CSE 332 Summer 2024
Lecture 9: hashing

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Dictionary (Map) ADT

- Contents:
  - Sets of key+value pairs
  - Keys must be comparable
- Operations:
  - insert(key, value)
    - Adds the (key,value) pair into the dictionary
    - If the key already has a value, overwrite the old value
      - Consequence: Keys cannot be repeated
  - find(key)
    - Returns the value associated with the given key
  - delete(key)
    - Remove the key (and its associated value)

Dictionary Data Structures

Data Structure	Time to insert	Time to find	Time to delete
Unsorted Array	$\Theta(1)$	$\Theta(n)$	$\Theta(n)$
Unsorted Linked List	$\Theta(1)$	$\Theta(n)$	$\Theta(n)$
Sorted Array	$\Theta(n)$	$\Theta(\log n)$	$\Theta(n)$
Sorted Linked List	$\Theta(n)$	$\Theta(n)$	$\Theta(n)$
Heap	$\Theta(\log n)$	$\Theta(n)$	$\Theta(n)$
Binary Search Tree	$\Theta(\text{height})$	$\Theta(\text{height})$	$\Theta(\text{height})$
AVL Tree	$\Theta(\log n)$	$\Theta(\log n)$	$\Theta(\log n)$

BSTs and AVL Trees

- Binary Search Tree:
  - A binary tree where for each node, all keys in its left subtree are smaller and all keys in its right subtree are larger
  - Find:
    - If it matches, return the value.
    - If the search key is less than the current node, look left. If it’s greater, look right.
    - If we reach an empty spot, find was unsuccessful
  - Insert:
    - Do a find, if it was successful then update the value
    - If it was unsuccessful, add a new node to the empty spot we found.
  - Delete:
    - If the deleted node is a leaf, just remove it
    - If the deleted node had one child, replace it with that one child
    - If the deleted node had 2 children, replace it with the largest key to the left
- AVL Tree:
  - A binary search tree where for each node, the height of its left subtree and the height of its right subtree are off by at most 1.
  - Find:
    - Same as BST
  - Insert:
    - Do a BST insert, then rotate if tree is unbalanced (apply one LL, RR, LR, RL case)
  - Delete:
    - Do a BST delete, then rotate if the tree is unbalanced (apply LL, RR, LR, RL cases as needed from leaf to root)

Other Tree-based Dictionaries

- Red-Black Trees
  - Similar to AVL Trees in that we add shape rules to BSTs
  - More “relaxed” shape than an AVL Tree
    - Trees can be taller (though not asymptotically so)
    - Needs to move nodes less frequently
  - This is what Java’s TreeMap uses!
- Tries
  - Similar to a Huffman Tree
  - Requires keys to be sequences (e.g. Strings)
  - Combines shared prefixes among keys to save space
  - Often used for text-based searches
    - Web search
    - Genomes

Next topic: Hash Tables

Data Structure	Time to insert	Time to find	Time to delete
Unsorted Array	$\Theta(n)$	$\Theta(n)$	$\Theta(n)$
Unsorted Linked List	$\Theta(n)$	$\Theta(n)$	$\Theta(n)$
Sorted Array	$\Theta(n)$	$\Theta(\log n)$	$\Theta(n)$
Sorted Linked List	$\Theta(n)$	$\Theta(n)$	$\Theta(n)$
Binary Search Tree	$\Theta(\text{height})$	$\Theta(\text{height})$	$\Theta(\text{height})$
AVL Tree	$\Theta(\log n)$	$\Theta(\log n)$	$\Theta(\log n)$
Hash Table (Worst case)	$\Theta(n)$	$\Theta(n)$	$\Theta(n)$
Hash Table (Average)	$\Theta(1)$	$\Theta(1)$	$\Theta(1)$

