CSE 332: Data Structures & Parallelism

Lecture 8: Dictionaries; Binary Search Trees

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Administrative

- EX02 – On Priority Queues, Due TONIGHT Friday Oct 10
- EX03 – On Recurrences, Due Fri Dec 17
- Mid Quarter Survey, coming soon!
- “Meet the Staff” activity
  - Sometime during the first 4 weeks of class, visit a CSE 332 office hour (in person or on zoom)
- Lecture MegaThread in Ed Discussion

Today

- Dictionaries
- Binary Trees
- Binary Search Trees

Where we are

Studying the absolutely essential ADTs of computer science and classic data structures for implementing them

ADTs so far:

1. Stack: **`push`**, **`pop`**, **`isEmpty`**, …
2. Queue: **`enqueue`**, **`dequeue`**, **`isEmpty`**, …
3. Priority queue: **`insert`**, **`deleteMin`**, …

Next:

4. Dictionary (a.k.a. Map): associate keys with values
   – probably the most common, way more than priority queue

Key: UWNetID Value: favorite color

The Dictionary (a.k.a. Map) ADT

Data:

must be unique

- *set* of (key, value) *pairs*
- keys ~~must be~~ *comparable*

for some Dictionary implementations

Operations:

- **`insert(key,val)`:**
  - places (key,val) in map
  (If key already used, overwrites existing entry)
- **`find(key)`:**
  - returns val associated with key
- **`delete(key)`**
  - …

insert(rea, Ruth Anderson)

find(jhsia)

Justin Hsia,…

- **rea**
  Ruth Anderson
  …
- **jhsia**
  Justin Hsia
  …

We will tend to emphasize the keys, but don't forget about the stored values!

Comparison: Set ADT vs. Dictionary ADT

The *Set* ADT is like a Dictionary without any values

- A key is *present* or not (no repeats)

For **`find`**, **`insert`**, **`delete`**, there is little difference

- In dictionary, values are "just along for the ride"
- So *same data-structure ideas* work for dictionaries and sets
  - Java HashSet implemented using a HashMap, for instance

Set ADT may have other important operations

- **`union`**, **`intersection`**, **`is_subset,`** etc.
- Notice these are binary operators on sets
- We will want different data structures to implement these operators

A Modest Few Uses for Dictionaries

Any time you want to store information according to some key and be able to retrieve it efficiently – a dictionary is the ADT to use!

- Lots of programs do that!

• Networks:	router tables
• Operating systems:	page tables
• Compilers:	symbol tables
• Databases:	dictionaries with other nice properties
• Search:	inverted indexes, phone directories, …
• Biology:	genome maps
• …	

Simple implementations

- Worst Case
- Assume Arrays have enough space

For dictionary with *n* key/value pairs

	`insert`	`find`	`delete`
Unsorted linked-list	$O(N)$	$O(N)$	$O(N)$
Unsorted array	$O(N)$	$O(N)$	$O(N)$
Sorted linked list	$O(N)$	$O(N)$	$O(N)$
Sorted array	$O(N)$	$O(\log N)$	$O(N)$

We'll see a Binary Search Tree (BST) probably does better, but not in the worst case unless we keep it balanced

Simple implementations

For dictionary with *n* key/value pairs

	`insert`	**`find`**	**`delete`**
• Unsorted linked-list	O(n) *	$O(n)$	$O(n)$
• Unsorted array	O(n)*	$O(n)$	$O(n)$
• Sorted linked list	$O(n)$	$O(n)$	$O(n)$
• Sorted array	$O(n)$	$O($**`log`** $n)$	$O(n)$

We'll see a Binary Search Tree (BST) probably does better, but not in the worst case unless we keep it balanced

*Note: If we insert the same key more than once (say to update the value associated with a key), we must first check to see if it exists, before inserting it, so our running time takes at least as long as a find operation.

Lazy Deletion (e.g. in a sorted array)

10	12	24	30	41	42	44	45	50
✓	✗	✓	✓	✓	✓	✗	✓	✓

A *general technique* for making **`delete`** as fast as **`find`**:

- Instead of actually removing the item just mark it deleted
- No need to shift values, etc.

Plusses:

- Simpler
- Can do removals later in batches
- If re-added soon thereafter, just unmark the deletion

Minuses:

- Extra *space* for the "is-it-deleted" flag
- Data structure full of deleted nodes wastes *space*
- **`find`** *O*(**`log`** *m*) *time* where *m* is data-structure size (m >= n)
- May complicate other operations

Better Dictionary data structures

Will spend the next several lectures looking at dictionaries with two different data structures

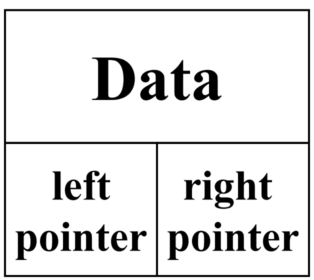
1. AVL trees
   – Binary search trees with *guaranteed balancing*
2. Hashtables
   – Not tree-like at all

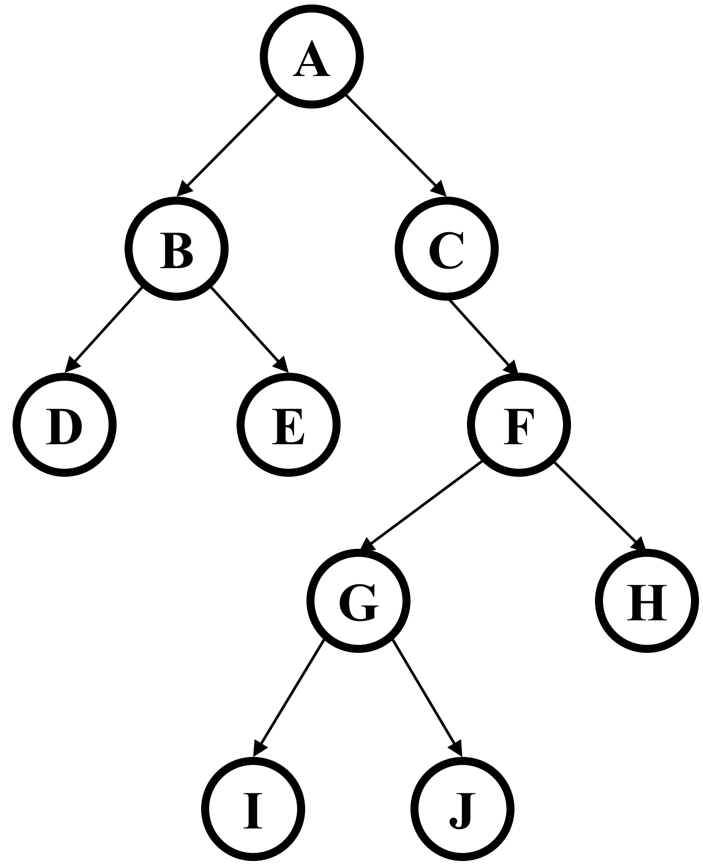
Skipping: Other balanced trees (B-trees, red-black, splay)

