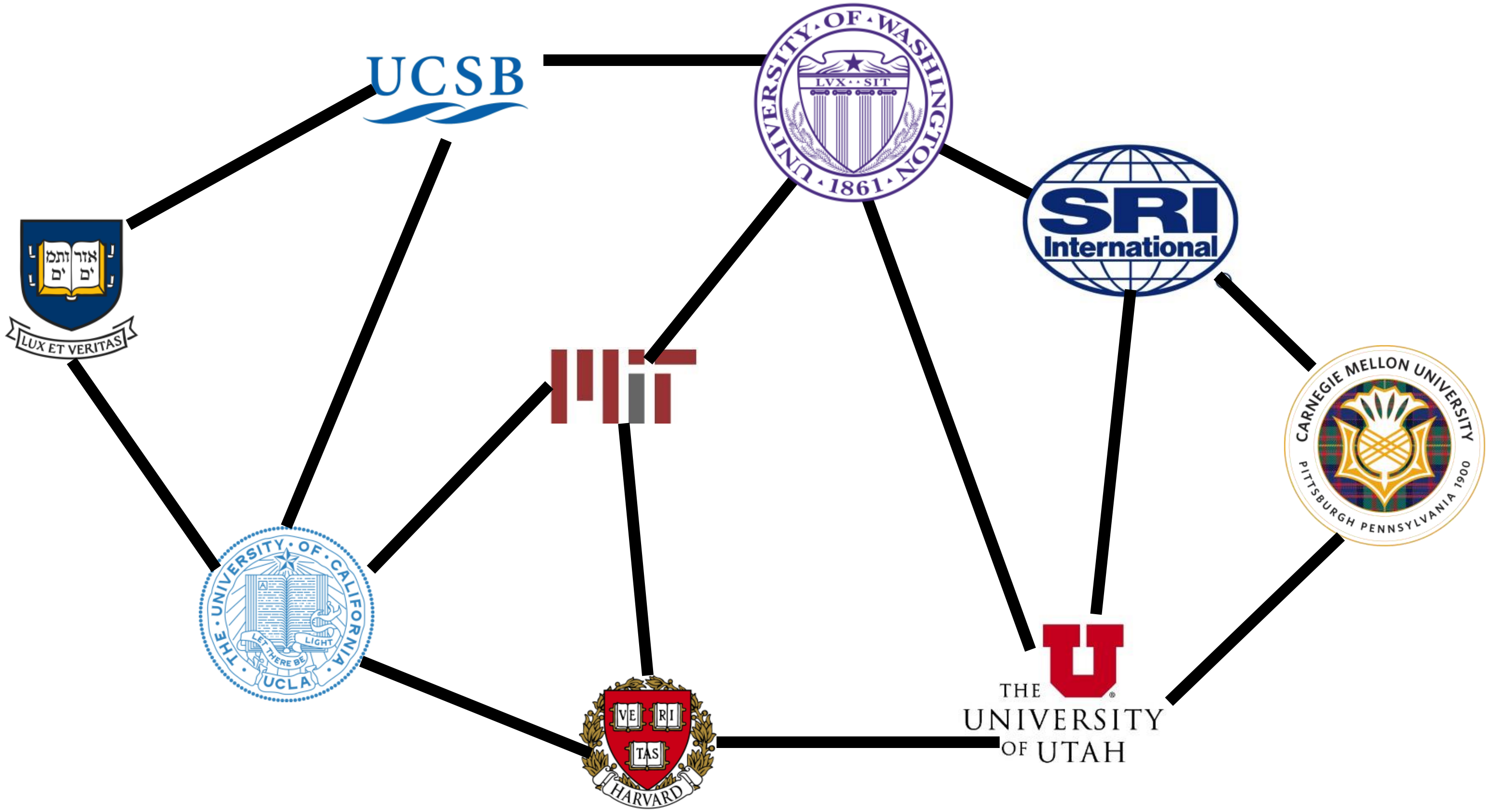
CSE 332 Summer 2024
Lecture 14: Graphs

Nathan Brunelle

http://www.cs.uw.edu/332

ARPANET



UCSB
UNIVERSITY OF WASHINGTON
LVX SIT
1861
SRI
International
LUX ET VERITAS
THE UNIVERSITY OF CALIFORNIA
LET THERE BE LIGHT
UCLA
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HARVARD
THE UNIVERSITY OF UTAH
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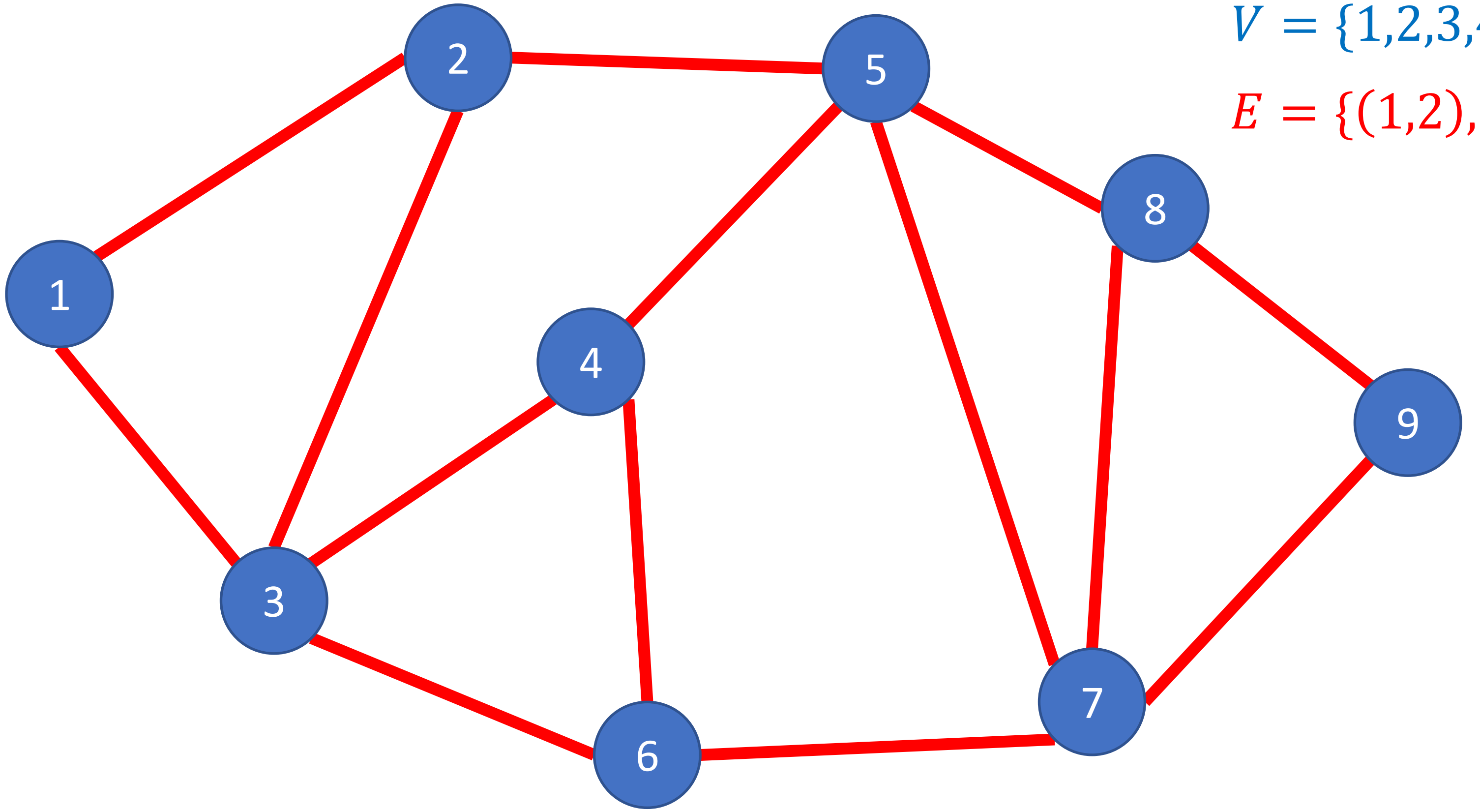
Undirected Graphs