Dictionary (Map) ADT

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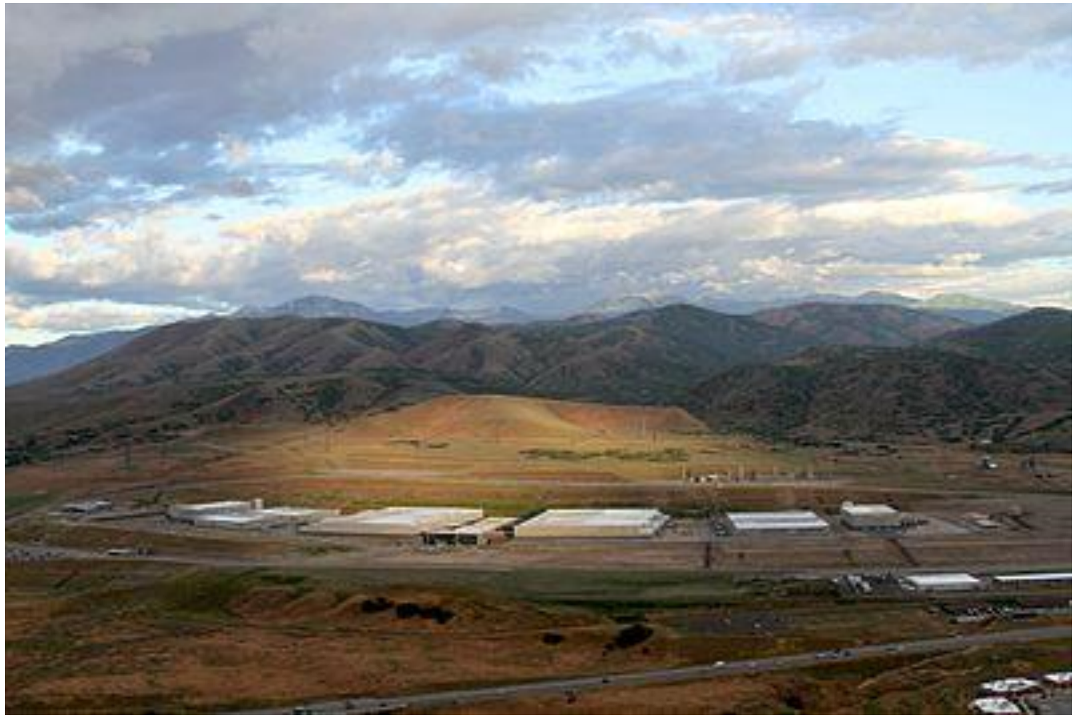
The Best Data Structure!

- Think of every key as a number
- Give each key its own index in an array

```
insert(key, value){
    arr[key]=value;
}
find(key){
    return arr[key];
}
delete(key){
    arr[key] = null;
}
```

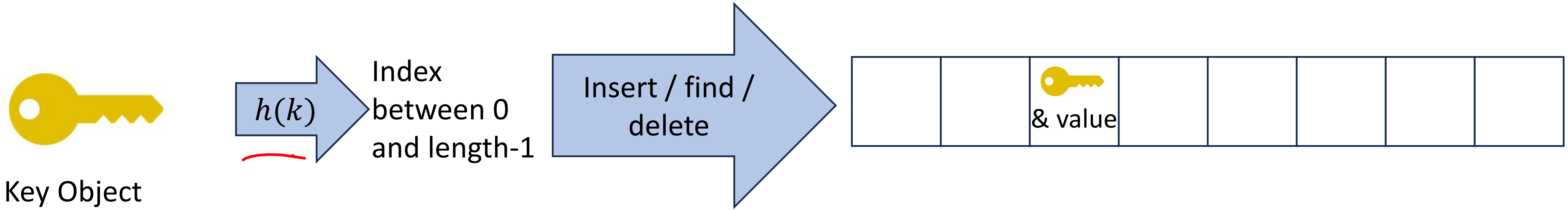
Problem?

exabytes



Hash Tables

- Idea:
  - Have a small array to store information
  - Use a **hash function** to convert the key into an index
    - Hash function should “scatter” the keys, behave as if it randomly assigned keys to indices
  - Store key at the index given by the hash function
  - Do something if two keys map to the same place (should be very rare)
    - Collision resolution



Example

0	1	2	3	4	5	6	7	8	9

- Key: Phone Number
- Value: People
- Table size: 10
- $h(phone)$ = number as an integer % 10
- $h(8675309) = 9$

What Influences Running time?

- How long hashing itself takes
- Likelihood of collisions
  - Size of the array vs number of values in the array
  - “quality” of our hash function
- What we do when we have a collision

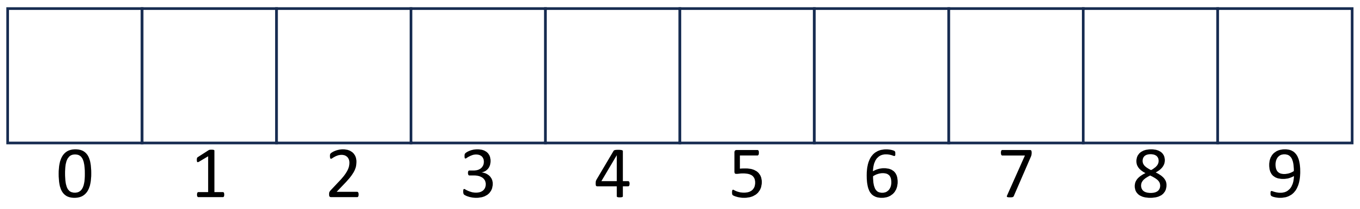
Properties of a “Good” Hash

- Definition: A hash function maps objects to integers

- Should be very efficient
  - Time to calculate the hash should be negligible
- Should “randomly” scatter objects
  - Even similar objects should hash to arbitrarily different values
- Should use the entire table
  - There should not be any indices in the table that nothing can hash to
  - Picking a table size that is prime helps with this
- Should use things needed to “identify” the object
  - Use only fields you would check for a .equals method be included in calculating the hash
    - $\{\text{fields used for hashing}\} \subseteq \{\text{fields used for}\ .\text{equals}\}$
  - More fields typically leads to fewer collisions, but less efficient calculation