Binary Trees

- Binary tree is empty or
  - a root *(with data)*
  - a left subtree *(maybe empty)*
  - a right subtree *(maybe empty)*

- Representation:

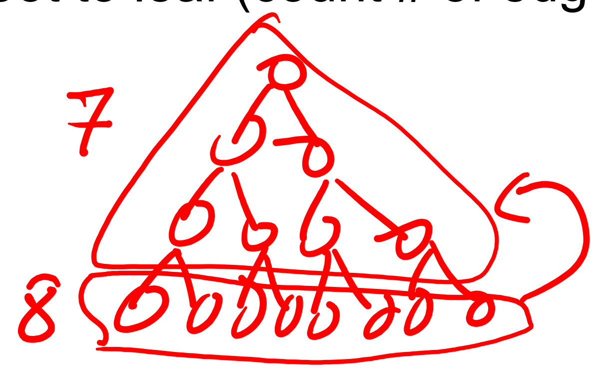




- For a dictionary, data will include a key and a value

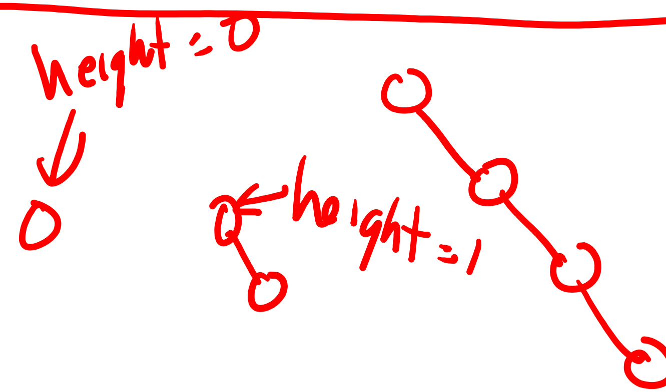
Binary Tree: Some Numbers

Recall: height of a tree = longest path from root to leaf (count # of edges)



For binary tree of height *h*:

- max # of leaves: 2^h
- max # of nodes: $2^{h+1} - 1$
- min # of leaves: 1
- min # of nodes: $h+1$



Binary Trees: Some Numbers

Recall: height of a tree = longest path from root to leaf (count edges)

For binary tree of height *h*:

- max # of leaves: 2^h
- max # of nodes: $2^{(h+1)} - 1$
- min # of leaves: 1
- min # of nodes: $h + 1$

For n nodes, we cannot do better than O(`log` n) height, and we want to avoid O(n) height

Calculating height

What is the height of a tree with root **`root`**?

```
int treeHeight(Node root) {

            ???


}
```

Calculating height

What is the height of a tree with root **`r`**?

```
int treeHeight(Node root) {
  if(root == null)
    return -1;
  return 1 + max(treeHeight(root.left),
                 treeHeight(root.right));
}
```

Running time for tree with *n* nodes: *O*(*n*) – single pass over tree

Note: non-recursive is painful – need your own stack of pending nodes; much easier to use recursion's call stack

Tree Traversals

A *traversal* is an order for visiting all the nodes of a tree

- *Pre-order*: root, left subtree, right subtree

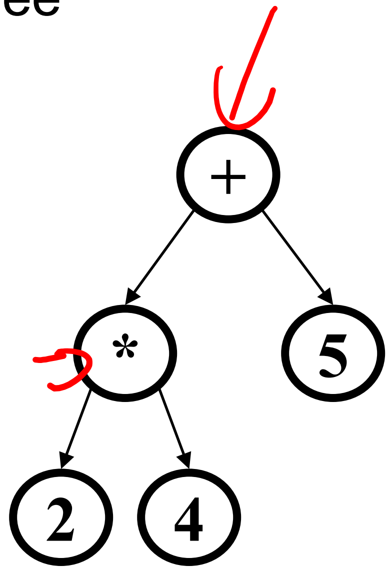
  + * 2 4 5

- *In-order*: left subtree, root, right subtree

  2 * 4 + 5

- *Post-order*: left subtree, right subtree, root

  2 4 * 5 +



(an expression tree)

Tree Traversals

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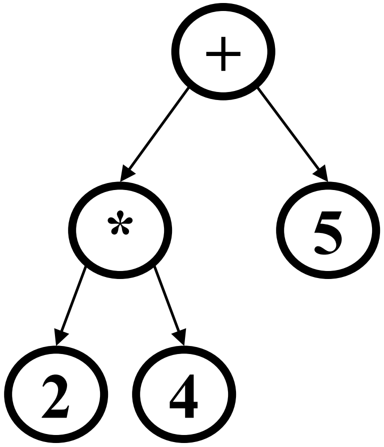
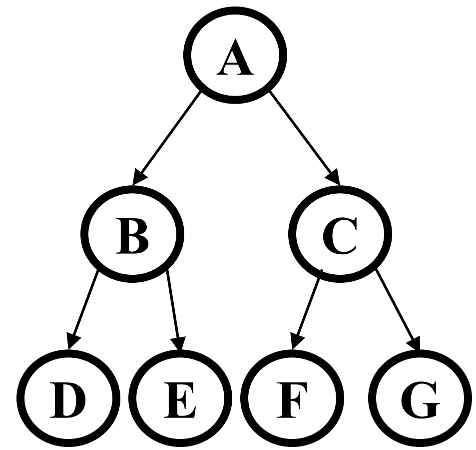


(an expression tree)

More on traversals

```
void inOrdertraversal(Node t){
    if(t != null) {
pre   traverse(t.left);
      process(t.element);
      traverse(t.right);
post }
}
```