Vertices/Nodes

Definition: $G = (V, E)$

Edges



$V = \{1,2,3,4,5,6,7,8,9\}$

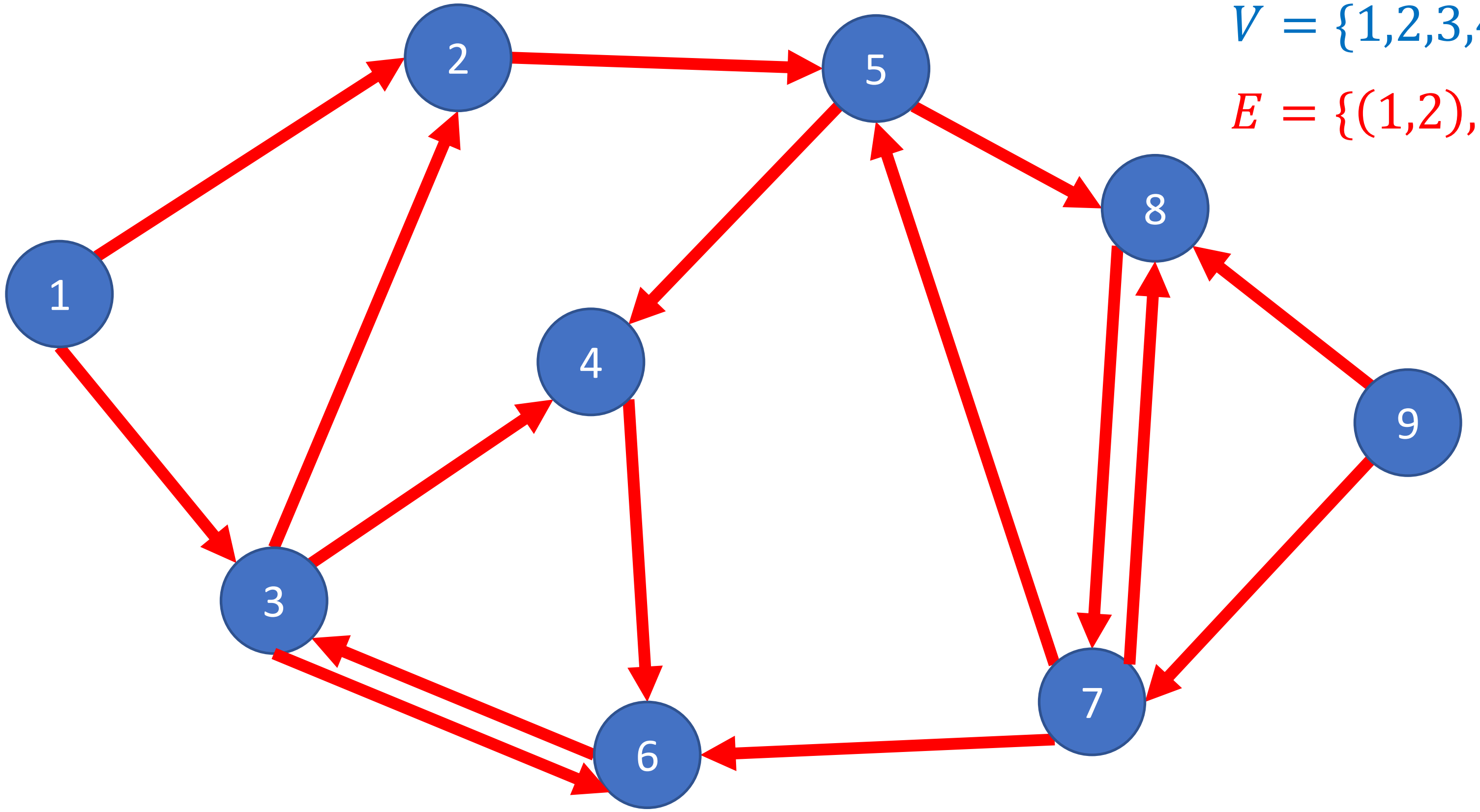
$E = \{(1,2), (2,3), (1,3), \dots\}$

Directed Graphs

Vertices/Nodes

Definition: $G = (V, E)$

Edges

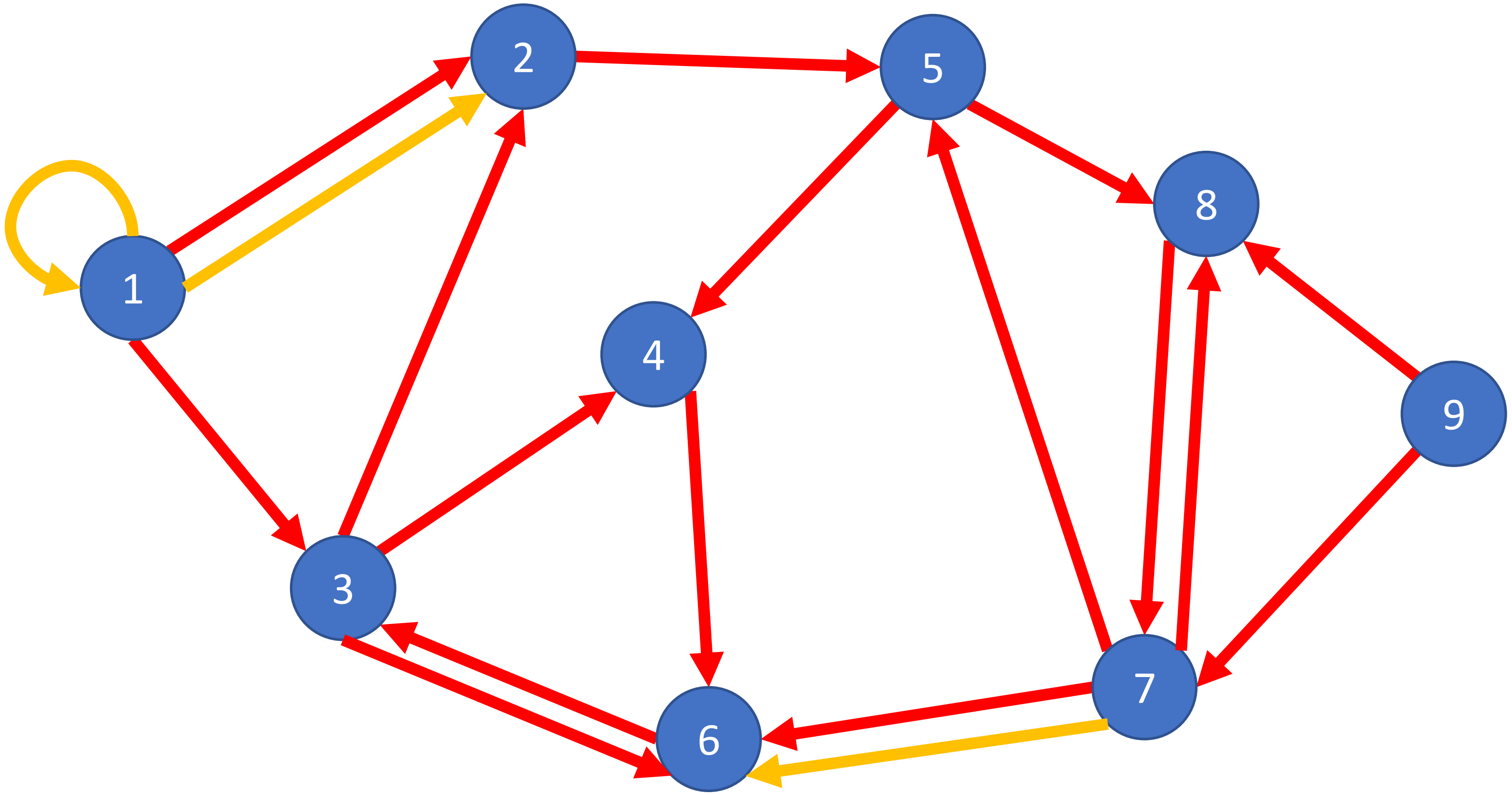


$V = \{1,2,3,4,5,6,7,8,9\}$

$E = \{(1,2), (2,3), (1,3), \ldots\}$

Self-Edges and Duplicate Edges

Some graphs may have duplicate edges (e.g. here we have the edge (1,2) twice).
Some may also have self-edges (e.g. here there is an edge from 1 to 1).
Graph with Neither self-edges nor duplicate edges are called **simple graphs**



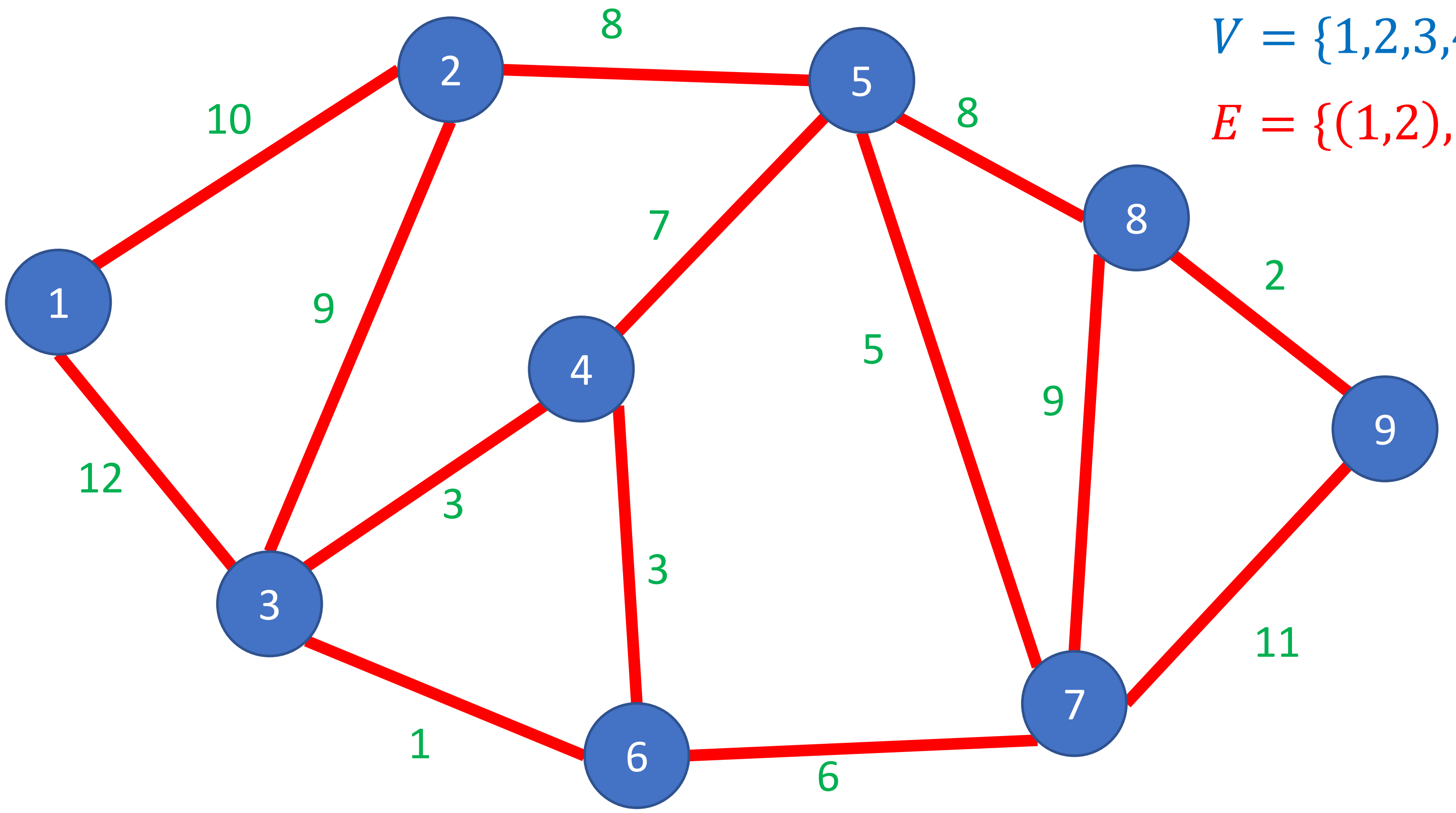
Weighted Graphs

Vertices/Nodes

Definition: $G = (V, E)$

Edges

$w(e)$ = weight of edge e



$V = \{1,2,3,4,5,6,7,8,9\}$

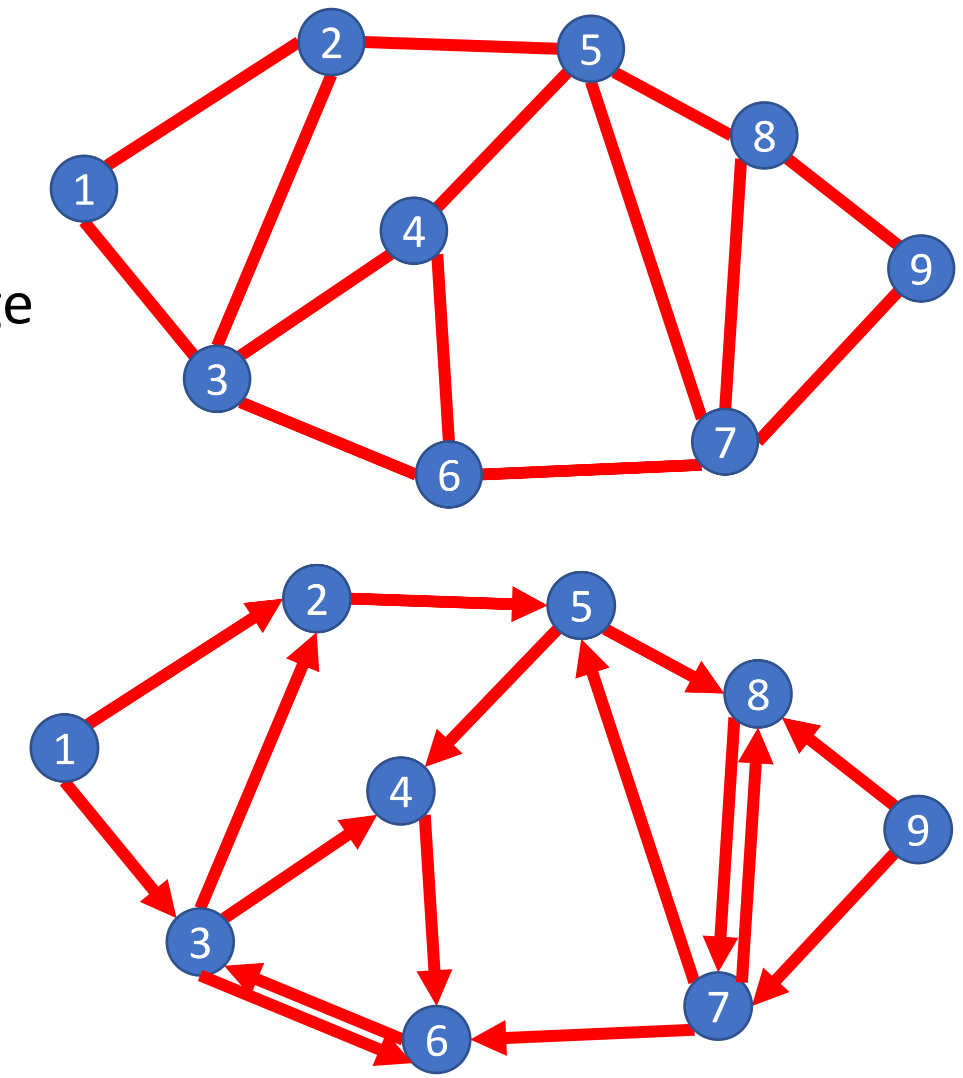
$E = \{(1,2), (2,3), (1,3), \ldots\}$

Graph Applications

- For each application below, consider:
  - What are the nodes, what are the edges?
  - Is the graph directed?
  - Is the graph simple?
  - Is the graph weighted?
- Facebook friends
- Twitter followers
- Java inheritance
- Airline Routes