A Bad Hash (and phone number trivia)

- $h(phone) =$ the first digit of the phone number
  - Assume 10-digit format
  - No US phone numbers start with 1 or 0
  - If we’re sampling from this class, 2 is by far the most likely

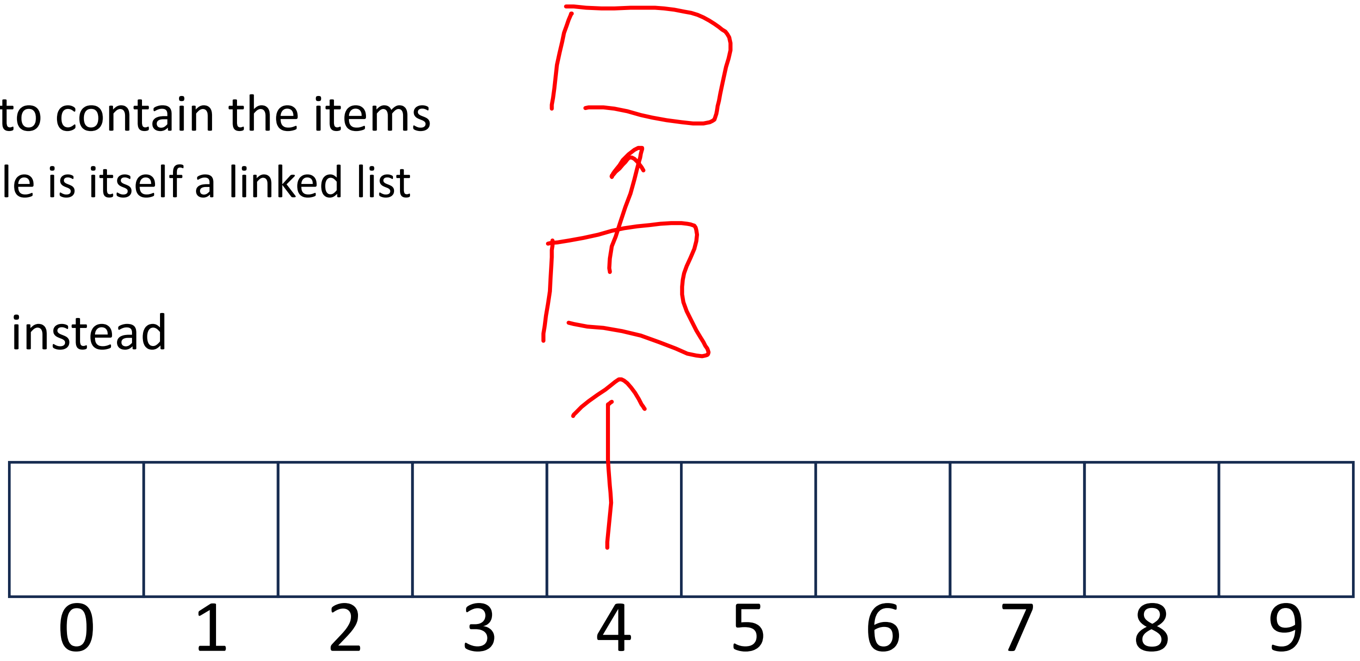


Compare These Hash Functions (for strings)

- Let $s = s_0 s_1 s_2 \dots s_{m-1}$ be a string of length m
  - Let $a(s_i)$ be the ascii encoding of the character s_i
- $h_1(s) = a(s_0)$
  - Way more items map to $h(e)$ than to $h(q)$
- $h_2(s) = \left(\sum_{i=0}^{m-1} a(s_i)\right)$
  - Feels pretty random
  - Relatively efficient
  - Use the whole string
  - dog and god map to the same spot
  - Might be different likelihoods of even vs odd
- $h_3(s) = \left(\sum_{i=0}^{m-1} a(s_i) \cdot 37^i\right)$
  - Shares benefits with the last one
  - Slower than the last because there's more math
  - Uses character's position
  - 37 is prime (important?)
    - May be a problem if length is also 37

