CSE 332 : 21AU Final

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Instructions

- The allotted time is **110** minutes. Please do not turn the page until the staff says so.
- This is a closed-book and closed-notes exam. You are not permitted to access electronic devices.
- Read directions carefully, especially for problems that require you to show work or provide an explanation.
- We can only give partial credit for work that you've written down.
- Unless otherwise noted, every time we ask for an O, Ω, or Θ bound, it must be simplified and tight.
- For answers that involve bubbling ◯ or ☐, make sure to fill in the shape completely: ● or ■.
- If you run out of room on a page, indicate that the answer continues on the back of that page. Try to avoid writing on the very edges of the pages: we scan your exams and edges often get cropped off.
- Make sure you also get a copy of the formula sheet.

Advice

- If you feel like you're stuck on a problem, you may want to skip it and come back at the end if you have time.
- Look at the question titles on the cover page to see if you want to start somewhere other than problem 1.
- The TAs agree that problem 8 is the hardest. We intentionally put it at the end for that reason.
- Remember to take deep breaths.
- Every tree is a forest. You can do this.

Question	**Max points**
1. First-Half Facts	16
2. Graphs (Short Answers)	14
3. Sorting (Short Answers)	15
4. Concurrency	15
5. Finish the ForkJoin	13
6. Graph Modeling	10
7. P/NP	8
8. Better Parallel Quick Sort	9
9. Drawing	1
Total	**101**

1. First-Half Facts [16 points]

(a) Suppose you have a B-tree with parameters $M = 4$ and $L = 6$. What is the **BEST CASE** number of disk accesses needed to perform a `find` in the B-tree if its height is h. Give an exact formula (not a big-O description). (a) _____

(b) Given a min-heap in array form, what is the tightest **WORST CASE** runtime to transform the min-heap into a max-heap? (b) _____

(c) Consider a hashtable that uses quadratic probing as its collision resolution strategy and currently has a load factor of $99/100$. What is the **WORST CASE** runtime of an `insert` into this hashtable? (c) _____

(d) Consider a hashtable that uses separate chaining as its collision resolution strategy and has a maximum load factor of 200. What is the **AVERAGE CASE** runtime of an insertion into this hashtable? (d) _____

(e) Suppose you have an AVL tree where for every non-leaf node, the node's right subtree has a height one more than its left subtree. You `insert` a new key into this tree that is larger than all others. Give a tight, simplified big-O bound on the number of rotations required for this insertion. (e) _____

(f) What is the **BEST CASE** runtime for an insertion into a binary tree, where the key to be inserted is new? (f) _____

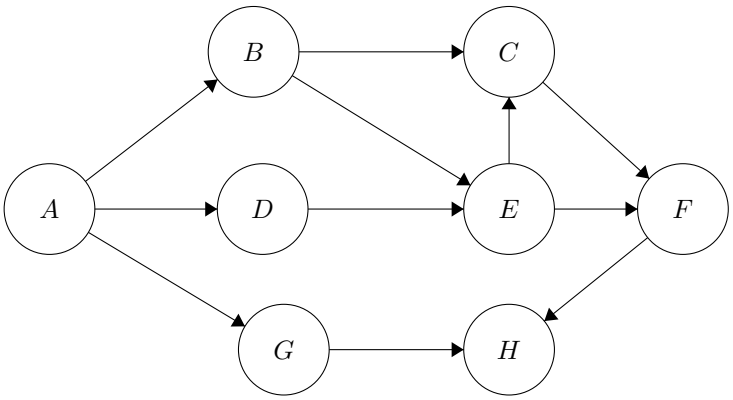
(g) What is the **BEST CASE** runtime for an insertion into an AVL tree, where the key to be inserted is new? (g) _____

(h) You are using a hash table with n key-value pairs. Your table uses separate chaining for collision resolution, and as the load factor has just reached 2, you need to resize. An array of appropriate size has already been allocated. What is a tight big-O for the **WORST CASE** time of the remaining resizing operations? (h) _____