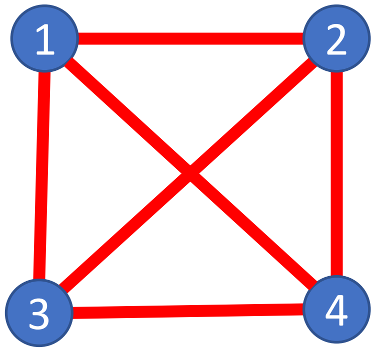
Some Graph Terms

- Adjacent/Neighbors
  - Nodes are adjacent/neighbors if they share an edge
- Degree
  - Number of edges “touching” a vertex
- Indegree
  - Number of incoming edges
- Outdegree
  - Number of outgoing edges

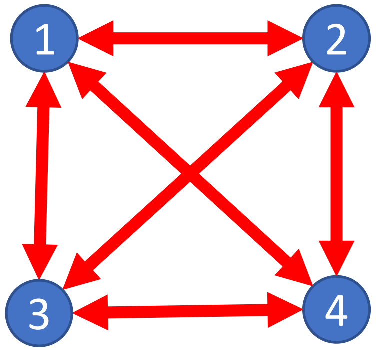


Definition: Complete Graph

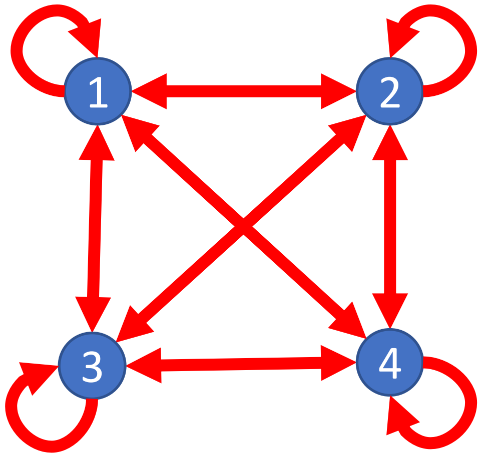
A Graph $G = (V, E)$ s.t. for any pair of nodes $v_1, v_2 \in V$ there is an edge from v_1 to v_2



Complete
Undirected Graph



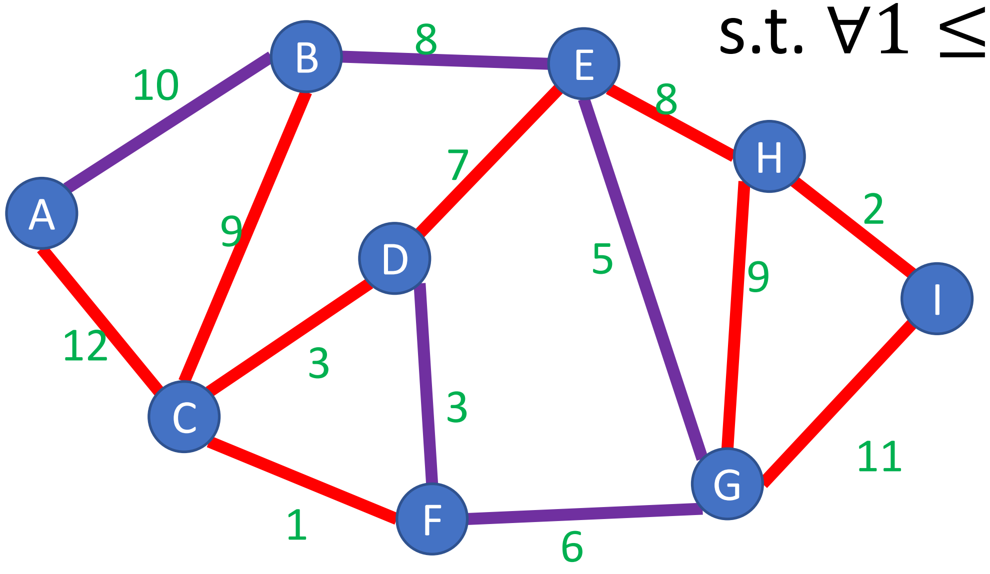
Complete
Directed Graph



Complete Directed
Non-simple Graph

Definition: Path

A sequence of nodes $(v_1, v_2, \dots, v_k)$
s.t. $\forall 1 \leq i \leq k-1,\ (v_i, v_{i+1}) \in E$

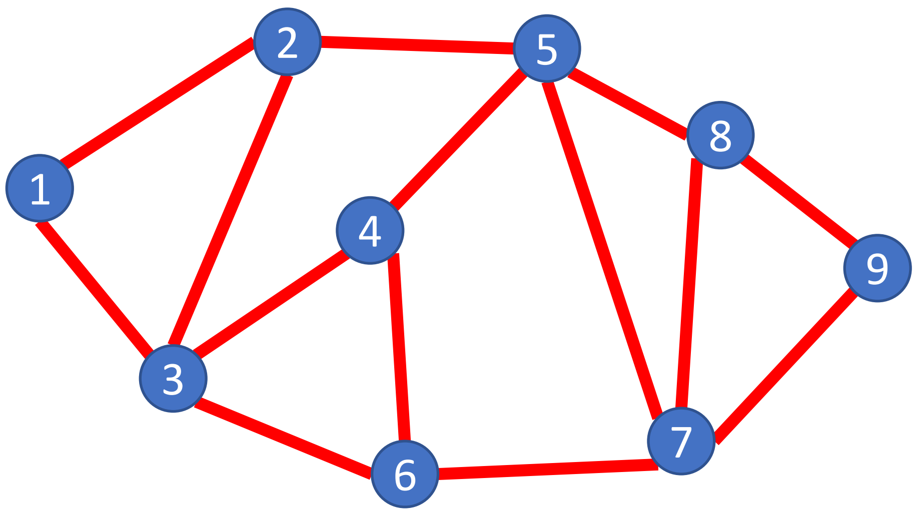


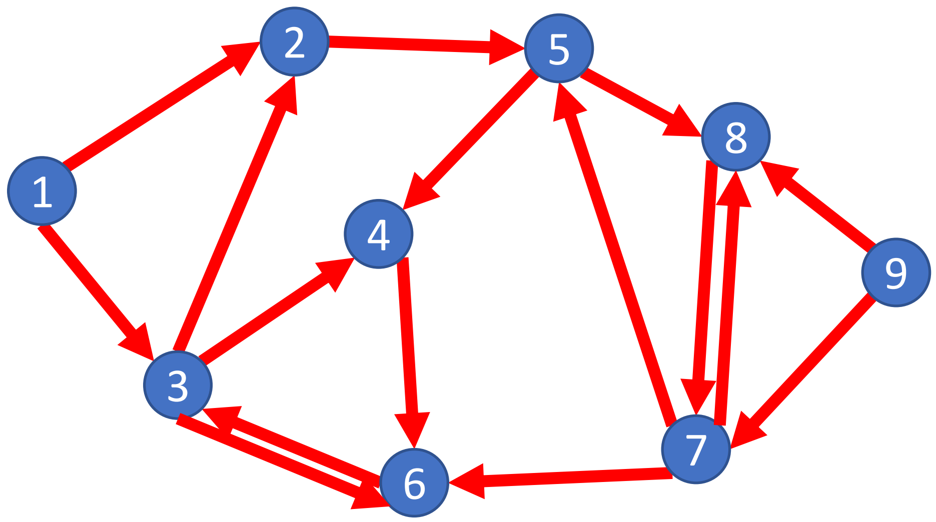
<u>Simple Path:</u>
A path in which each node appears at most once

<u>Cycle:</u>
A path which starts and ends in the same place

Definition: (Strongly) Connected Graph

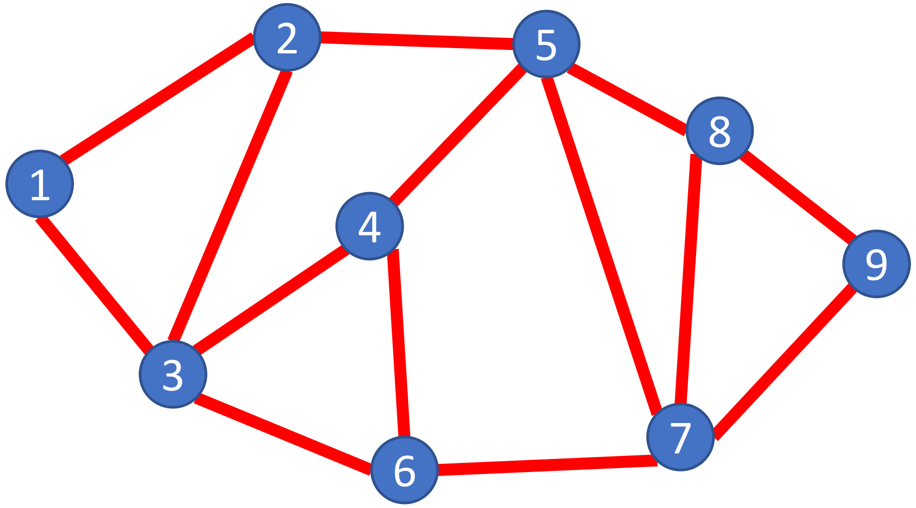
A Graph $G = (V, E)$ s.t. for any pair of nodes $v_1, v_2 \in V$ there is a path from v_1 to v_2



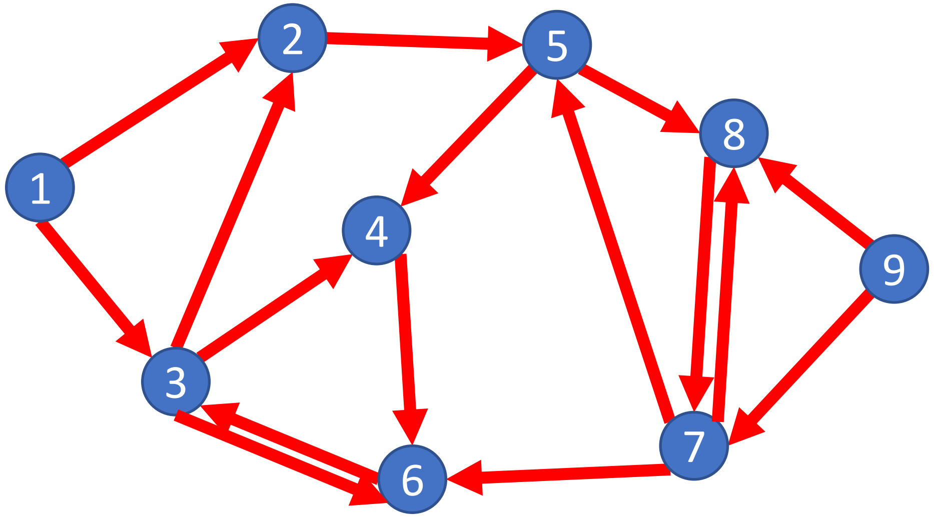


Definition: (Strongly) Connected Graph

A Graph $G = (V, E)$ s.t. for any pair of nodes $v_1, v_2 \in V$ there is a path from v_1 to v_2