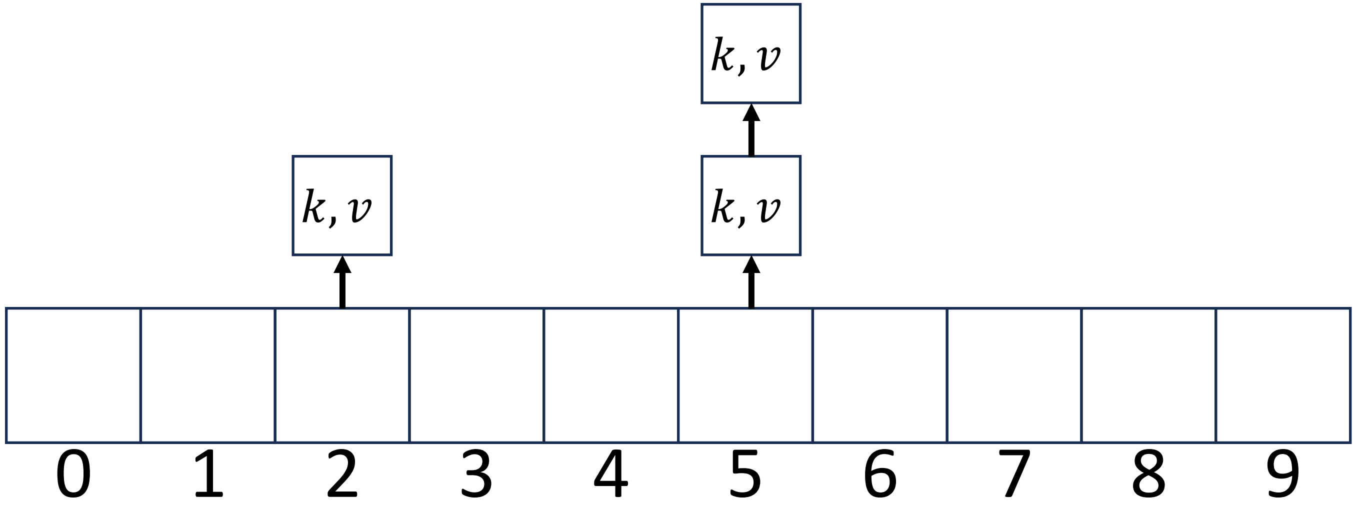
Collision Resolution

- A Collision occurs when we want to insert something into an already-occupied position in the hash table
- 2 main strategies:
  - Separate Chaining
    - Use a secondary data structure to contain the items
      - E.g. each index in the hash table is itself a linked list
  - Open Addressing
    - Use a different spot in the table instead
      - Linear Probing
      - Quadratic Probing
      - Double Hashing



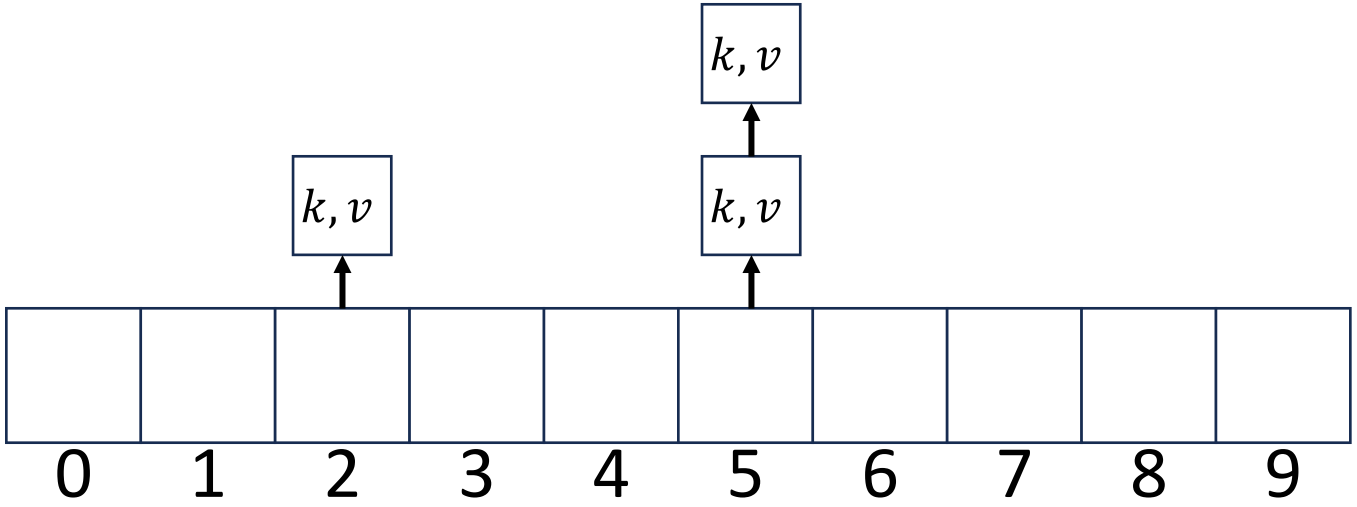
Separate Chaining Insert

- To insert k, v:
  - Compute the index using $i = h(k) \% length$
  - Add the key-value pair to the data structure at $table[i]$