2. Graphs (Short Answers) [14 points]

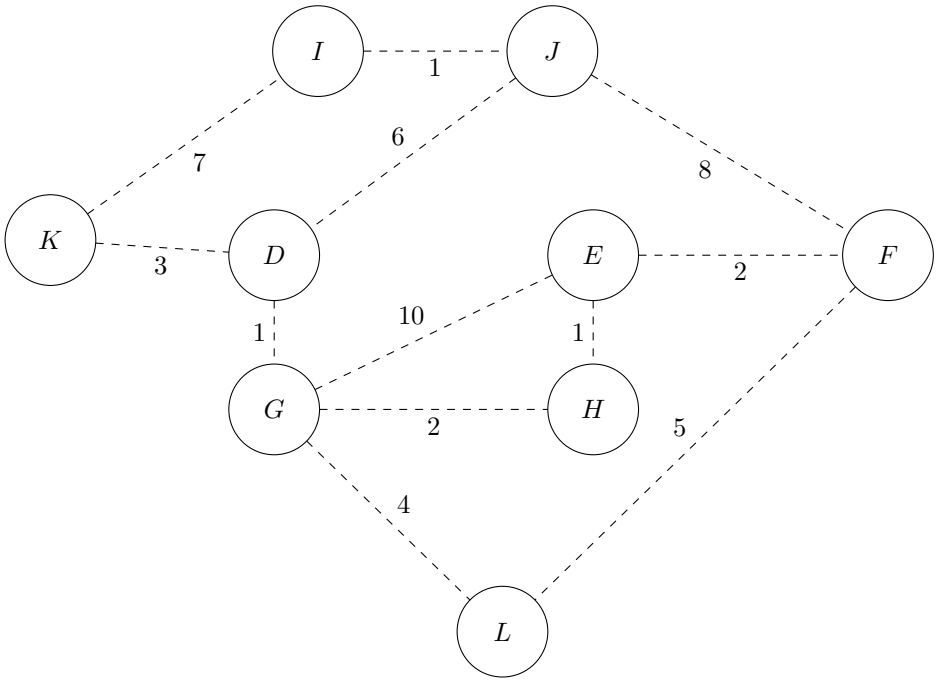
Graph Basics [10 of 14 points]

(a) Give a valid topological sort ordering of the graph below. [2 points]



Answer:

(b) **Color/trace through** the edges of an MST of the graph below. You do not need to show your work. [3 points]



(c) Which kind of graph search (BFS or DFS) can be used to (efficiently) find strongly connected components of a graph? [1 point]

Bubble one: ◯ BFS ◯ DFS

(d) Which kind of graph search (BFS or DFS) can be used to (efficiently) find shortest paths in an unweighted graph? [1 point]

Bubble one: ◯ BFS ◯ DFS

(e) Which kind of graph search (BFS or DFS) can also be implemented recursively (hint: remember that recursion uses the call *stack*)? [1 point]

Bubble one: ◯ BFS ◯ DFS

(f) Suppose vertex x has k neighbors in a graph with n vertices and m edges. If you use an *adjacency list* representation of your graph (where the lists are doubly-linked lists), what is the big-O running time of listing all neighbors of x? [1 point]

$O(\quad)$

(g) Answer the same question, but for an *adjacency matrix*. [1 point]

$O(\quad)$

A Modified Graph Algorithm [4 of 14 points]

Imagine you have a graph where you are guaranteed that all edges have the weight of either 2, 4, or 6.

You design a modified version of Prim's Algorithm, where instead of using a heap to get the vertex that is reached by the next smallest edge to add to the MST, you create four sets of vertices: a set for vertices not seen yet, a set for vertices reachable by an edge of weight 2, a set for vertices reachable by an edge of weight 4, and a set for vertices reachable by an edge of weight 6. As you find edges that reach a vertex in a smaller weight, you move the vertex between the sets.

You will implement your sets using hash tables. Since you control the keys (the names of the vertices), do your analysis assuming there are no collisions in any of your tables.