Sometimes order doesn't matter

- Example: sum all elements

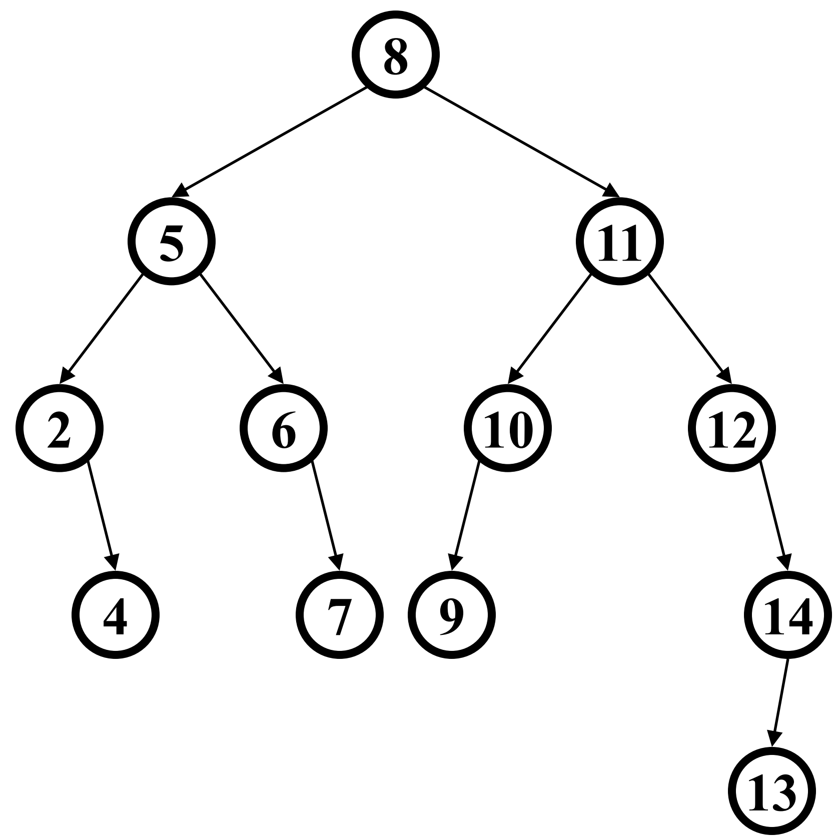
Sometimes order matters

- Example: print tree with parent above indented children (pre-order)
- Example: evaluate an expression tree (post-order)

A
    B
        D
        E
    C
        F
        G

Binary Search Tree

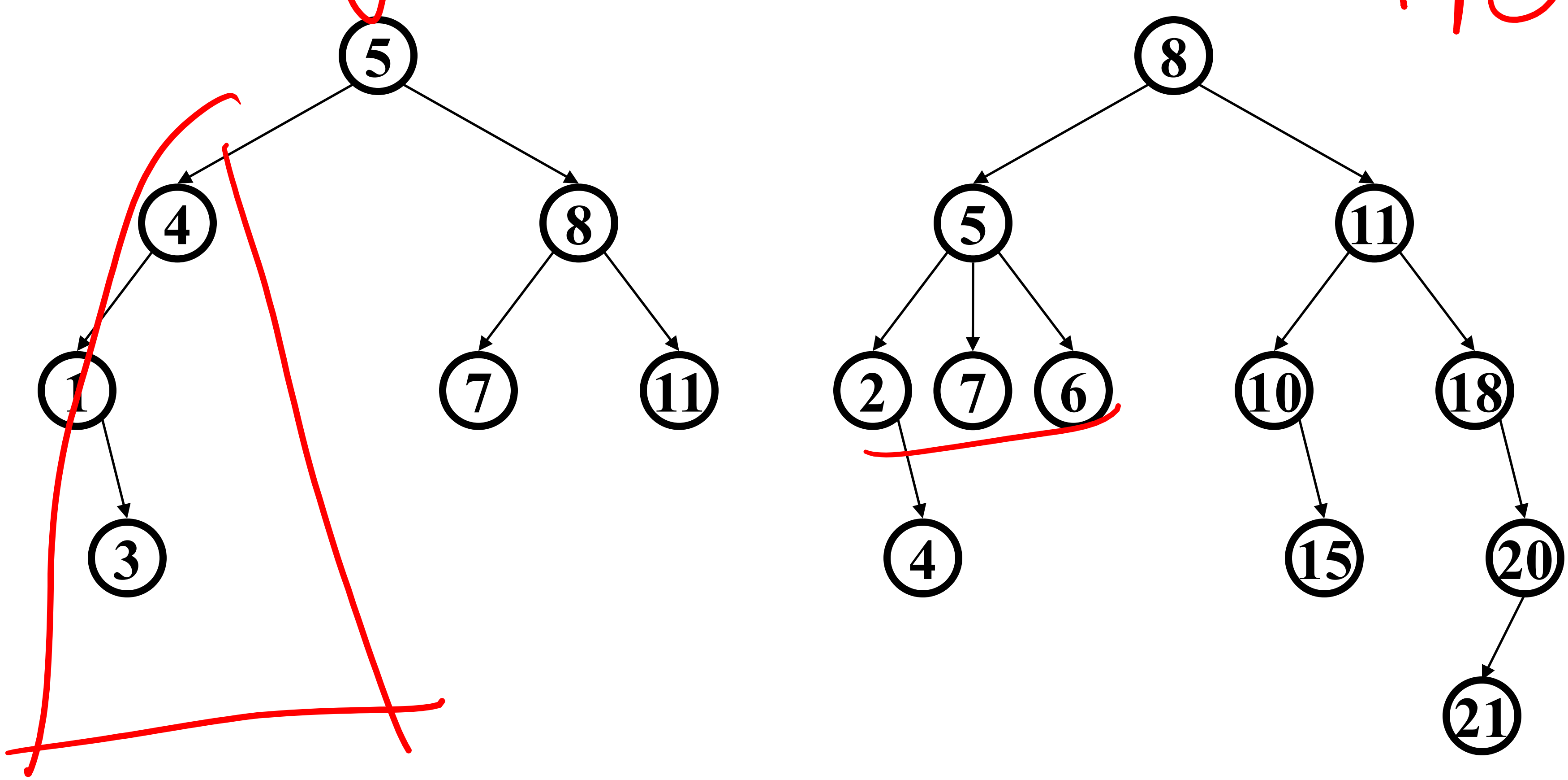
- Structural property ("binary")
  - each node has ≤ 2 children
  - result: keeps operations simple
- Order property
  - all keys in left subtree smaller than node's key
  - all keys in right subtree larger than node's key
  - result: easy to find any given key



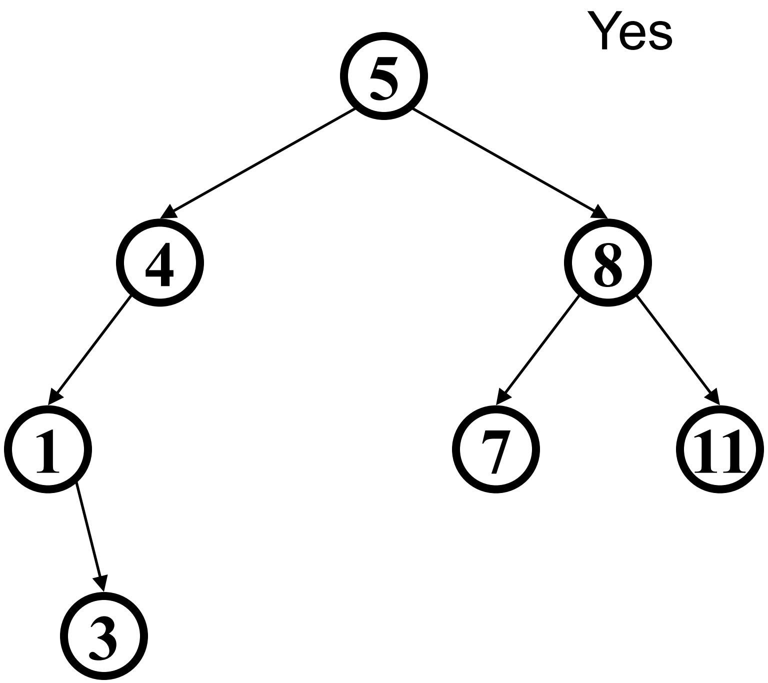
Are these BSTs?