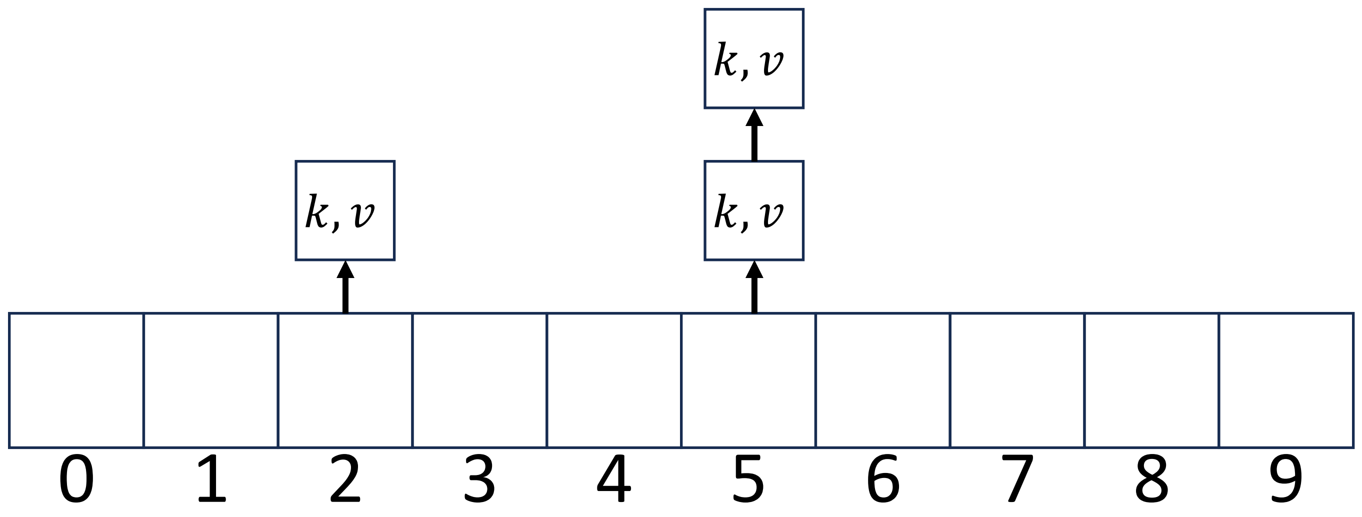
Separate Chaining Find

- To find k:
  - Compute the index using $i = h(k)\ \%\ length$
  - Call find with the key on the data structure at $table[i]$



Separate Chaining Delete

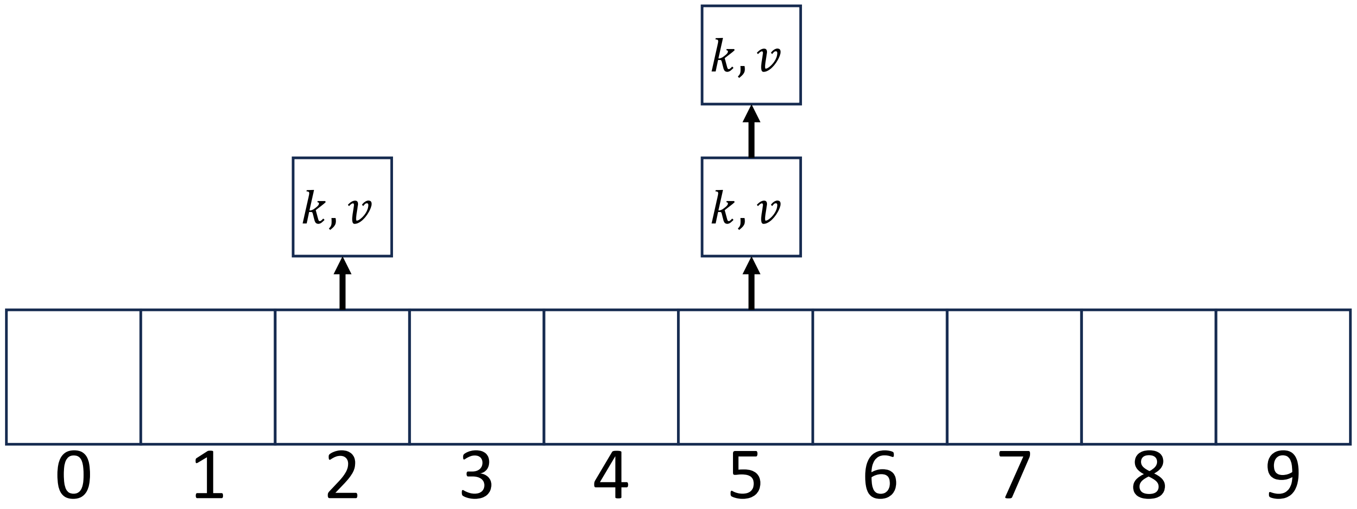
- To delete k:
  - Compute the index using $i = h(k)\ \%\ length$
  - Call delete with the key on the data structure at $table[i]$