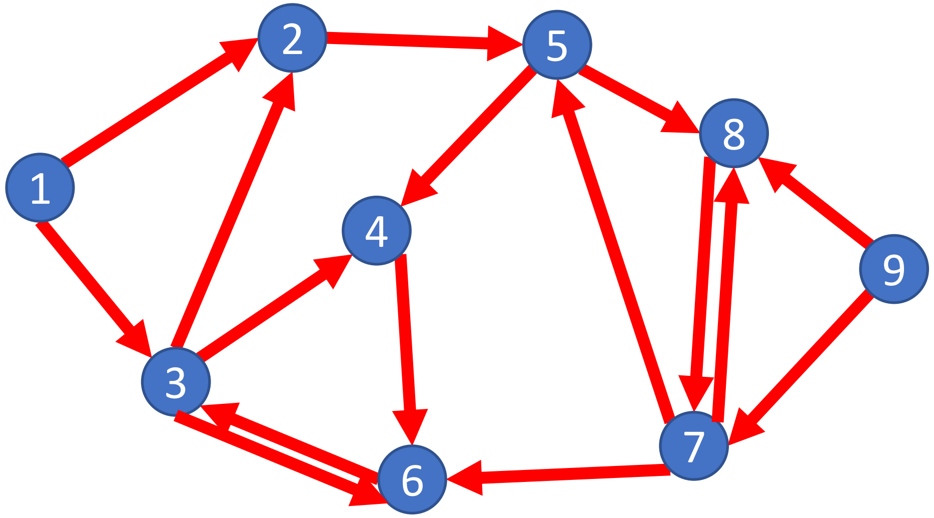
Connected



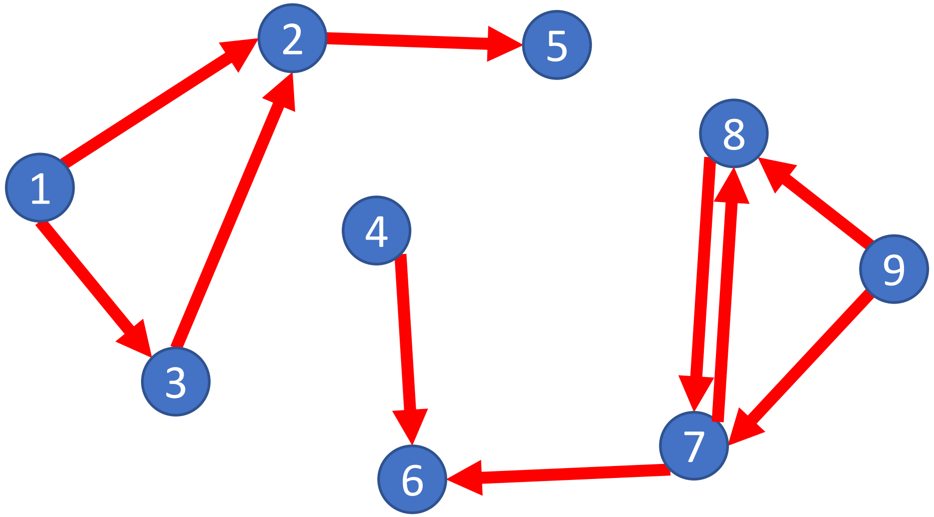
Not (strongly) Connected

Definition: Weakly Connected Graph

A Graph $G = (V, E)$ s.t. for any pair of nodes $v_1, v_2 \in V$ there is a path from v_1 to v_2 ignoring direction of edges



Weakly Connected



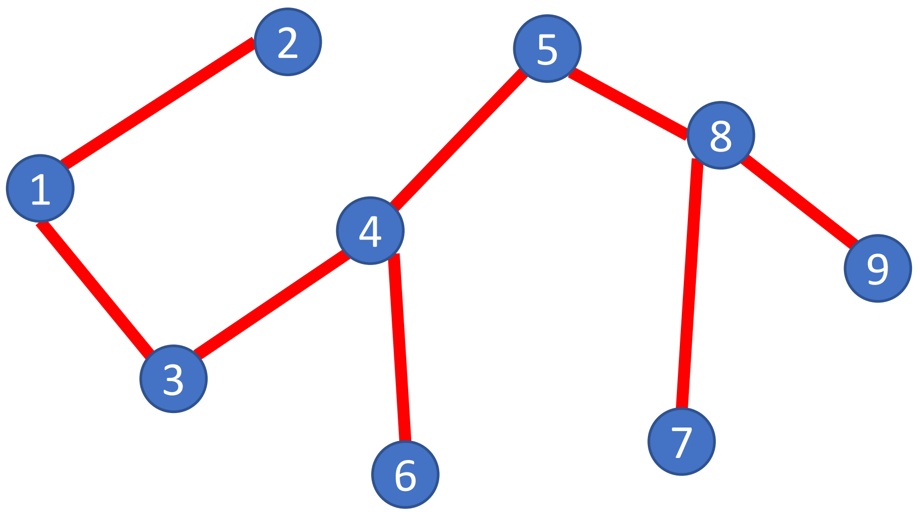
Not Weakly Connected

Graph Density, Data Structures, Efficiency

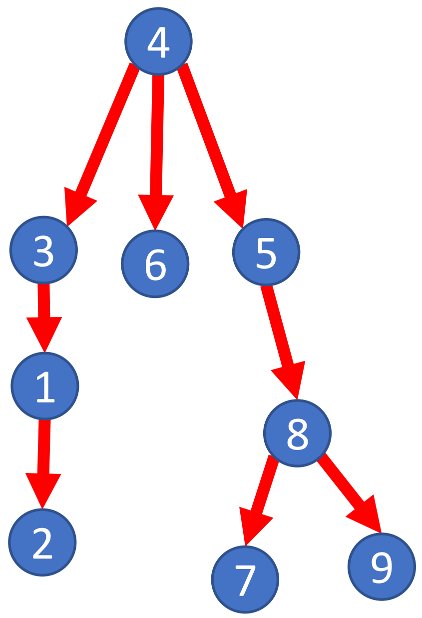
- The maximum number of edges in a graph is $\Theta(|V|^2)$:
  - Undirected and simple: $\frac{|V|(|V|-1)}{2}$
  - Directed and simple: $|V|(|V| - 1)$
  - Direct and non-simple (but no duplicates): $|V|^2$
- If the graph is connected, the minimum number of edges is $|V| - 1$
- If $|E| \in \Theta(|V|^2)$ we say the graph is **dense**
- If $|E| \in \Theta(|V|)$ we say the graph is **sparse**
- Because $|E|$ is not always near to $|V|^2$ we do not typically substitute $|V|^2$ for $|E|$ in running times, but leave it as a separate variable
  - However, $\log(|E|) \in \Theta(\log(|V|))$

Definition: Tree

A Graph $G = (V, E)$ is a tree if it is undirect, connected, and has no cycles (i.e. is acyclic). Often one node is identified as the "root"



A Tree

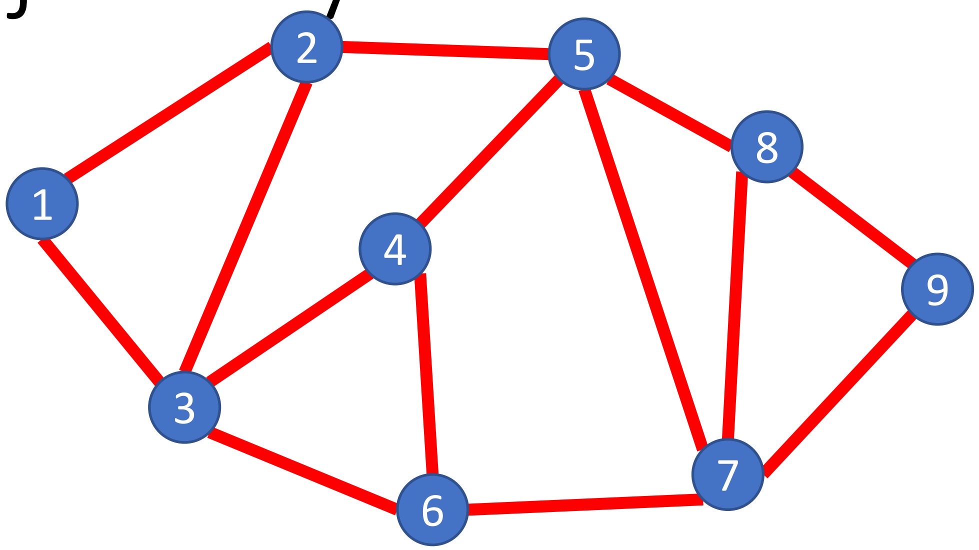


A Rooted Tree

Graph Operations

- To represent a Graph (i.e. build a data structure) we need:
  - Add Edge
  - Remove Edge
  - Check if Edge Exists
  - Get Neighbors (incoming)
  - Get Neighbors (outgoing)

Adjacency List



1	2	3		
2	1	3	5	
3	1	2	4	6
4	3	5	6	
5	2	4	7	8
6	3	4	7	
7	5	6	8	9
8	5	7	9	
9	7	8		

<u>Time/Space Tradeoffs</u>

Space to represent: $\Theta(n+m)$

Add Edge (v,w): $\Theta(\deg(v))$

Remove Edge (v,w): $\Theta(\deg(v))$

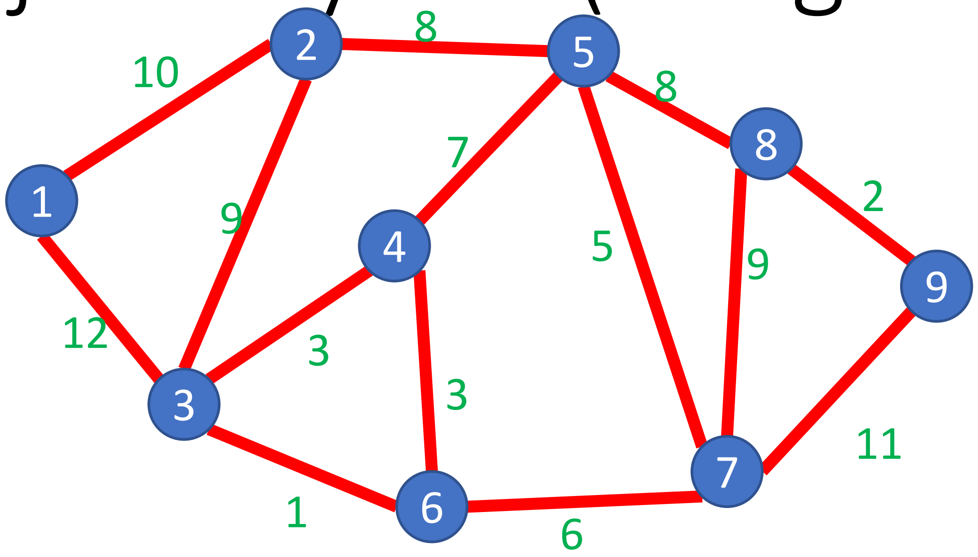
Check if Edge (v,w) Exists: $\Theta(\deg(v))$

Get Neighbors (incoming): $\Theta(n+m)$

Get Neighbors (outgoing): $\Theta(\deg(v))$

$|V| = n$
$|E| = m$

Adjacency List (Weighted)



1	2	3		
2	1	3	5	
3	1	2	4	6
4	3	5	6	
5	2	4	7	8
6	3	4	7	
7	5	6	8	9
8	5	7	9	
9	7	8		

Time/Space Tradeoffs

Space to represent: $\Theta(n+m)$

Add Edge (v,w): $\Theta(\deg(v))$

Remove Edge (v,w): $\Theta(\deg(v))$

Check if Edge (v,w) Exists: $\Theta(\deg(v))$

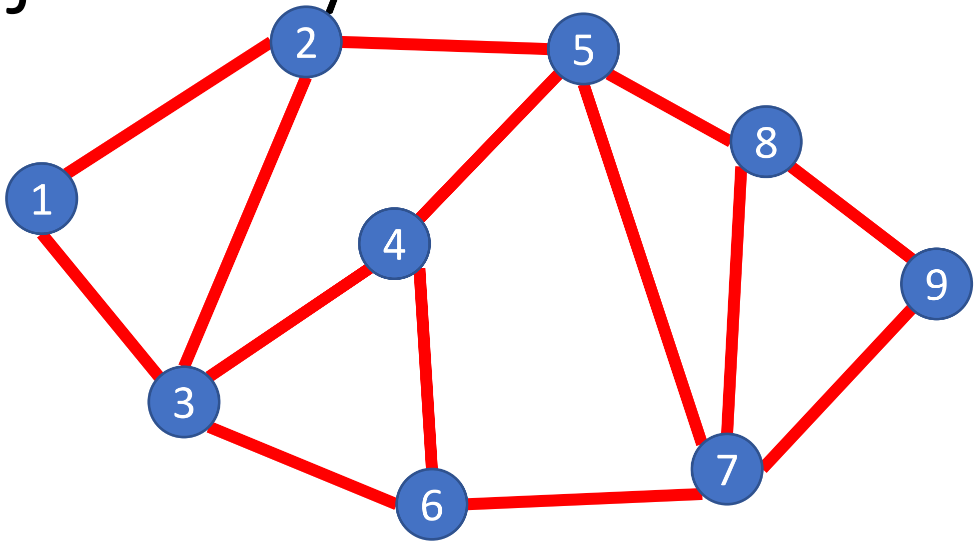
Get Neighbors (incoming): $\Theta(n+m)$

Get Neighbors (outgoing): $\Theta(\deg(v))$

$|V| = n$

$|E| = m$

Adjacency Matrix



	A	B	C	D	E	F	G	H	I
A		1	1						
B	1		1		1				
C	1	1		1		1			
D			1		1	1			
E		1		1			1	1	
F			1	1			1		
G					1	1		1	1
H					1		1		1
I							1	1	