(a) Suppose you are moving a vertex from a set with p vertices to a set with q vertices. What is the **WORST CASE** running time to move the vertex from one set to the other? [2 points]

$O(\quad)$

(b) Let your graph have m edges and n vertices. What is the **WORST CASE** overall running time of this version of Prim's? [2 points]

$O(\quad)$

3. Sorting (Short Answers) [15 points]

Which Sort? [9 of 15 points]

For each of the following scenarios choose exactly one of the following sorts as the best option, and give a 1-2 sentence explanation of why it is the best sort for the given scenario.

The sorts available to you are:

- Heapsort
- Mergesort
- Quicksort (using median-of-3 for pivot selection)

You will use each sort once.

(a) Every night you get a dataset consisting of many (separate) lists. You want a sorting algorithm which will sort each of the lists (sequentially, one after the other). There is no pattern to the lists (you can think of them as randomly ordered). Your goal is to finish sorting all the lists as soon as possible, so you care about constant factors.

(b) You are writing code for an IoT device. Because its memory is so limited, it gets a list that takes up more than half of its memory. To ensure the device has good uptime, you care about sorting the list with the best-possible worst-case time.

(c) You are finishing writing backend code for a new gradebook to be used by CSE instructors. You know that instructors often need to sort across multiple columns (e.g. to alphabetize, they'll sort by last name then first name).

Sorting Lower Bounds [6 of 15 points]

Recall that we showed a sorting lower-bound in class (a limit on how fast a sorting algorithm could be). For each of the following, you should state either

- "impossible" if the claim contradicts the lower-bound from class.
- "possible" if the claim does not contradict the lower-bound from class (whether you know how to write the described algorithm or not!).

Also include 1-3 sentences explaining your answer. Your explanation should relate to the sorting lower-bound ("I've never heard of that algorithm" or "Oh, that's radix sort" aren't good explanations, because they don't relate to the sorting lower-bound).

(a) Sashu tells you she has designed a comparison-based algorithm that finds the median of a list of n elements in $O(n)$ time.

Bubble one: ◯ Impossible ◯ Possible

Explanation:

(b) Allen shows you his new comparison-based sorting algorithm. The **BEST CASE** running time is $\Theta(n \log \log n)$

Bubble one: ◯ Impossible ◯ Possible

Explanation:

(c) Aashna shows you a comparison-based sorting algorithm that has a **WORST CASE** running time of $O(n \log^*(n!))$.

Bubble one: ◯ Impossible ◯ Possible

Explanation:

4. Concurrency [15 points]

The Allen school is hoping to prevent students from having to wait in a physical line for the career fairs and has implemented its own better virtual queue that students can use to join online. Assume the `ConcurrentLinkedQueues` are THREAD-SAFE (that is, multiple threads may operate on them simultaneously without any concurrency issues in the objects themselves.

You should also assume the queues have enough space and operations on them will not throw an exception.

```
1  class CareerFairQueue {
2      private int maxQueueSize = 20;
3      private Queue<String> names = new ConcurrentLinkedQueue<>();
4      private Queue<String> numbers = new ConcurrentLinkedQueue<>();
5
6      public String joinQueue(String name, String number) {
7
8          if (names.size() < this.maxQueueSize) {
9              names.add(name);
10             numbers.add(number);
11             return "joined queue";
12         } else {
13             return "unable to join queue, queue is full";
14         }
15     }
16
17     public String getNextStudent() {
18         if (names.isEmpty()) {
19             return null;
20         } else {
21             String nextName = names.remove();
22             String nextNumber = numbers.remove();
23             return nextName + " " + nextNumber;
24         }
25     }
26 }
```

(a) Does the `CareerFairQueue` class as shown above have (bubble all that apply):

☐ a race condition ☐ potential for deadlock ☐ a data race ☐ none of these

If there are any problems, give an example of when they could occur. Be specific, and use the line numbers above.