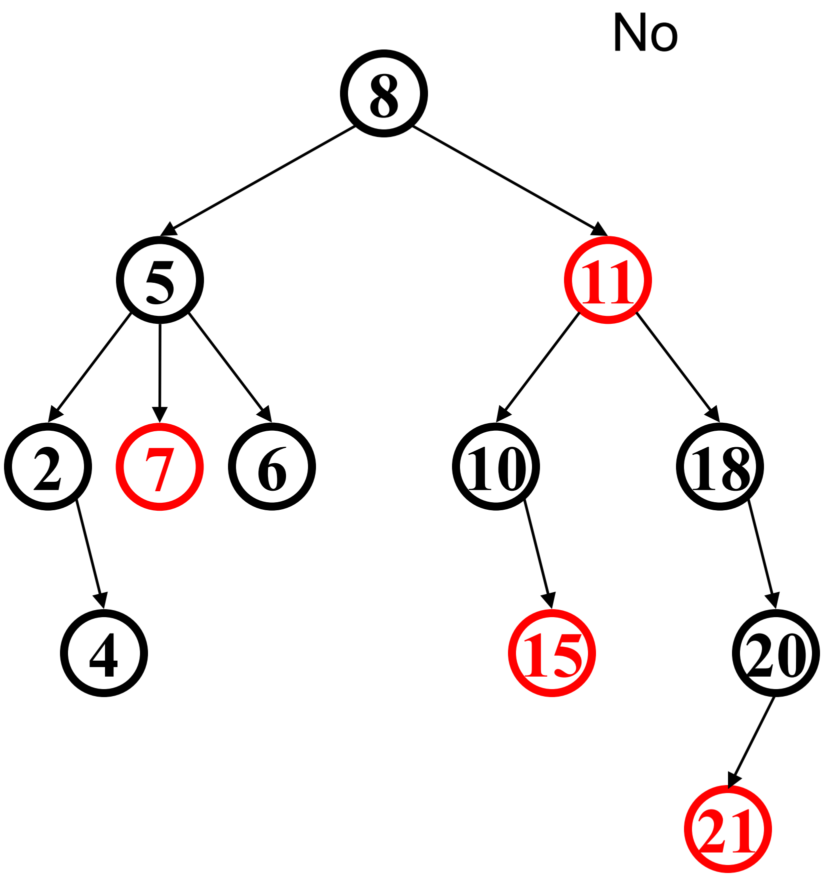
yes

no

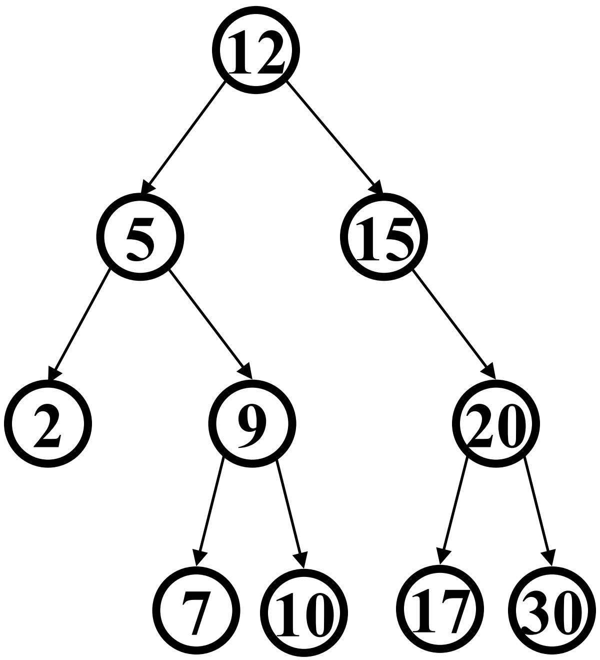


Are these BSTs?



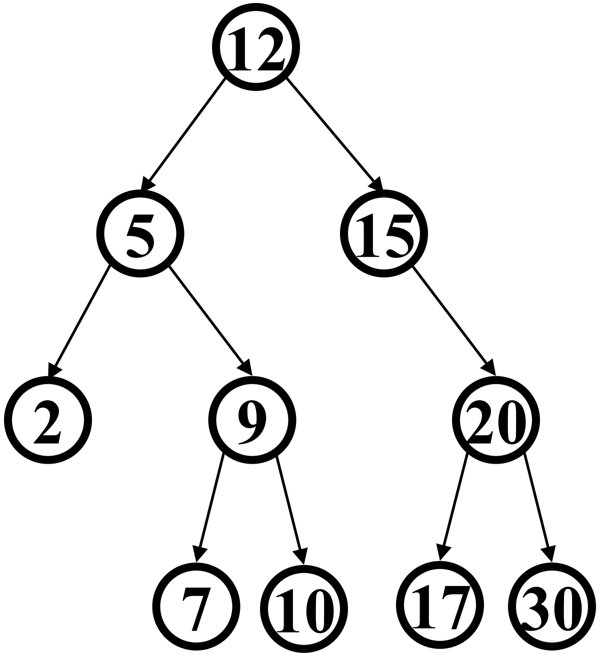


Find in BST, Recursive




```
Data find(Key key, Node root){
 if(root == null)
   return null;
 if(key < root.key)
   return find(key,root.left);
 if(key > root.key)
   return find(key,root.right);
 return root.data;
}
```

Find in BST, Iterative




```
Data find(Key key, Node root){
 while(root != null
        && root.key != key) {
  if(key < root.key)
     root = root.left;
  else(key > root.key)
     root = root.right;
 }
 if(root == null)
     return null;
 return root.data;
}
```

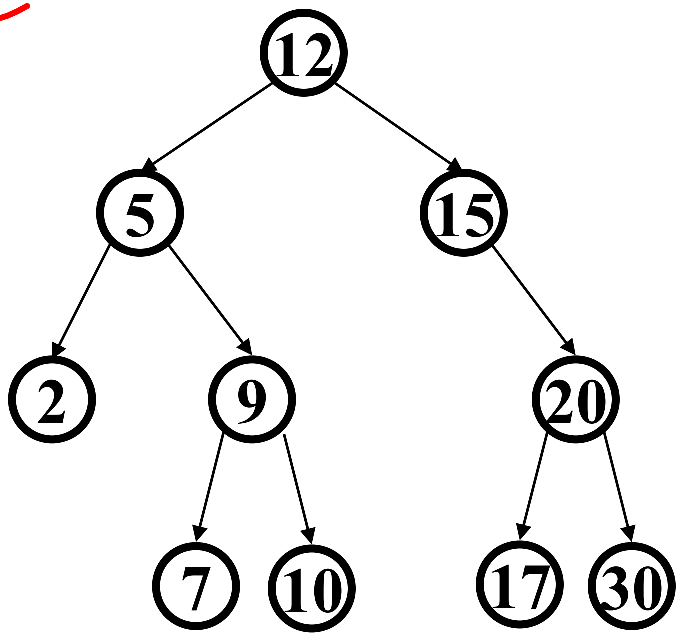
Other "finding operations"