<u>Time/Space Tradeoffs</u>

Space to represent: $\Theta(?)$

Add Edge (v, w): $\Theta(?)$

Remove Edge (v, w): $\Theta(?)$

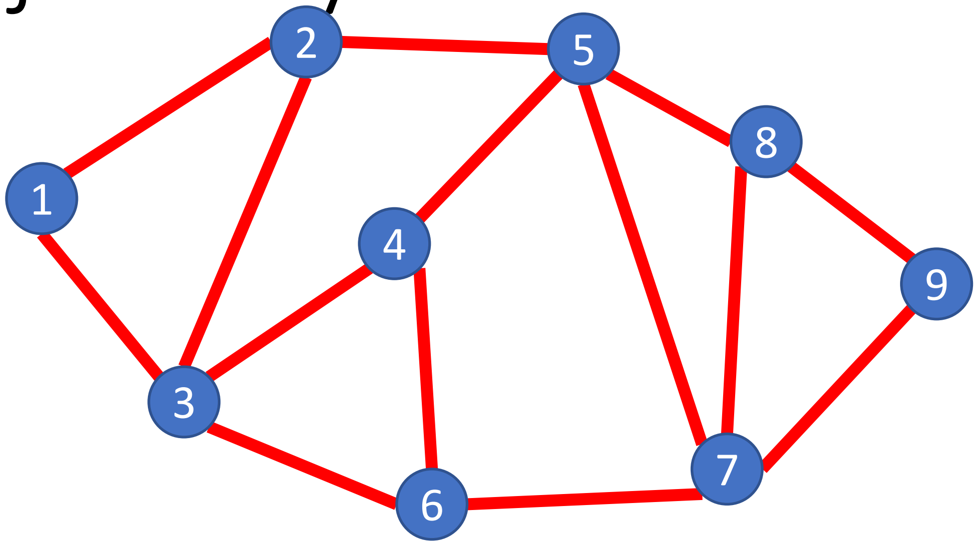
Check if Edge (v, w) Exists: $\Theta(?)$

Get Neighbors (incoming): $\Theta(?)$

Get Neighbors (outgoing): $\Theta(?)$

$|V| = n$
$|E| = m$

Adjacency Matrix



	A	B	C	D	E	F	G	H	I
A		1	1						
B	1		1		1				
C	1	1		1		1			
D			1		1	1			
E		1		1			1	1	
F			1	1			1		
G					1	1		1	1
H					1		1		1
I							1	1	

<u>Time/Space Tradeoffs</u>

Space to represent: $\Theta(n^2)$

Add Edge (v, w): $\Theta(1)$

Remove Edge (v, w): $\Theta(1)$

Check if Edge (v, w) Exists: $\Theta(1)$

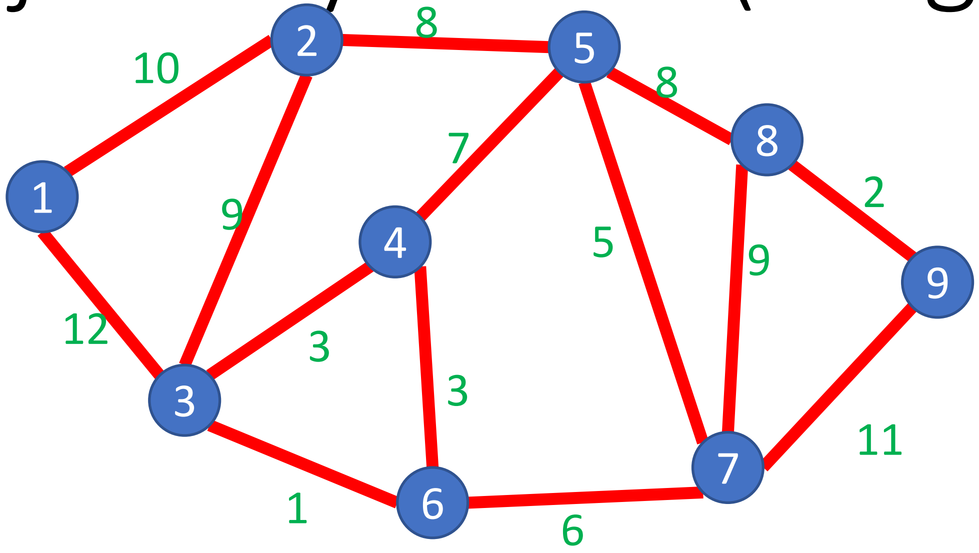
Get Neighbors (incoming): $\Theta(n)$

Get Neighbors (outgoing): $\Theta(n)$

$|V| = n$

$|E| = m$

Adjacency Matrix (weighted)



	A	B	C	D	E	F	G	H	I
A		1	1						
B	1		1		1				
C	1	1		1		1			
D			1		1	1			
E		1		1			1	1	
F			1	1			1		
G					1	1		1	1
H					1		1		1
I							1	1	

<u>Time/Space Tradeoffs</u>

Space to represent: $\Theta(n^2)$

Add Edge (v, w): $\Theta(1)$

Remove Edge (v, w): $\Theta(1)$

Check if Edge (v, w) Exists: $\Theta(1)$

Get Neighbors (incoming): $\Theta(n)$

Get Neighbors (outgoing): $\Theta(n)$

$|V| = n$

$|E| = m$

Comparison

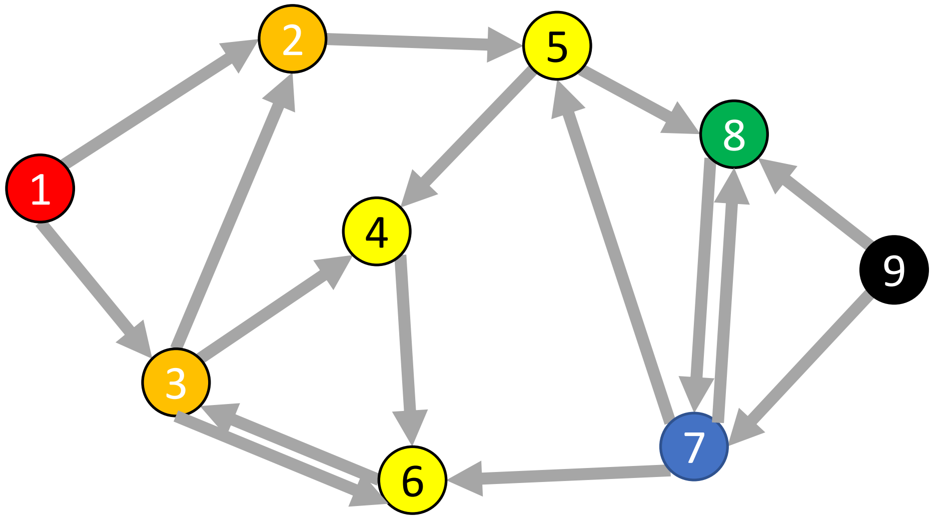
- Adjacency List:
  - Less memory when $|E| < |V|^2$
  - Operations with running time linear in degree of source node
    - Add an edge
    - Remove an edge
    - Check for edge
    - Get neighbors
- Adjacency Matrix:
  - Similar amount of memory when $|E| \approx |V|^2$
  - Constant time operations:
    - Add an edge
    - Remove an edge
    - Check for an edge
  - Operations running with linear time in $|V|$
    - Get neighbors

Adjacency List is more common in practice:

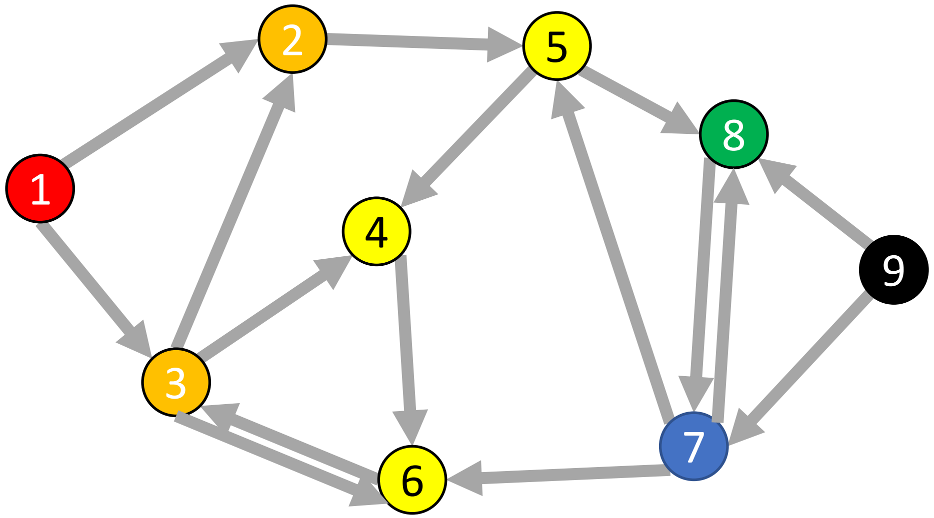
- Most graphs have $|E| \ll |V|^2$
  - Saves memory
  - Most nodes will have small degree
- Getting neighbors is a common operation
- Adjacency Matrix may be better if the graph is "dense" or if its edges change a lot

Breadth-First Search

- Input: a node s
- Behavior: Start with node s, visit all neighbors of s, then all neighbors of neighbors of s, …
- Visits every node reachable from s in order of distance
- Output:
  - How long is the shortest path?
  - Is the graph connected?