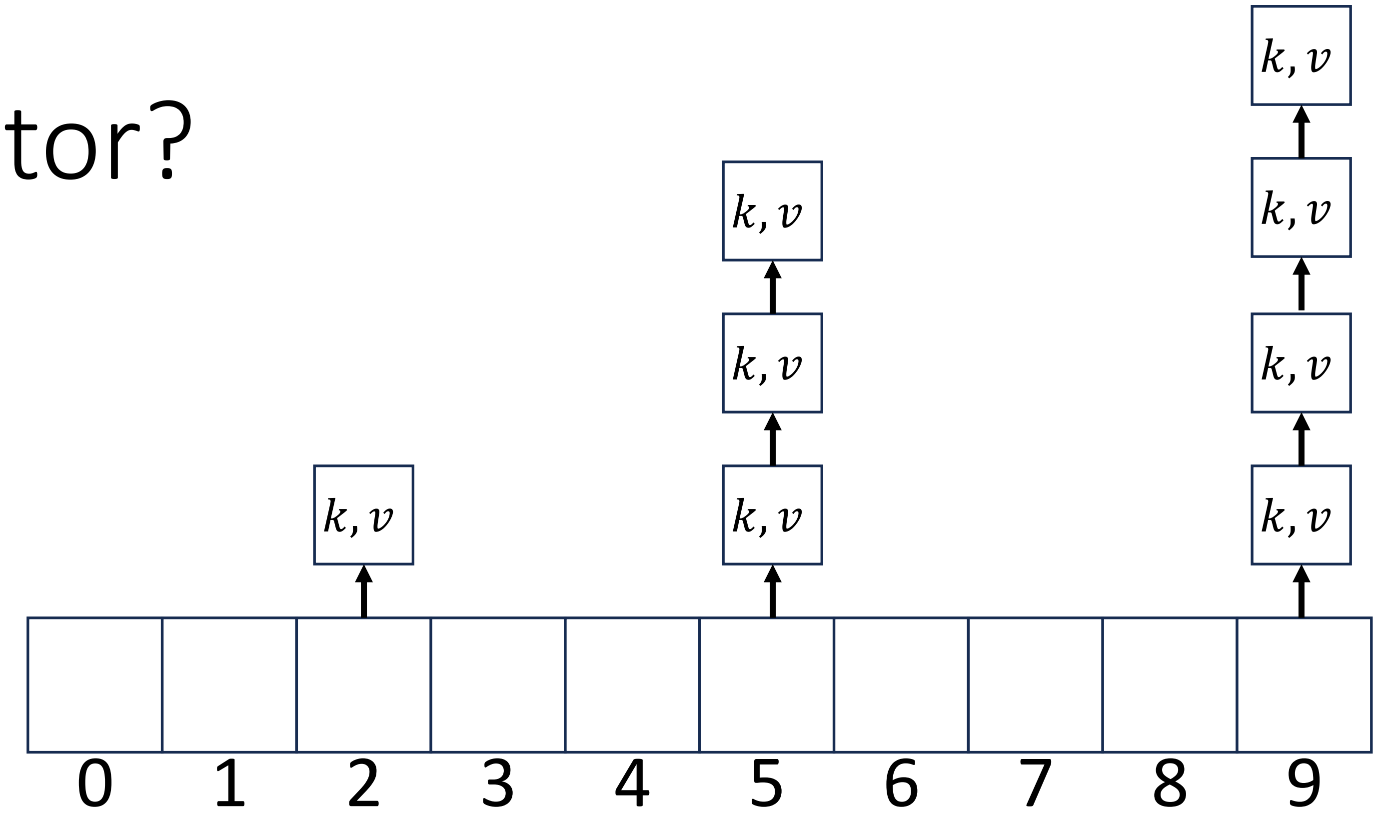
Formal Running Time Analysis

- The **load factor** of a hash table represents the average number of items per "bucket"
  - $\lambda = \frac{n}{length}$
- Assume we have a has table that uses a linked-list for separate chaining
  - What is the expected number of comparisons needed in an unsuccessful find?
  - What is the expected number of comparisons needed in a successful find?
- How can we make the expected running time $\Theta(1)$?

