text{key}) + i^2)\ \%\ \text{TableSize}$

0	
1	
2	
3	
4	
5	
6	
7	7
8	48
9	9

Problem 1b

Insert 7, 9, 48, **8**, 37, 57 into an empty table.

- $(h(8) + 0^2)\ \%\ 10 = 8$
- $(h(8) + 1^2)\ \%\ 10 = 9$
- $(h(8) + 2^2)\ \%\ 10 = 2$

Quadratic Probing

i^{th} probe: $(h(\text{key}) + i^2)\ \%\ \text{TableSize}$

0	
1	
2	8
3	
4	
5	
6	
7	7
8	48
9	9

Problem 1b

Insert 7, 9, 48, 8, **37**, 57 into an empty table.

- $(h(37) + 0^2)\ \%\ 10 = 7$
- $(h(37) + 1^2)\ \%\ 10 = 8$
- $(h(37) + 2^2)\ \%\ 10 = 1$

Quadratic Probing

i^{th} probe: $(h(\text{key}) + i^2)\ \%\ \text{TableSize}$

0	
1	37
2	8
3	
4	
5	
6	
7	7
8	48
9	9

Problem 1b

Insert 7, 9, 48, 8, 37, **57** into an empty table.

- $(h(57) + 0^2)\ \%\ 10 = 7$
- $(h(57) + 1^2)\ \%\ 10 = 8$
- $(h(57) + 2^2)\ \%\ 10 = 1$
- $(h(57) + 3^2)\ \%\ 10 = 6$

Quadratic Probing

i^{th} probe: $(h(\text{key}) + i^2)\ \%\ \text{TableSize}$

0	
1	37
2	8
3	
4	
5	
6	57
7	7
8	48
9	9

Problem 1c

Separate Chaining

Use a linked list for each slot

Problem 1c

Insert 7, 9, 48, 8, 37, 57 into an empty table.

Separate Chaining

Use a linked list for each slot

0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Problem 1c

Insert 7, 9, 48, 8, 37, 57 into an empty table.

- $h(7) \% 10 = 7$

Separate Chaining

Use a linked list for each slot

0	
1	
2	
3	
4	
5	
6	
7	7
8	
9	

Problem 1c

Insert 7, **9**, 48, 8, 37, 57 into an empty table.

- $h(9) \,\%\, 10 = 9$

Separate Chaining

Use a linked list for each slot

0	
1	
2	
3	
4	
5	
6	
7	7
8	
9	9

Problem 1c

Insert 7, 9, **48**, 8, 37, 57 into an empty table.

- $h(48) \% 10 = 8$

Separate Chaining

Use a linked list for each slot

0	
1	
2	
3	
4	
5	
6	
7	7
8	48
9	9

Problem 1c

Insert 7, 9, 48, **8**, 37, 57 into an empty table.

- $h(8) \% 10 = 8$

Separate Chaining

Use a linked list for each slot

Problem 1c

Insert 7, 9, 48, 8, **37**, 57 into an empty table.

- $h(37) \% 10 = 7$

Separate Chaining

Use a linked list for each slot

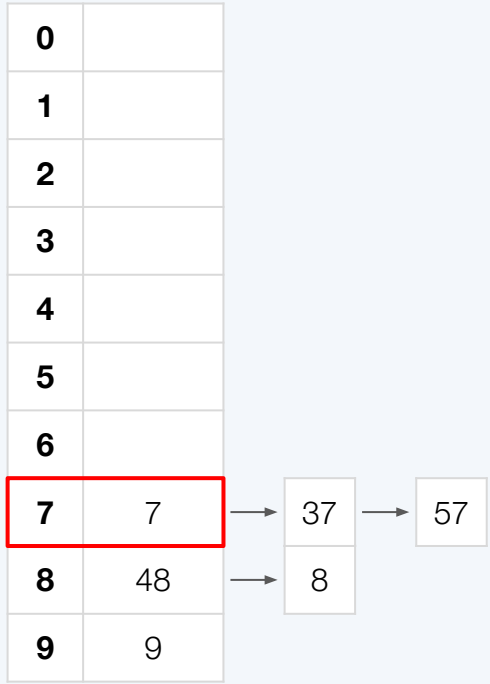
Problem 1c

Insert 7, 9, 48, 8, 37, **57** into an empty table.

- $h(57) \% 10 = 7$

Separate Chaining

Use a linked list for each slot



Problem 2

Problem 2a

Describe double hashing.

- On the i^{th} probe, we check the slot with index $(h(\text{key}) + i \cdot g(\text{key}))$ % TableSize.
- The first hash function h determines the location where we initially try to place the item.
- If there is a collision, then the second hash function g determines the probing step size (i.e. $1 \cdot g(\text{key})$, $2 \cdot g(\text{key})$, ... distance away from the initial location).

Problem 2b

List two disadvantages of quadratic probing.

1. If the table is more than half full (i.e. load factor > 0.5), then we are not guaranteed to find a location to insert an item.
2. Suffers from secondary clustering since items that initially hash to the same location resolve the collision identically.

Describe how double hashing fixes one of these disadvantages.

A good second hash function prevents secondary clustering since items that initially hash to the same location will likely resolve the collision differently.

- Items that have the same value for the first hash function f, will likely have different values for the second hash function g, leading to different probing step sizes.

Problem 2c

Compare open addressing with separate chaining.

Open Addressing	Separate Chaining
Handles collisions by searching for an open slot within the table itself.	Handles collisions by adding elements to a chain at the corresponding index.
Can use less memory since all elements are stored within the table itself.	Uses more memory since additional data structures are needed. Worse memory locality.
Linear probing suffers from primary clustering, but is guaranteed to find an open slot.	Average runtime: $\mathcal{O}(1 + \lambda)$
Quadratic probing suffers from secondary clustering, and is only guaranteed to find an empty slot when $\lambda < 0.5$.	Best-case runtime: $\mathcal{O}(1)$
Double hashing does not suffer from clustering, but requires an additional hash function (computationally expensive).	Worst-case runtime: $\mathcal{O}(n)$

Thank You!