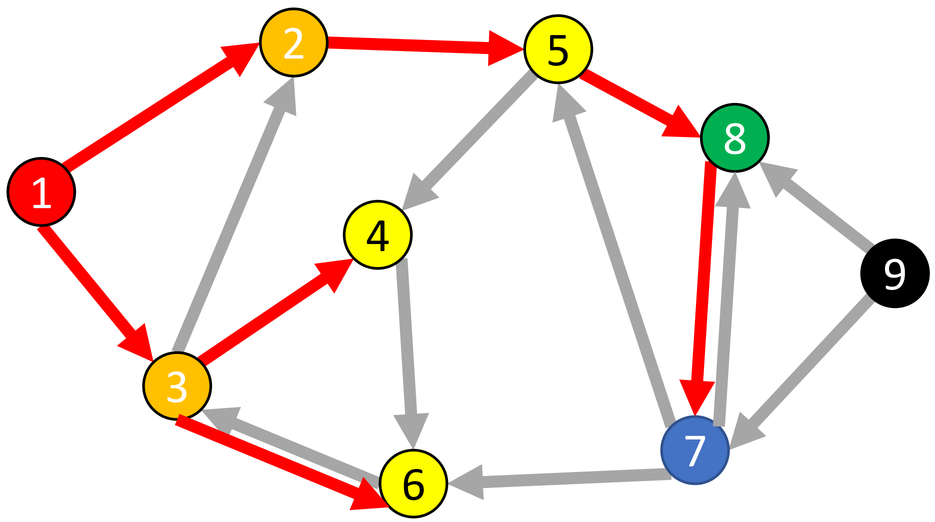
BFS



Running time: $\Theta(|V| + |E|)$

```
void bfs(graph, s){
    found = new Queue();
    found.enqueue(s);
    mark s as “visited”;
    While (!found.isEmpty()){
        current = found.dequeue();
        for (v : neighbors(current)){
            if (! v marked “visited”){
                mark v as “visited”;
                found.enqueue(v);
            }
        }
    }
}
```

Shortest Path (unweighted)



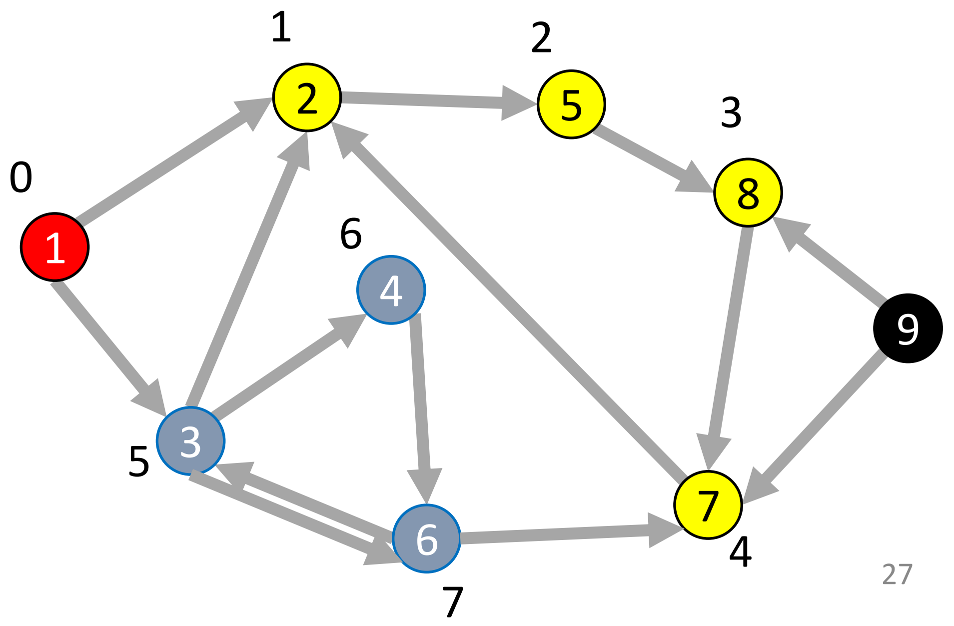
Idea: when it's seen, remember its "layer" depth!

```
int shortestPath(graph, s, t){
        found = new Queue();
        layer = 0;
        found.enqueue(s);
        mark s as "visited";
        While (!found.isEmpty()){
                current = found.dequeue();
                layer = depth of current;
                for (v : neighbors(current)){
                        if (! v marked "visited"){
                                mark v as "visited";
                                depth of v = layer + 1;
                                found.enqueue(v);
                        }
                }
        }
        return depth of t;
}
```

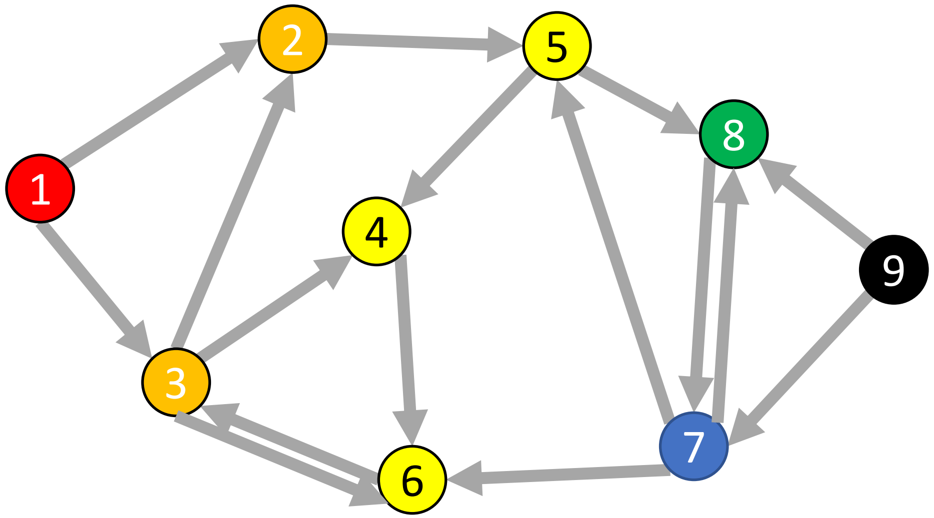
Depth-First Search

Depth-First Search

- Input: a node s
- Behavior: Start with node s, visit one neighbor of s, then all nodes reachable from that neighbor of s, then another neighbor of s,…
  - Before moving on to the second neighbor of s, visit everything reachable from the first neighbor of s
- Output:
  - Does the graph have a cycle?
  - A **topological sort** of the graph.



DFS (non-recursive)

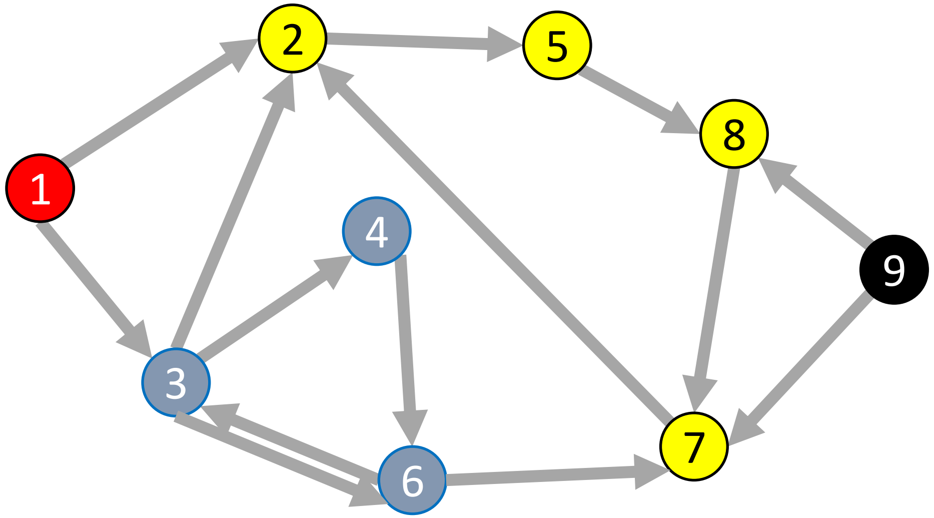


Running time: $\Theta(|V| + |E|)$

```
void dfs(graph, s){
        found = new Stack();
        found.pop(s);
        mark s as "visited";
        While (!found.isEmpty()){
                current = found.pop();
                for (v : neighbors(current)){
                        if (! v marked "visited"){
                                mark v as "visited";
                                found.push(v);
                        }
                }
        }
}
```

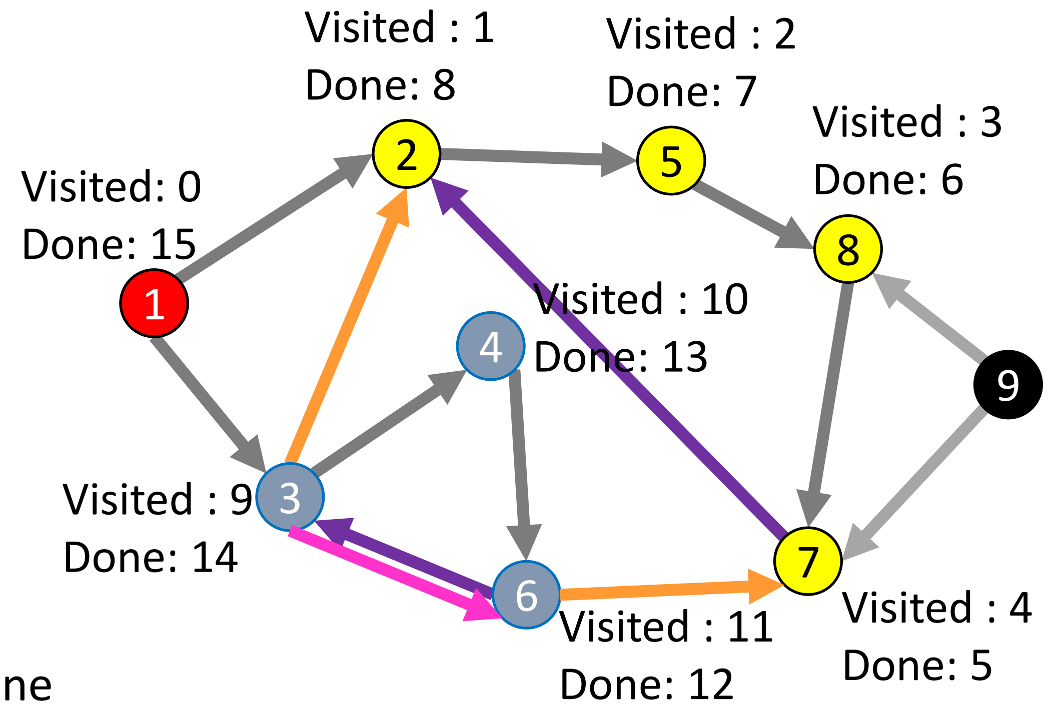
DFS Recursively (more common)

```
void dfs(graph, curr){
        mark curr as “visited”;
        for (v : neighbors(current)){
                if (! v marked “visited”){
                        dfs(graph, v);
                }
        }
        mark curr as “done”;
}
```



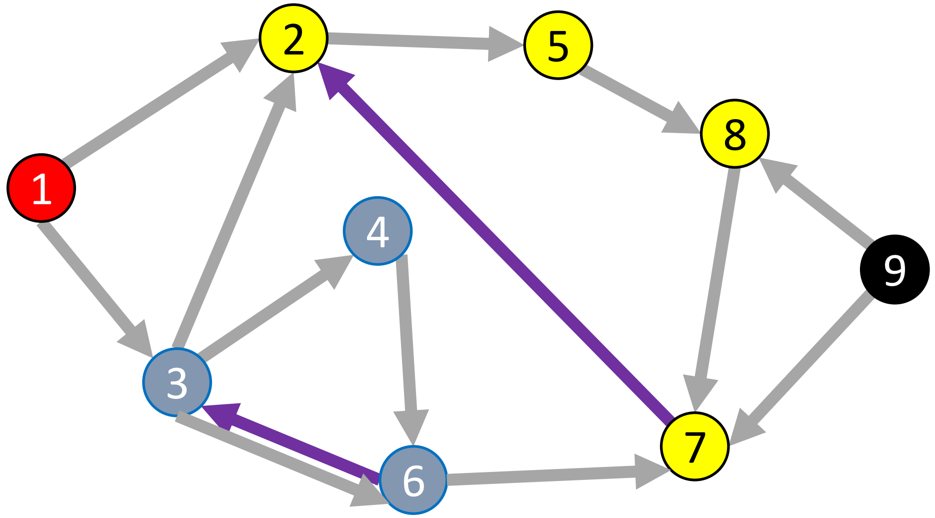
Using DFS

- Consider the "visited times" and "done times"
- Edges can be categorized:
  - Tree Edge
    - (a, b) was followed when pushing
    - (a, b) when b was unvisited when we were at a
  - Back Edge
    - (a, b) goes to an "ancestor"
    - a and b visited but not done when we saw (a, b)
    - $t_{visited}(b) < t_{visited}(a) < t_{done}(a) < t_{done}(b)$
  - Forward Edge
    - (a, b) goes to a "descendent"
    - b was visited and done between when a was visited and done
    - $t_{visited}(a) < t_{visited}(b) < t_{done}(b) < t_{done}(a)$
  - Cross Edge
    - (a, b) goes to a node that doesn't connect to a
    - b was seen and done before a was ever visited
    - $t_{done}(b) < t_{visited}(a)$