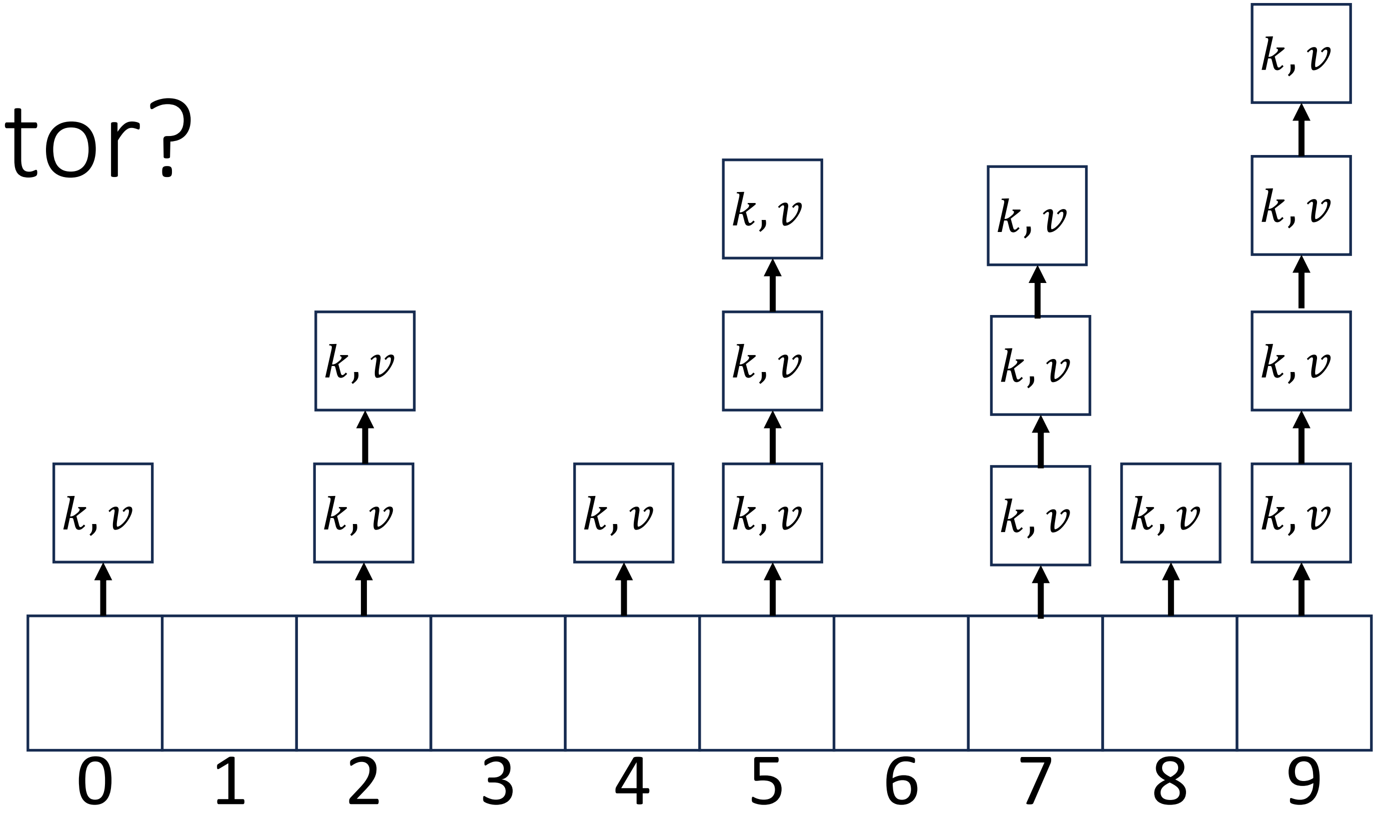
Load Factor?



Load Factor?



Load Factor?



