CSE 332 Winter 2024
Lecture 5: Priority Queues

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http://www.cs.uw.edu/332
ADT: Queue

• What is it?
  • A “First In First Out” (FIFO) collection of items

• What Operations do we need?
  • Enqueue
    • Add a new item to the queue
  • Dequeue
    • Remove the “oldest” item from the queue
  • Is_empty
    • Indicate whether or not there are items still on the queue
ADT: Priority Queue

• What is it?
  • A collection of items and their “priorities”
  • Allows quick access/removal to the “top priority” thing

• What Operations do we need?
  • insert(item, priority)
    • Add a new item to the PQ with indicated priority
    • Usually, smaller priority value means more important
  • deleteMin
    • Remove and return the “top priority” item from the queue
  • Is_empty
    • Indicate whether or not there are items still on the queue

• Note: the “priority” value can be any type/class so long as it’s comparable (i.e. you can use “<” or “compareTo” with it)
Priority Queue, example

```java
PriorityQueue PQ = new PriorityQueue();
PQ.insert(5,5)
PQ.insert(6,6)
PQ.insert(1,1)
PQ.insert(3,3)
PQ.insert(8,8)
Print(PQ.deleteMin)
Print(PQ.deleteMin)
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```

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Applications?
- to do list
- string
- loading things on web
- class registration
- callback
- emails
- processor
Thinking through implementations

<table>
<thead>
<tr>
<th>Data Structure</th>
<th>Worst case time to insert</th>
<th>Worst case time to deleteMin</th>
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<tbody>
<tr>
<td>Unsorted Array</td>
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<td></td>
</tr>
<tr>
<td>Unsorted Linked List</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sorted Array</td>
<td>$O(n)$</td>
<td>$10\cdot n$</td>
</tr>
<tr>
<td>Sorted Linked List</td>
<td>$O(n)$</td>
<td>$1$</td>
</tr>
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Note: Assume we know the maximum size of the PQ in advance
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*Somewhat sorted*
Trees for Heaps

- **Binary Trees:**
  - The branching factor is 2
  - Every node has \( \leq 2 \) children

- **Complete Tree:**
  - All "layers" are full, except the bottom
  - Bottom layer filled left-to-right
Heap – Priority Queue Data Structure

- Idea: We need to keep some ordering, but it doesn’t need to be entirely sorted
- $\Theta(\log n)$ worst case for deleteMin and insert
Heap – Priority Queue Data Structure

• Idea: We need to keep some ordering, but it doesn’t need to be entirely sorted
• $\Theta(\log n)$ worst case for deleteMin and insert
Challenge!

• What is the maximum number of total nodes in a binary tree of height $h$?
  • $2^{h+1} - 1$
  • $\Theta(2^h)$

• If I have $n$ nodes in a binary tree, what is its minimum height?
  • $\Theta(\log n)$

• Heap Idea:
  • If $n$ values are inserted into a complete tree, the height will be roughly $\log n$
  • Ensure each insert and deleteMin requires just one “trip” from root to leaf
(Min) Heap Data Structure

- Keep items in a complete binary tree
- Maintain the “(Min) Heap Property” of the tree
  - Every node’s priority is ≤ its children’s priority
  - Max Heap Property: every node’s priority is ≥ its children’s priority
- Where is the min?
- How do I insert?
- How do I deleteMin?
- How to do it in Java?
Heap Insert

insert(item){
    put item in the “next open” spot (keep tree complete)
    while (item.priority < parent(item).priority){
        swap item with parent
    }
}
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}

1

3

4

5

7

9

1.5

5

2

6

1

3

4

5

6

7

9
Heap Insert

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```
Heap deleteMin

deleteMin(){
    min = root
    br = bottom-right item
    move br to the root
    while(br > either of its children){
        swap br with its smallest child
    }
    return min
}
Heap deleteMin

deleteMin()
{
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}
Percolate Up and Down (for a Min Heap)

• Goal: restore the “Heap Property”

• Percolate Up:
  • Take a node that may be smaller than a parent, repeatedly swap with a parent until it is larger than its parent

• Percolate Down:
  • Take a node that may be larger than one of its children, repeatedly swap with smallest child until both children are larger

• Worst case running time of each:
  • $\Theta(\log n)$
Representing a Heap

- Every complete binary tree with the same number of nodes uses the same positions and edges
- Use an array to represent the heap
- Index of root:
- Parent of node $i$:
- Left child of node $i$:
- Right child of node $i$:
- Location of the leaves:
Insert Psuedocode

```plaintext
insert(item){
    if(size == arr.length - 1){resize();}
    size++;
    arr[i] = item;
    percolateUp(i)
}
```
Percolate Up

```java
percolateUp(int i){
    int parent = i/2; \index of parent
    int val = arr[i]; \value at current location
    while(i > 1 && arr[i].priority < arr[parent].priority){ \until location is root or heap property holds
        arr[i] = arr[parent]; //move parent value to this location
        arr[parent] = val; //put current value into parent’s location
        i = parent; //make current location the parent
        parent = i/2; //update new parent
    }
}
```
DeleteMin Pseudo code

def deleteMin():
    theMin = arr[1];
    arr[1] = arr[size];
    size--;
    percolateDown(1);
    return theMin;
}
Percolate Down

percolateDown(int i) {
    int left = i * 2;   // index of left child
    int right = i * 2 + 1;  // index of right child
    Item val = arr[i];   // value at location
    while (left <= size) {   // until location is leaf
        int toSwap = right;
        if (right > size || arr[left].priority < arr[right].priority) {
            toSwap = left;   // swap with left
        }
        // now toSwap has the smaller of left/right, or left if right does not exist
        if (arr[toSwap].priority < val.priority) {
            arr[i] = arr[toSwap];
            arr[toSwap] = val;  // swap parent with smaller child
            i = toSwap;  // update current node to be smaller child
            left = i * 2;
            right = i * 2 + 1;
        }
    } else { return; }   // if we don’t swap, then heap property holds
}
Other Operations

• Increase Key
  • Given the index of an item in the PQ, make its priority value larger
    • Min Heap: Then percolate down
    • Max Heap: Then percolate up

• Decrease Key
  • Given the index of an item in the PQ, make its priority value smaller
    • Min Heap: Then percolate up
    • Max Heap: Then percolate down

• Remove
  • Given the item at the given index from the PQ
Aside: Expected Running time of Insert
Building a Heap From “Scratch”

• Suppose we had $n$ items and wanted to “heapify” them

Violate Heap Property!

Two ways for “fix” the heap:
1) Percolate Up
2) Percolate Down
Floyd’s buildHeap method

• Working towards the root, one row at a time, percolate down

```java
buildHeap(){
    for(int i = size; i>0; i--){
        percolateDown(i);
    }
}
```
Floyd’s buildHeap method

• Suppose we had $n$ items and wanted to “heapify” them

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- Suppose we had \( n \) items and wanted to “heapify” them

```java
buildHeap()
{
    for(int i = size; i > 0; i--)
    {
        percolateDown(i);
    }
}
```
How long did this take?

• Worst case running time of buildHeap:
• No node can percolate down more than the height of its subtree
  • When i is a leaf:
  • When i is second-from-last level:
  • When i is third-from-last level:
• Overall Running time:

```java
buildHeap()
{
    for(int i = size; i>0; i--){
        percolateDown(i);
    }
}
```