- Find *minimum* node

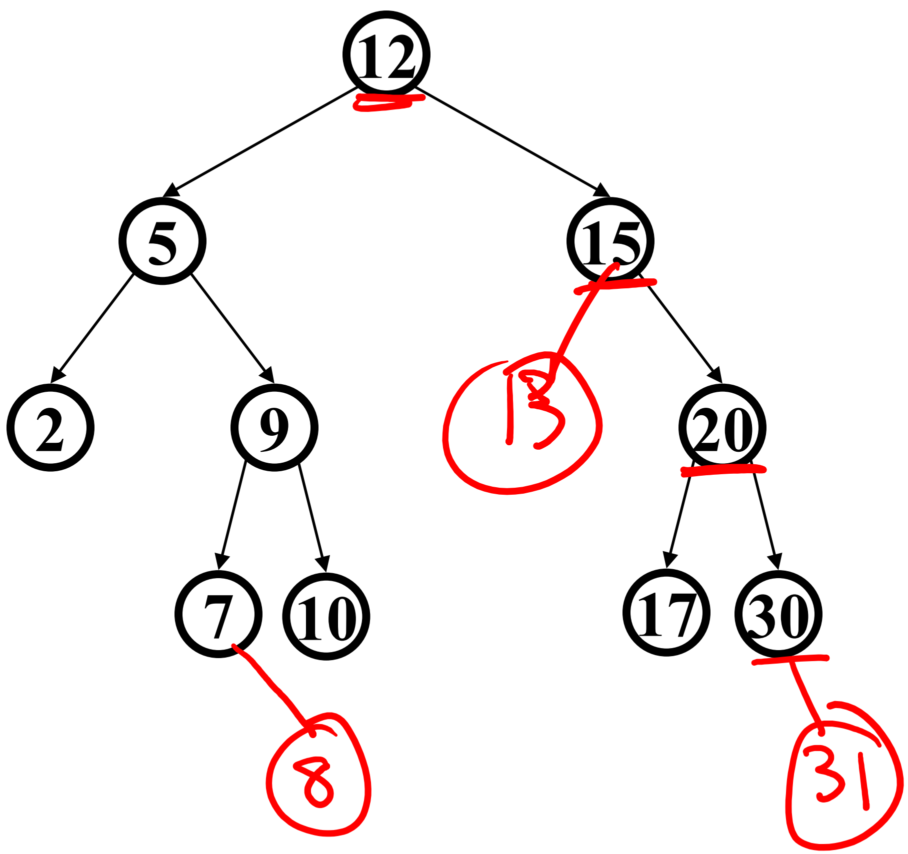
go left

- Find *maximum* node

go right



Insert in BST

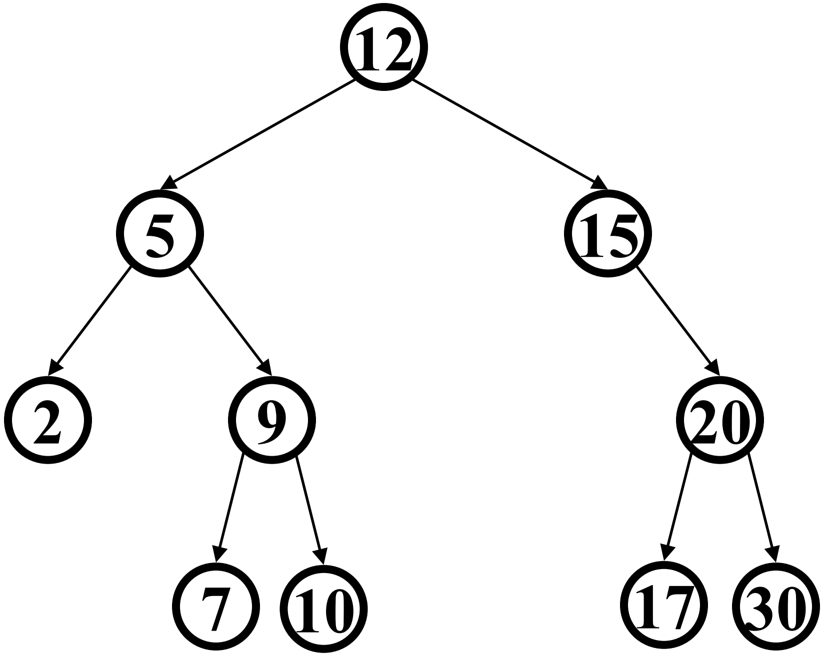



```
insert(13)
insert(8)
insert(31)
```

(New) insertions happen only at leaves – easy!

1. Find
2. Create a new node

Deletion in BST



Why might deletion be harder than insertion?

Deletion

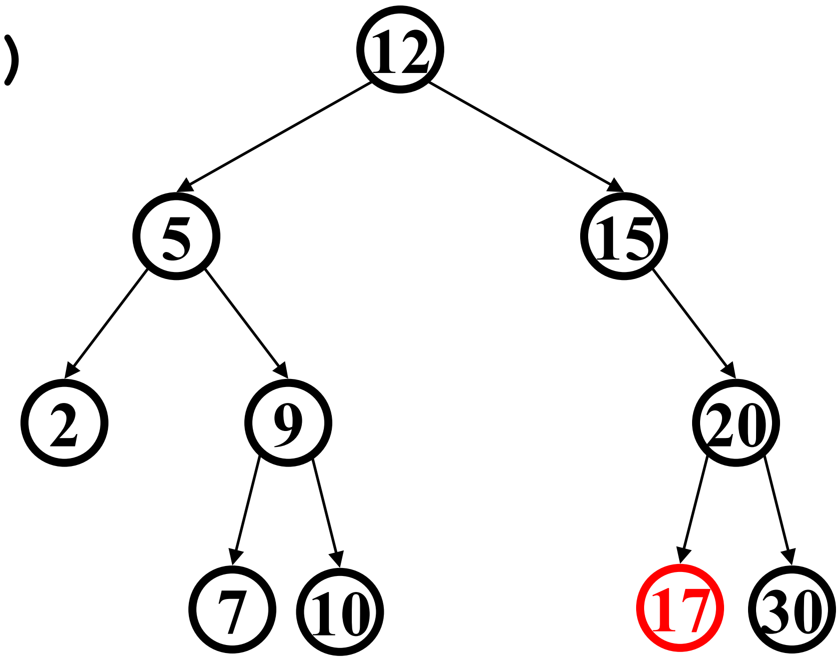
- Removing an item disrupts the tree structure

- Basic idea:
  - `find` the node to be removed,
  - Remove it
  - “fix” the tree so that it is still a binary search tree

- Three cases:
  - node has no children (leaf)
  - node has one child
  - node has two children