(b) The Allen School decides to add one more method to the class to increase the maxQueueSize.

```
1  public void increaseMaxQueueSize(int amount) {
2      maxQueueSize += amount;
3  }
```

Does adding this method **cause any new** (bubble all that apply):

☐ a race condition ☐ potential for deadlock ☐ a data race ☐ none of these

If there are any NEW problems, give an example of when they could occur. Be specific, and use the line numbers on this page and the prior page. Remember this code goes inside the class on the previous page (even though the line numbers for this snippet start over at 1).

(c) On the next page, we will have another copy of the code for the `CareerFairQueue` including the `increaseMaxQueueSize` method we added. Insert (and use) locks on the next page to allow the most concurrent access while avoiding all of the potential problems listed above. For full credit you must allow the most concurrent access possible without introducing any errors. Create locks as needed, but do not create extraneous locks.

DO NOT use `synchronized`. You should create re-entrant lock objects and can call the locking methods as follows:

```
ReentrantLock lock = new ReentrantLock();
lock.acquire();
lock.release();
```

Answer this question in the next page.

```
// Fix this code with locks!
class CareerFairQueue {
    private int maxQueueSize = 20;
    private Queue < String > names = new ConcurrentLinkedQueue <>();
    private Queue < String > numbers = new ConcurrentLinkedQueue<>();


    public String joinQueue(String name, String number) {


        if (names.size() < this.maxQueueSize) {


            names.add(name);


            numbers.add(number);


            return "joined queue";

        } else {


            return "unable to join queue, queue is full";
        }
    }

    public String getNextStudent() {


        if (names.isEmpty()) {


            return null;
        } else {


            String nextName = names.remove();


            String nextNumber = numbers.remove();


            return nextName + " " + nextNumber;
        }
    }

    public void increaseMaxQueueSize(int amount) {


        maxQueueSize += amount;


    }
}
```

5. Finish the ForkJoin [13 points]

Call a number **very odd** if it is both (1) odd itself and (2) stored at an odd-index of the array. You wish to write fork-join code that will return the **product** of all very odd numbers in an array. If there are no very odd numbers, you should return 1. You may assume the final answer is small enough to be stored in an `int`.

For example, if your array is $[5, \mathbf{3}, 2, \mathbf{3}, 4, 6, 7, \mathbf{5}]$ you should return $3 \cdot 3 \cdot 5 = 45$. Notice that the 6 was not part of the final answer because the number itself is not odd.

Finish the code snippets below to give fork-join code with work $O(n)$ and span $O(\log n)$.

```
import java.util.concurrent.ForkJoinPool;
import java.util.concurrent.RecursiveTask;
import java.util.concurrent.RecursiveAction;

class Main {
    public static final ForkJoinPool fjPool = new ForkJoinPool();

    // returns the product of very odd numbers in input.
    public static int ProductVeryOdd(int[] input) {


        return fj.invoke(new VeryOddTask(_____________________________________________));
    }


    public static class VeryOddTask extends ____________________________________________________ {
        int[] input;
        int lo;
        int hi;

        public VeryOddTask(int[] input, int lo, int hi) {
            this.input = input;
            this.lo = lo;
            this.hi = hi;
        }

        public Integer compute() {
            // sequential case, don't use a cutoff. Only one element should be processed here.

            if (_____________________________________________) {

                if (_____________________________________________)

                    return _____________________________________________;

                else return _____________________________________________;


            }



            // parallel case on next page
```

```
47            // parallel case (you're still in the ”compute” method)
48            else {
49
50              int mid = ____________________________________________________________;
51
52              VeryOddTask left = ____________________________________________________________;
53
54              VeryOddTask right = ____________________________________________________________;
55
56              //finish the method here.
57
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86
87
88
89
90
91
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96
97            } // end of else branch
98          } // end of compute method
99        } // end of VeryOddTask class
```

6. Graph Modeling [10 points]

With the new light rail station, the Northgate area is being converted to an outdoor mall, but renovations aren't complete yet. The planning commission has a goal that pedestrians must be able to get from any shop to any other using sidewalks. Currently none of the sidewalks have been completed. You have a map of all the shops, along with the potential locations where sidewalks can be added. Every potential sidewalk also contains the length (in feet) of sidewalk required to connect those two shops.

