CSE332: Data Structures & Parallelism

Lecture 3: Algorithm Analysis

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Administrative

- Survey – Due **TONIGHT**! Monday 9/29
- EX0 – Due next Friday 10/03
- Ex1 – released later tonight, due next Monday 10/06
- "Meet the Staff" activity
  - Sometime during the first 4 weeks of class, visit a CSE 332 office hour (in person or on zoom)
  - Tell the staff member you want to get checked off
  - You do not have to have a question about course content
  - We just want to meet you!
- Lecture MegaThread in Ed Lessons
  - We will have one of these for each lecture
  - Feel free to ask questions there during or after lecture!

Today – Algorithm Analysis Day 2

- ~~What do we care about?~~
- ~~How to compare two algorithms~~
- ~~Analyzing Code~~
- Asymptotic Analysis (previous slide deck)
- Big-Oh Definition (previous slide deck)
- Big-Oh Proofs

Proving Big-O, Formally

- Big-O is an $\exists c, n_0 \forall n$ statement.
- I.e., an exists statement with a “forall” inside.

- How do you prove an exists statement?

- How do you prove a for-all statement?

Proving Big-O, Formally (answers)

- Big-O is an $\exists c, n_0 \forall n$ statement.
- I.e., an exists statement with a “forall” inside.

- How do you prove an exists statement?
  - Show the c, n_0 that will work. Give specific values.
- How do you prove a for-all statement?
  - Introduce an arbitrary n.

Using the Definition

- Let's show: $10n^2 + 15n$ is $O(n^2)$

Using the Definition (2)

- Let's show: $10n^2 + 15n$ is $O(n^2)$

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Scratch work:
$10n^2 \leq 10n^2$
$15n \leq 15n^2$ **for** $n \geq 1$
$10n^2 + 15n \leq 25n^2$ **for** $n \geq 1$

Proof:
Take $c = 25$ **and** $n_0 = 1$**. For an arbitrary** $n \geq n_0$**, we have**
The inequality $10n^2 \leq 10n^2$ **is always true. The inequality** $15n \leq 15n^2$ **is true for** $n \geq 1$**, as the right hand side is a factor of** n **more than the right hand side.**
As long as both inequalities are true we can add them, thus
$10n^2 + 15n \leq 25n^2$ **holds as long as** $n \geq 1$**.**
This is exactly the inequality we needed to show.

Writing Proofs

- Where did that $c = 25, n_0 = 1$ come from?
- That was some "scratch work" – the insight isn't explained in the final proof
  - You just say "Consider"
- Don't try to skip the scratch work when drafting your big-O proofs.
  - But it won't appear in your final version.

Be sure you're arguing in correct logical order---you only assert something is true when you know it. Often that's the reverse of the scratch work order.

Don't just choose $c = 10^{10}, n_0 = 10^5$. That will be technically correct, but proofs are acts of communication; that won't convince your reader if they didn't already believe the claim; smaller values with algebra are more convincing than overkill.