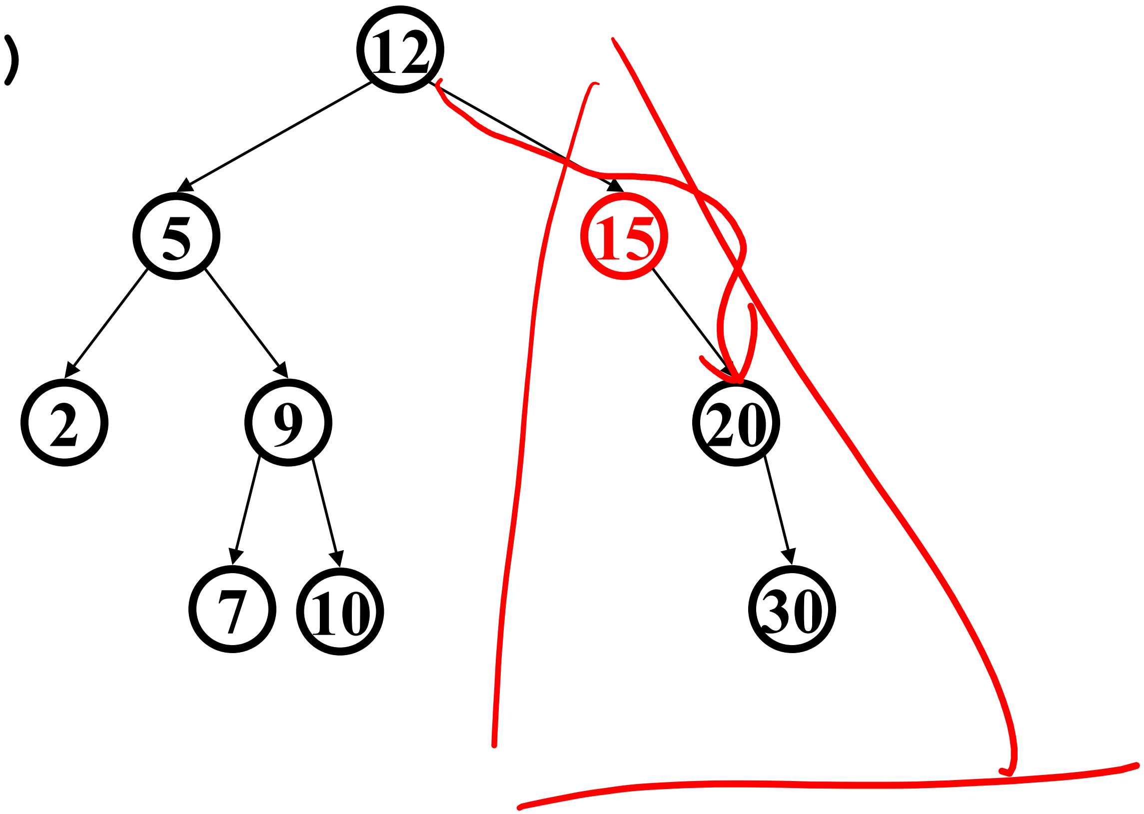
Deletion – The Leaf Case

`delete(17)`



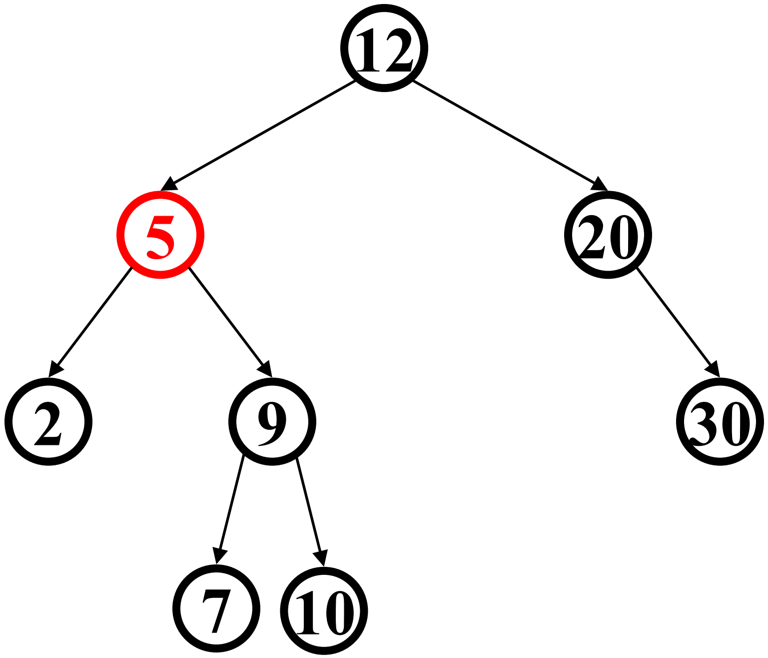
Deletion – The One Child Case

`delete(15)`



Deletion – The Two Child Case

`delete(5)`



What can we replace 5 with?

Deletion – The Two Child Case

Idea: Replace the deleted node with a value guaranteed to be between the two child subtrees

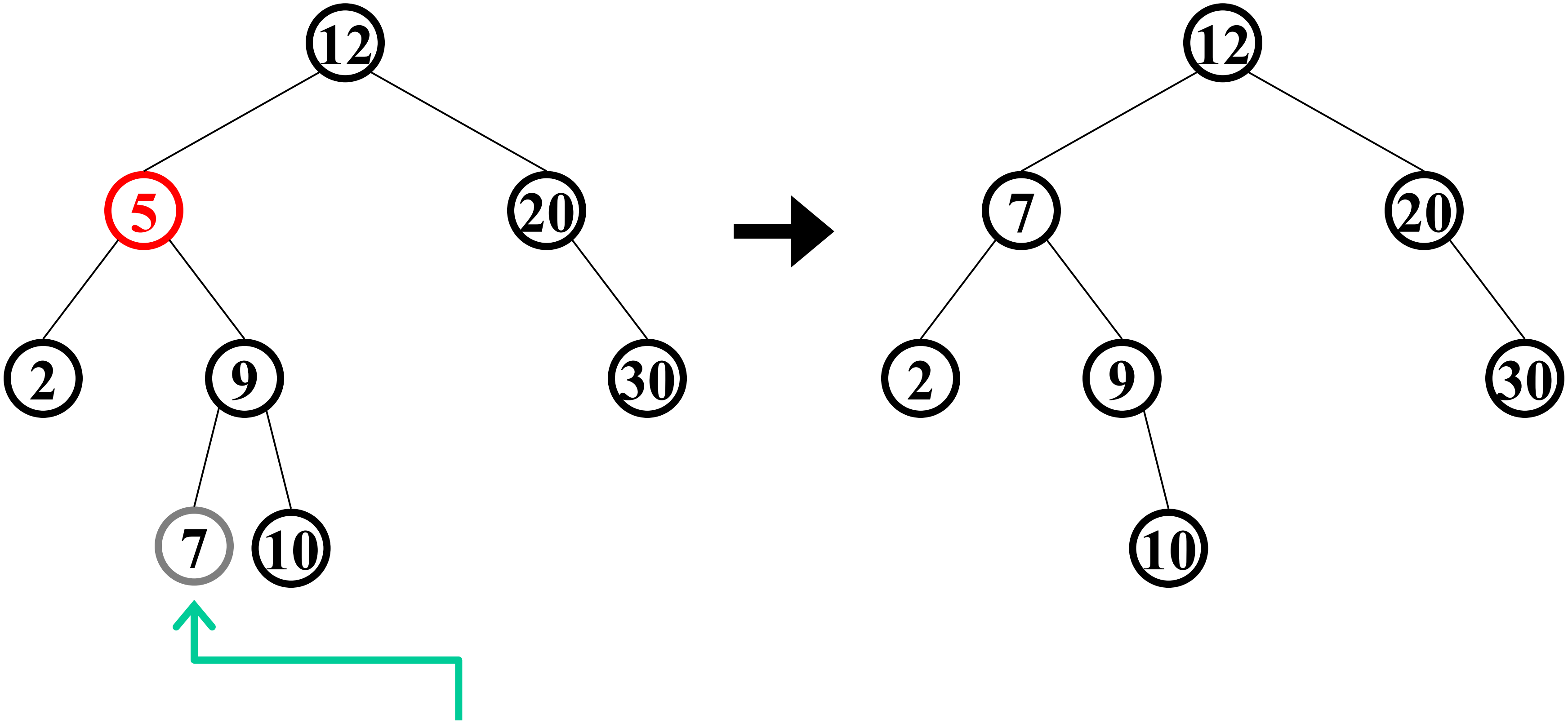
Options:

- *successor* from right subtree: **`findMin(node.right)`**
- *predecessor* from left subtree: **`findMax(node.left)`**
  - These are the easy cases of predecessor/successor

Now delete the original node containing *successor* or *predecessor*

- Leaf or one child case – easy cases of delete!

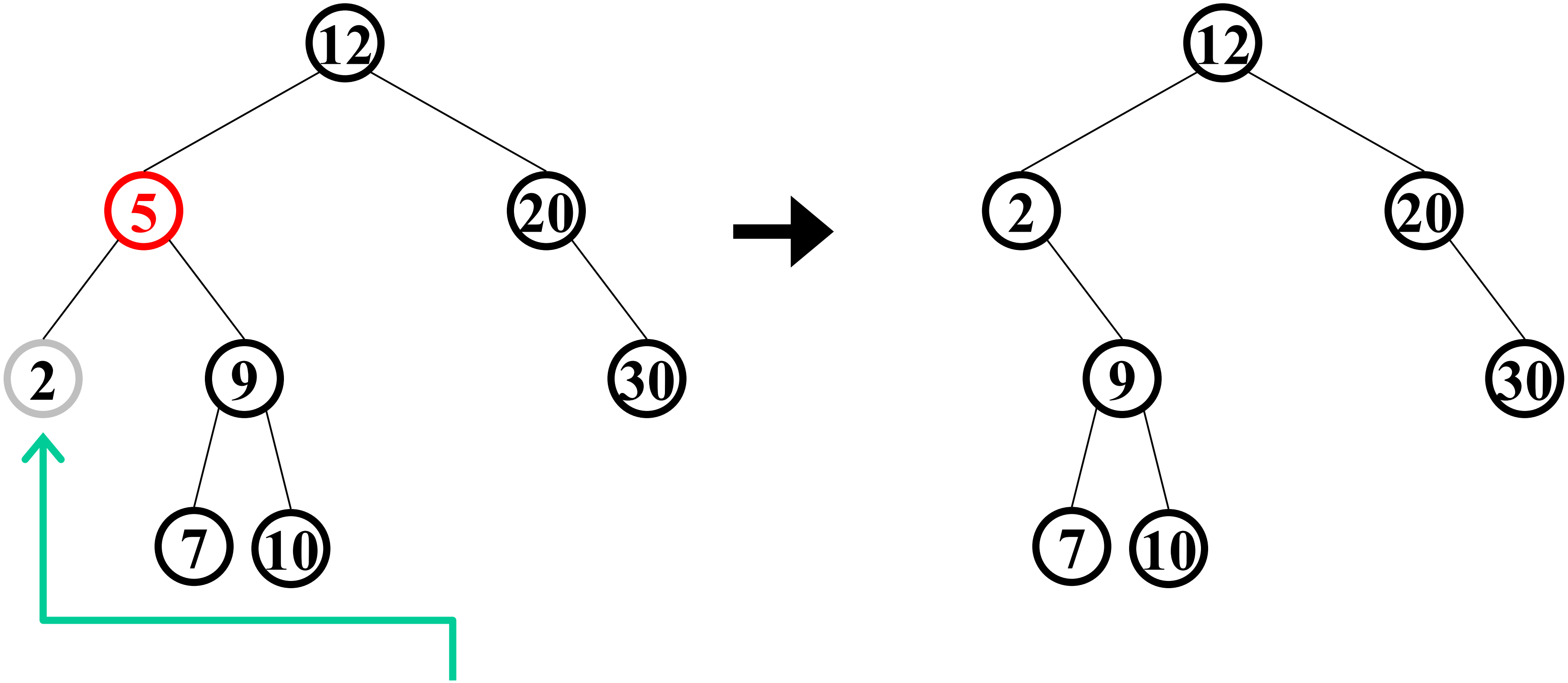
Delete Using Successor



findMin(right sub tree) → 7

`delete(5)`

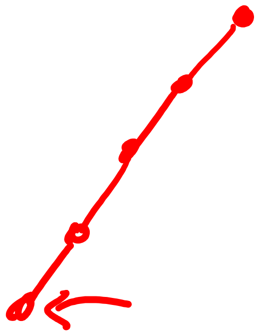
Delete Using Predecessor



findMax(left sub tree) → 2

`delete(5)`

Binary Search Trees as Dictionaries?



- Keys will have to be comparable…
- What is the worst case for insert and find?