You know that it takes $\ell(\ell - 5) + 10$ hours to make a sidewalk of length ℓ feet. Sadly, you can afford only one sidewalk construction team, and with the station already opened, you need to make a plan which will meet the commission's goal of connecting all the shops as soon as possible.

(a) What will the vertices of your graph be?

(b) What will the edges be? You should at least say whether your edges are directed or not and whether they're weighted or not.

(c) What algorithm will you run on your graph?

(d) How will you interpret the output of your algorithm? (i.e., what sidewalks are you building "in the real world" instead of just in graph terms). (1-3 sentences)

(e) Briefly (2-4 sentences) explain why your model works. You should at least address why you ran the algorithm you did (e.g., why are you looking for a shortest path/MST/topological ordering/etc.) and how you are incorporating the formula relating sidewalk length to time.

7. P vs. NP [10 points]

(a) What two words does "NP" stand for? [2 points]

(b) For each of the following problems, bubble all the categories which the problem is **known** to be in. [2 points each]

Given a directed graph, decide whether there is a tour (a walk that visits every vertex exactly once) of length at most k.	☐ P	☐ NP	☐ NP-complete	☐ None of these
Given an undirected graph, decide whether there is a spanning tree of weight at most k.	☐ P	☐ NP	☐ NP-complete	☐ None of these
Given an array of `int`s, sort the array.	☐ P	☐ NP	☐ NP-complete	☐ None of these

(c) Briefly (2-3 sentences) describe why designing a polynomial-time algorithm for 3-coloring would mean you also have a polynomial time algorithm for 3-SAT. Your answer should include both the mention of a complexity class and a description of what the 3-SAT algorithm would look like. [2 points]

8. Parallel QuickSort [9 points]

The TAs thinks this is the trickiest problem on the exam, which is why it's at the end. We've intentionally made it worth fewer points than expected for its difficulty, so don't be afraid to jump around and come back if you have time.

In class, we designed a parallel version of quick sort that used a heuristic to find a (usually) good pivot. In this problem, we'll modify the algorithm to find the true median to use as the pivot. For simplicity, throughout the problem assume all elements of the array are distinct, and that we have any needed auxiliary arrays already allocated.

To find the median, we'll run n reduces, where the i^{th} reduce will calculate "how many elements of `A` are smaller than $A[i]$"?

(a) What is the **span** of running all n of these reduces (include both the individual reduces and the process of forking/joining enough threads to run them)? Give your answer in *simplified* big-O. [1 point]

$O(\quad\quad)$

(b) What is the **work** of running all n of these reduces (include both the individual reduces and the process of forking/joining enough threads to run them)? Give your answer in *simplified* big-O. [1 point]

$O(\quad\quad)$

(c) Briefly (3-5 bullet-points or sentences) describe how, given `B` where `B[i]` contains the output of the i^{th} reduce, you can output `m` the index of the median of `A` (the original array). Assume all threads from the last step have already joined together into a single thread. You may not use concurrency primitives (e.g. no locks). Your process must have the best possible big-O span, but don't worry about constant factors. You may assume you already have extra auxiliary arrays if needed. [3 points]

(d) What is the work and span of your process? [1 point]

Work: $O(\quad\quad)$
Span: $O(\quad\quad)$

(e) Recall that the other non-recursive work of quicksort is to pack elements smaller than the pivot into one array and the elements greater-than-or-equal to the pivot into another. What is the span of **just that part** of the process (give a simplified, big-O) [2 points]

$O(\quad\quad)$

(f) Give a recurrence that describes the worst-case **span** of our new quicksort (using the exact median finding). You may ignore ceilings and floors in your recurrence, and use O-notation in the non-recursive work descriptions. [1 point]

Answer:

9. Drawing [1 point]

Draw what you would do in a world where you had just found an extremely efficient algorithm for 3-SAT. These help TA morale while grading! **Any non-blank page will receive the point**.

Gumball wishes you a happy Winter Break!