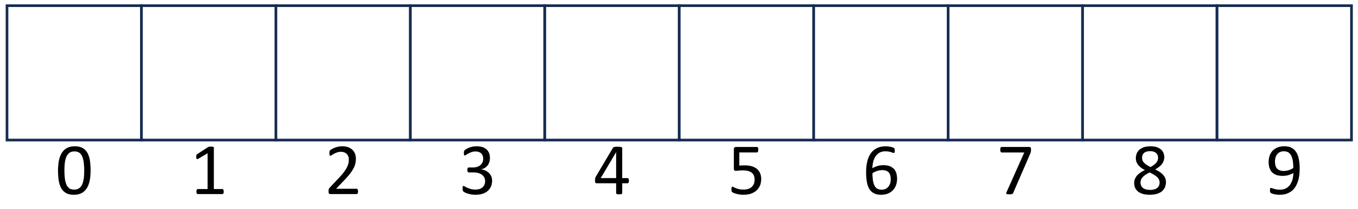
Collision Resolution: Linear Probing

- When there’s a collision, use the next open space in the table

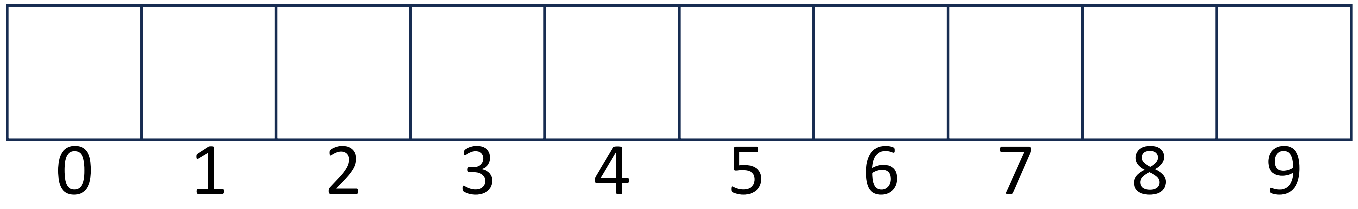
0	1	2	3	4	5	6	7	8	9

Linear Probing: Insert Procedure

- To insert k, v
  - Calculate $i = h(k)\ \%\ length$
  - If $table[i]$ is occupied then try $(i + 1)\%\ length$
  - If that is occupied try $(i + 2)\%\ length$
  - If that is occupied try $(i + 3)\%\ length$
  - …



Linear Probing: Find



Linear Probing: Find

- To find key k
  - Calculate $i = h(k)\ \%\ length$
  - If $table[i]$ is occupied and does not contain k then look at $(i + 1)\ \%\ length$
  - If that is occupied and does not contain k then look at $(i + 2)\ \%\ length$
  - If that is occupied and does not contain k then look at $(i + 3)\ \%\ length$
  - Repeat until you either find k or else you reach an empty cell in the table

0	1	2	3	4	5	6	7	8	9

Linear Probing: Delete

- Suppose A, B, C, D, and E all hashed to 3
- Now let's delete B

Before:

			A	B	C	D	E		
0	1	2	3	4	5	6	7	8	9

After:

0	1	2	3	4	5	6	7	8	9

Linear Probing: Delete

- Suppose A, B, and E all hashed to 3, and C and D hashed to 5
- Now let's delete B

Before:

			A	B	C	D	E		
0	1	2	3	4	5	6	7	8	9

After:

0	1	2	3	4	5	6	7	8	9

Linear Probing: Delete

- Suppose A and E hashed to 3, and B,C, and D hashed to 4
- Now let's delete B

Before:

			A	B	C	D	E		
0	1	2	3	4	5	6	7	8	9

After:

0	1	2	3	4	5	6	7	8	9

Linear Probing: Delete

- Let’s do this together!

0	1	2	3	4	5	6	7	8	9

Linear Probing: Delete

- To delete key k, where $h(k) = i$
  - Assume it is present
- Beginning at index i, probe until we find k (call this location index j)
- Mark j as empty (e.g. null), then continue probing while doing the following until you find another empty index
  - If you come across a key which hashes to a value $\leq j$ then move that item to index j and update j.

0	1	2	3	4	5	6	7	8	9

Linear Probing: Delete

- Option 1: Fill in with items that hashed to before the empty slot
- Option 2: “Tombstone” deletion. Leave a special object that indicates an object was deleted from there
  - The tombstone does not act as an open space when finding (so keep looking after its reached)
  - When inserting you can replace a tombstone with a new item

k, v		k, v	k, v		k, v				
0	1	2	3	4	5	6	7	8	9

Linear Probing + Tombstone: Find

- To find key k
  - Calculate $i = h(k)\ \%\ length$
  - While $table[i]$ has a tombstone or a key other than k, $i = (i + 1)\ \%\ length$
  - If you come across k return $table[i]$
  - If you come across an empty index, the find was unsuccessful

0	1	2	3	4	5	6	7	8	9

Linear Probing + Tombstone: Insert

- To insert k, v
  - Calculate $i = h(k)\ \%\ length$
  - While $table[i]$ has a key other than k, $i = (i+1)\ \%\ length$
    - If $table[i]$ has a tombstone, set $x = i$
      - That is where we will insert if the find is unsuccessful
  - If you come across k, set $table[i] = k, v$
  - If you come across an empty index, the find was unsuccessful
    - Set $table[x] = k, v$ if we saw a tombstone
    - Set $table[i] = k, v$ otherwise

0	1	2	3	4	5	6	7	8	9