	Insert	Find	Delete
Worse-Case	O(n)	O(n)	O(n)
Average-Case	O(log n)	O(log n)	O(log n)

Balanced BST

Observation

- BST: the shallower the better!
- For a BST with n nodes inserted in arbitrary order
  - Average height is $O(\texttt{log}\ n)$ – see text for proof
  - Worst case height is $O(n)$
- Simple cases such as inserting in key order lead to the worst-case scenario

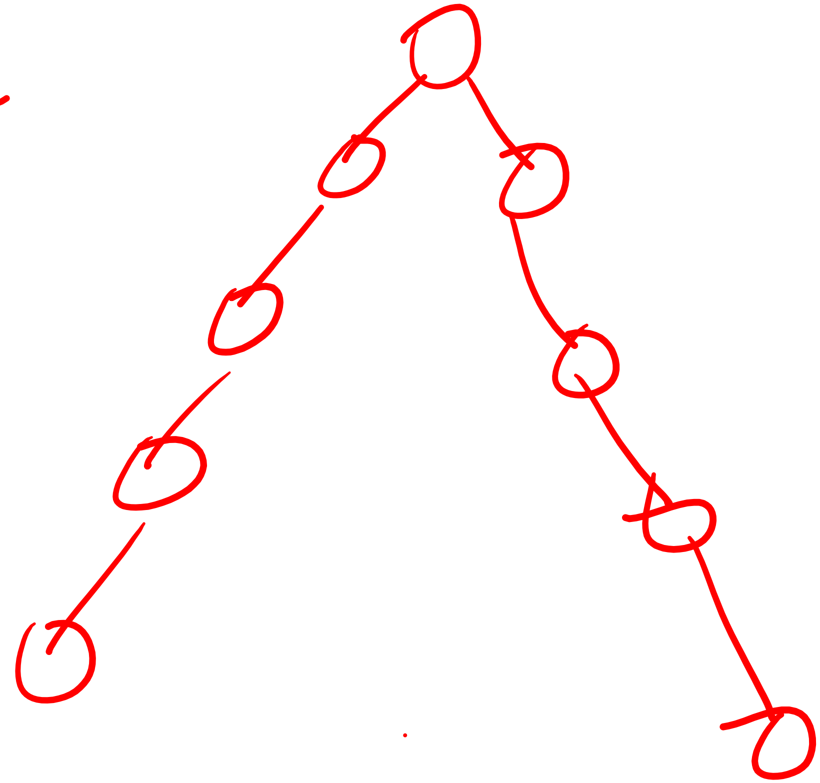
Solution: Require a **Balance Condition** that

1. ensures depth is always $O(\texttt{log}\ n)$ – strong enough!
2. is easy to maintain – not too strong!

Potential Balance Conditions

1. Left and right subtrees of the *root* have equal number of nodes

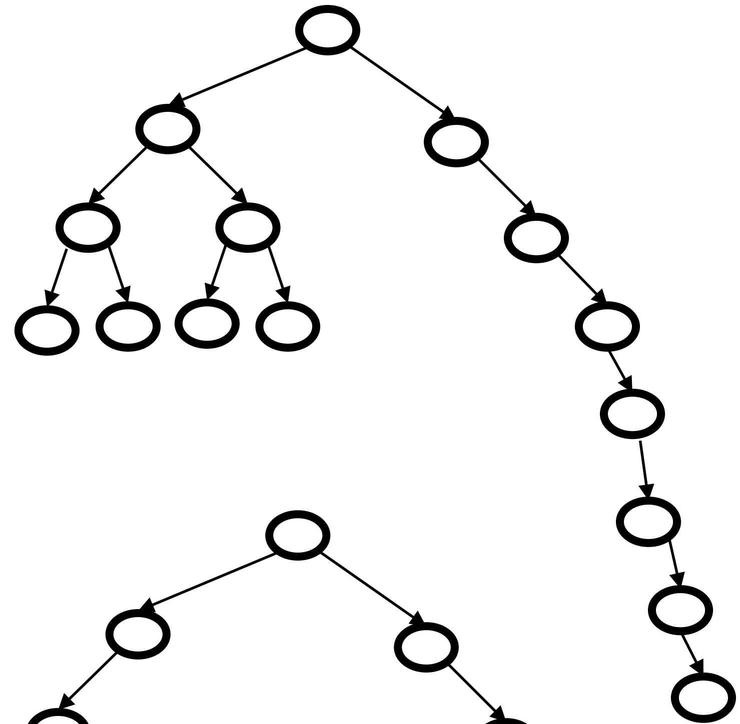
2. Left and right subtrees of the *root* have equal *height*



Potential Balance Conditions

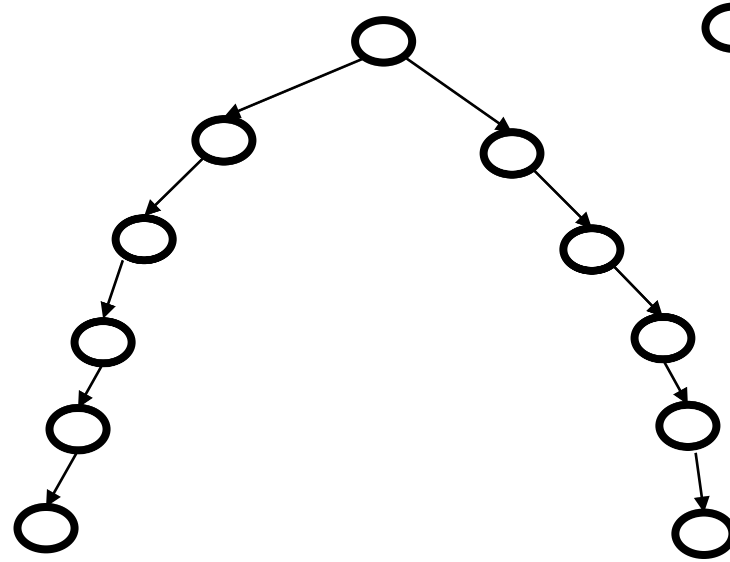
1. Left and right subtrees of the *root* have equal number of nodes

Too weak!
Height mismatch example:



2. Left and right subtrees of the *root* have equal *height*

Too weak!
Double chain example:



Potential Balance Conditions

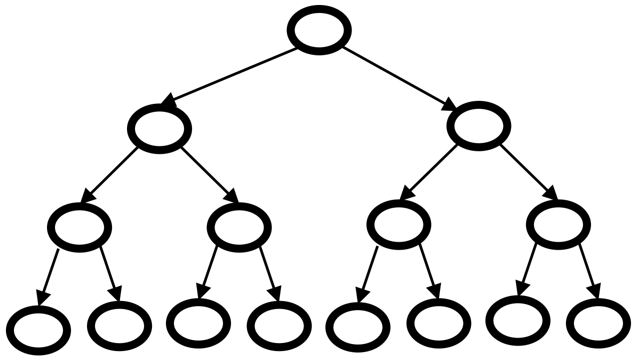
3. Left and right subtrees of every node have equal number of nodes

4. Left and right subtrees of every node have equal *height*

Potential Balance Conditions

3. Left and right subtrees of every node have equal number of nodes

   Too strong!
   Only perfect trees allowed



4. Left and right subtrees of every node have equal *height*

   Too strong!
   Only perfect trees allowed

The AVL Balance Condition

Left and right subtrees of *every node*
have *heights* **differing by at most 1**

Definition: **balance**(*node*) = height(*node*.left) – height(*node*.right)

AVL *property*: **for every node *x*, $-1 \leq$ balance(*x*) ≤ 1**

- Ensures small depth
  - Will prove this by showing that an AVL tree of height *h* must have a number of nodes *exponential* in *h*
- Easy (well, efficient) to maintain
  - Using single and double rotations