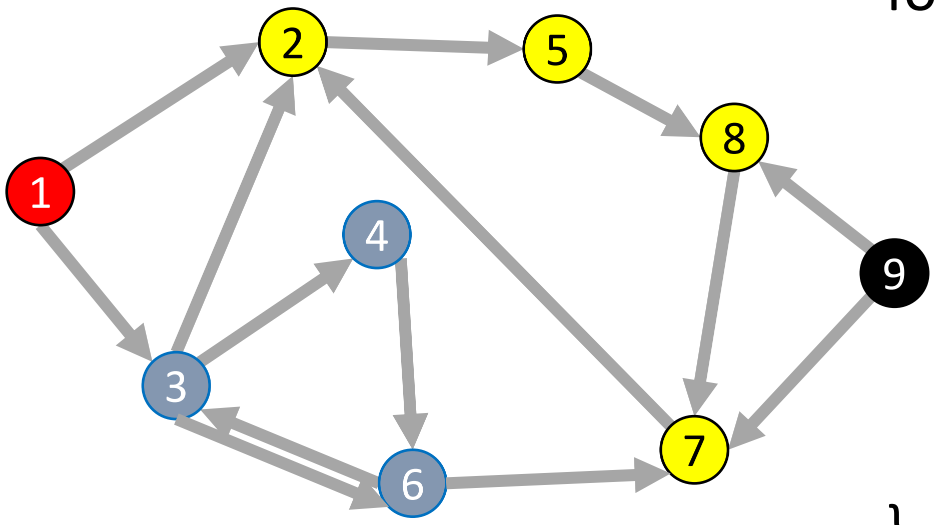
Back Edges

- Behavior of DFS:
  - “Visit everything reachable from the current node before going back”
- Back Edge:
  - The current node’s neighbor is an “in progress” node
  - Since that other node is “in progress”, the current node is reachable from it
  - The back edge is a path to that other node
  - **Cycle!**



Cycle Detection

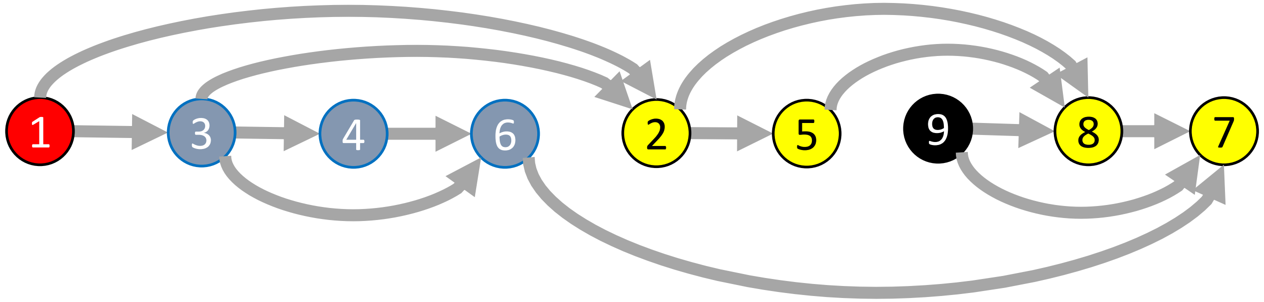
Idea: Look for a back edge!

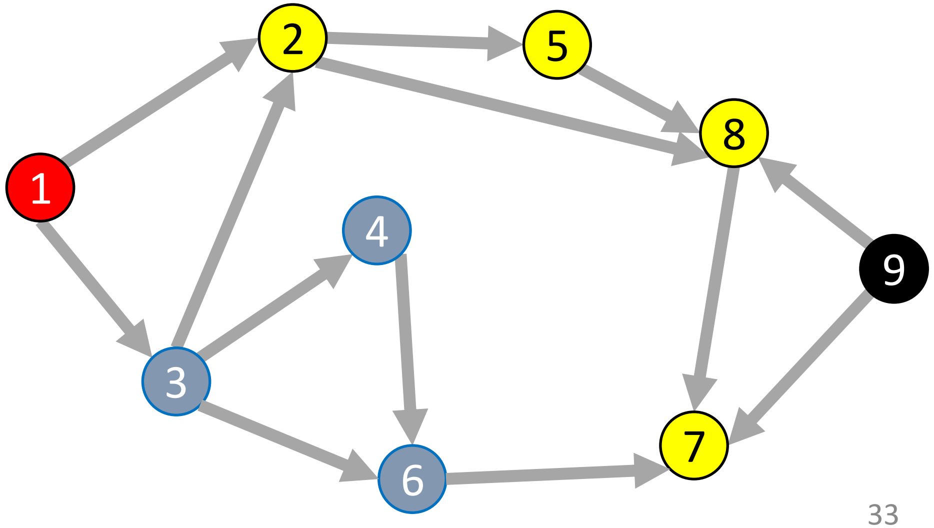



```
boolean hasCycle(graph, curr){
        mark curr as “visited”;
        cycleFound = false;
        for (v : neighbors(current)){
                if (v marked “visited” && ! v marked “done”){
                        cycleFound=true;
                }
                if (! v marked “visited” && !cycleFound){
                        cycleFound = hasCycle(graph, v);
                }
        }
        mark curr as “done”;
        return cycleFound;
}
```

Topological Sort

- A Topological Sort of a **directed acyclic graph** $\boldsymbol{G = (V, E)}$ is a permutation of V such that if $(u, v) \in E$ then u is before v in the permutation

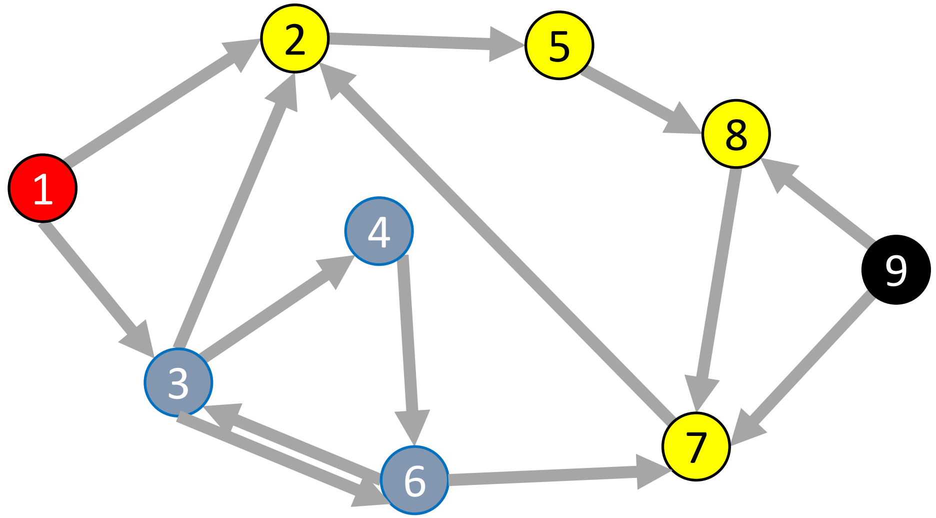




DFS Recursively

```
void dfs(graph, curr){
        mark curr as "visited";
        for (v : neighbors(current)){
                if (! v marked "visited"){
                        dfs(graph, v);
                }
        }
        mark curr as "done";
}
```

Idea: List in reverse order by "done" time

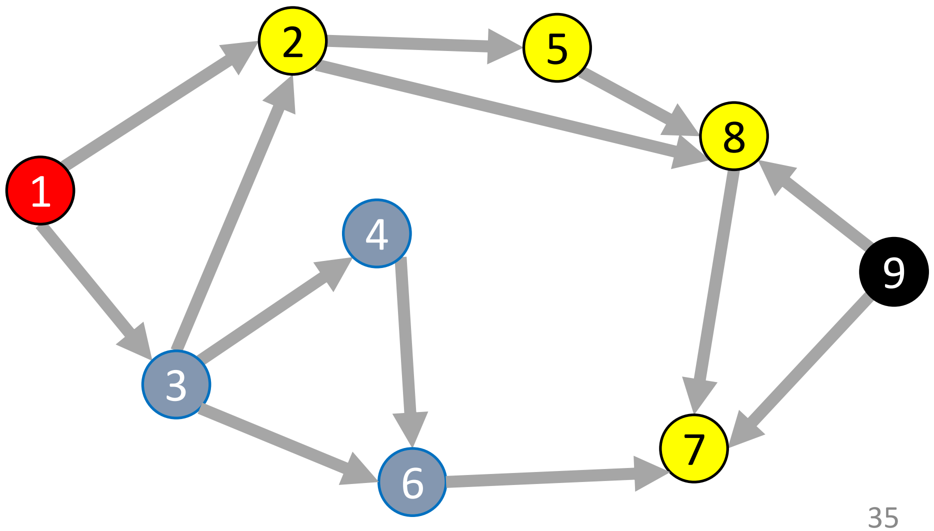


DFS: Topological sort

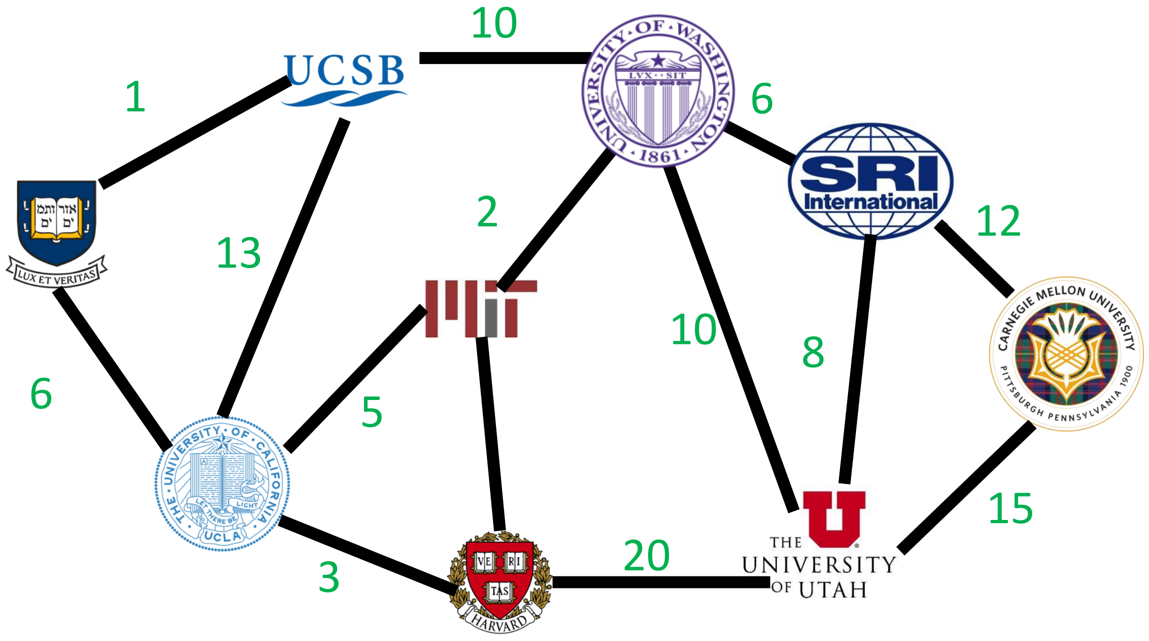
```
List topSort(graph){
        List<Nodes> done = new List<>();
        for (Node v : graph.vertices){
                if (!v.visited){
                        finishTime(graph, v, finished);
                }
        }
        done.reverse();
        return done;
}

void finishTime(graph, curr, finished){
        curr.visited = true;
        for (Node v : curr.neighbors){
                if (!v.visited){
                        finishTime(graph, v, finished);
                }
        }
        done.add(curr)
}
```

Idea: List in reverse order by "done" time

finished:



Single-Source Shortest Path



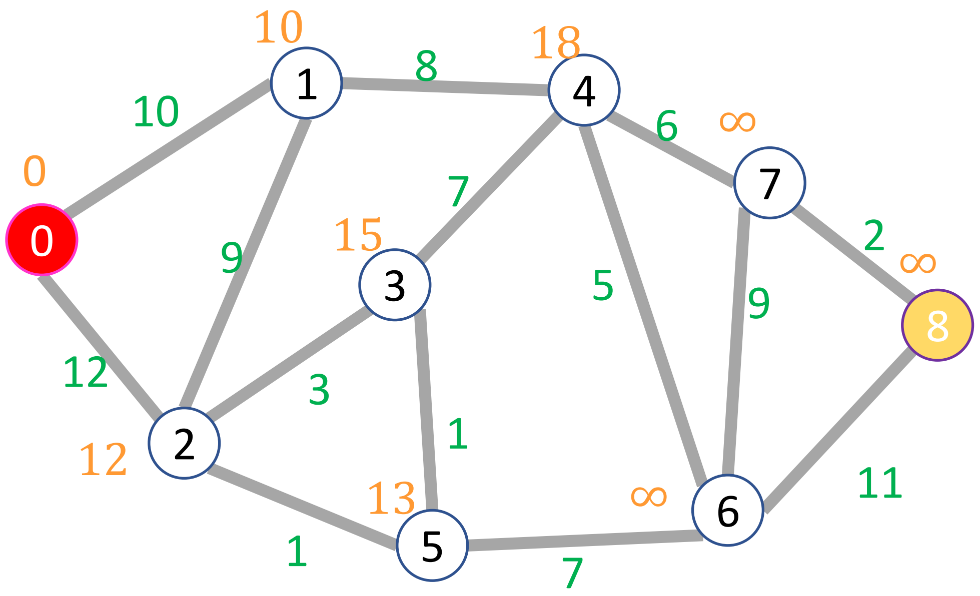
Find the quickest way to get from UVA to each of these other places

Given a graph $G = (V, E)$ and a start node $s \in V$, for each $v \in V$ find the least-weight path from $s \to v$ (call this weight $\delta(s, v)$)

(assumption: all edge weights are positive)

Dijkstra's Algorithm

- Input: graph with **no negative edge weights**, start node s, end node t
- Behavior: Start with node s, repeatedly go to the incomplete node "nearest" to s, stop when
- Output:
  - Distance from start to end
  - Distance from start to every node



Dijkstra’s Algorithm

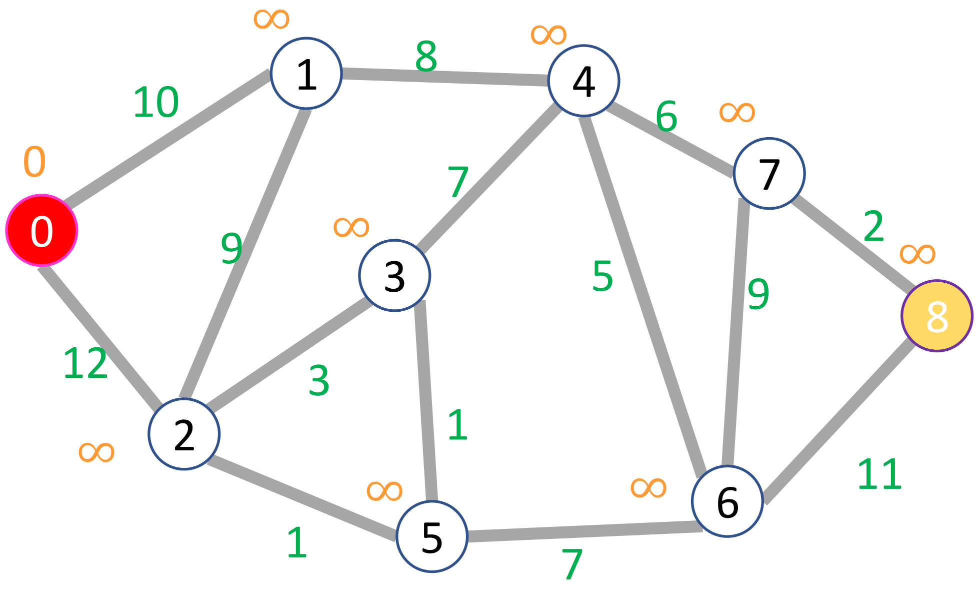
Start: 0

End: 8

Idea: When a node is the closest undiscovered thing to the start, we have found its shortest path

Node	Done?
0	F
1	F
2	F
3	F
4	F
5	F
6	F
7	F
8	F

Node	Distance
0	0
1	∞
2	∞
3	∞
4	∞
5	∞
6	∞
7	∞
8	∞



Dijkstra's Algorithm

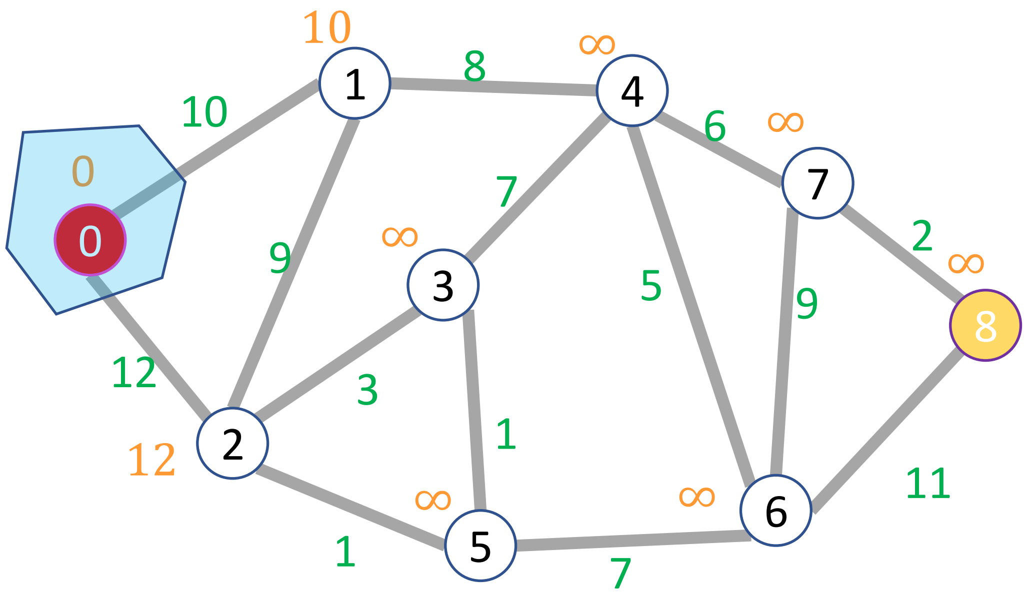
Start: 0

End: 8

Idea: When a node is the closest undiscovered thing to the start, we have found its shortest path

Node	Done?
0	T
1	F
2	F
3	F
4	F
5	F
6	F
7	F
8	F

Node	Distance
0	0
1	10
2	12
3	∞
4	∞
5	∞
6	∞
7	∞
8	∞



Dijkstra's Algorithm

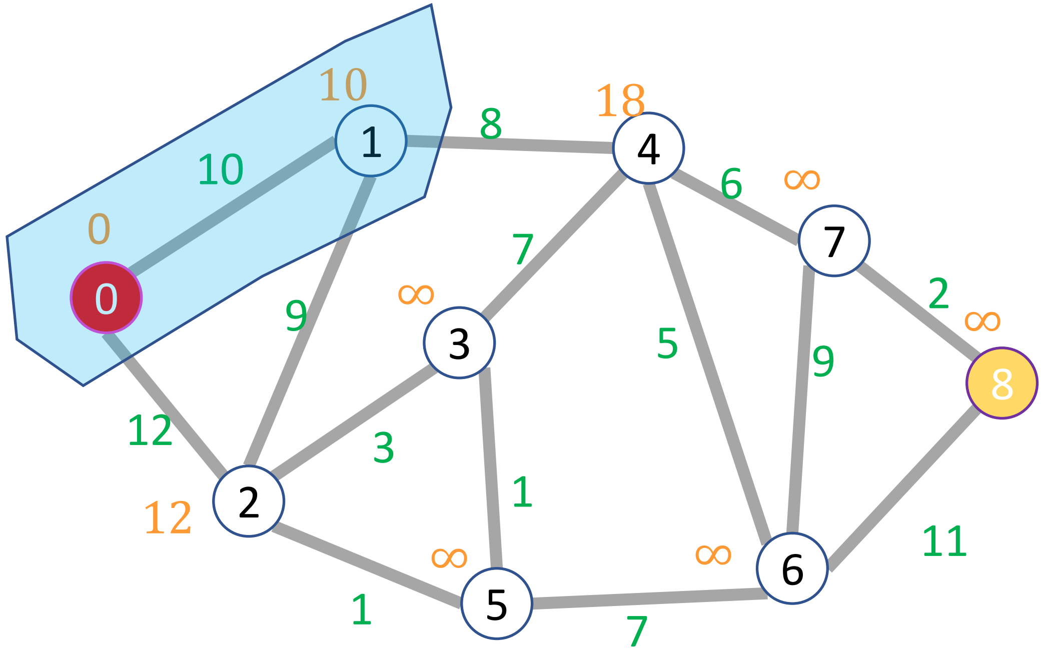
Start: 0

End: 8

Idea: When a node is the closest undiscovered thing to the start, we have found its shortest path

Node	Done?
0	T
1	T
2	F
3	F
4	F
5	F
6	F
7	F
8	F

Node	Distance
0	0
1	10
2	12
3	∞
4	18
5	∞
6	∞
7	∞
8	∞



Dijkstra's Algorithm

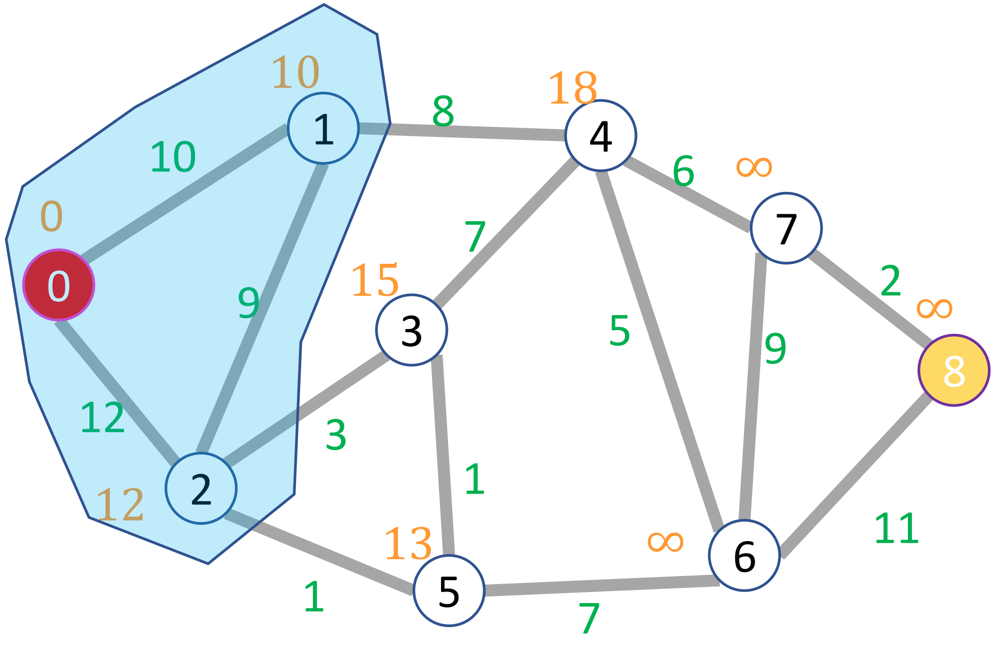
Start: 0

End: 8

Idea: When a node is the closest undiscovered thing to the start, we have found its shortest path

Node	Done?
0	T
1	T
2	T
3	F
4	F
5	F
6	F
7	F
8	F

Node	Distance
0	0
1	10
2	12
3	15
4	18
5	13
6	∞
7	∞
8	∞



Dijkstra's Algorithm

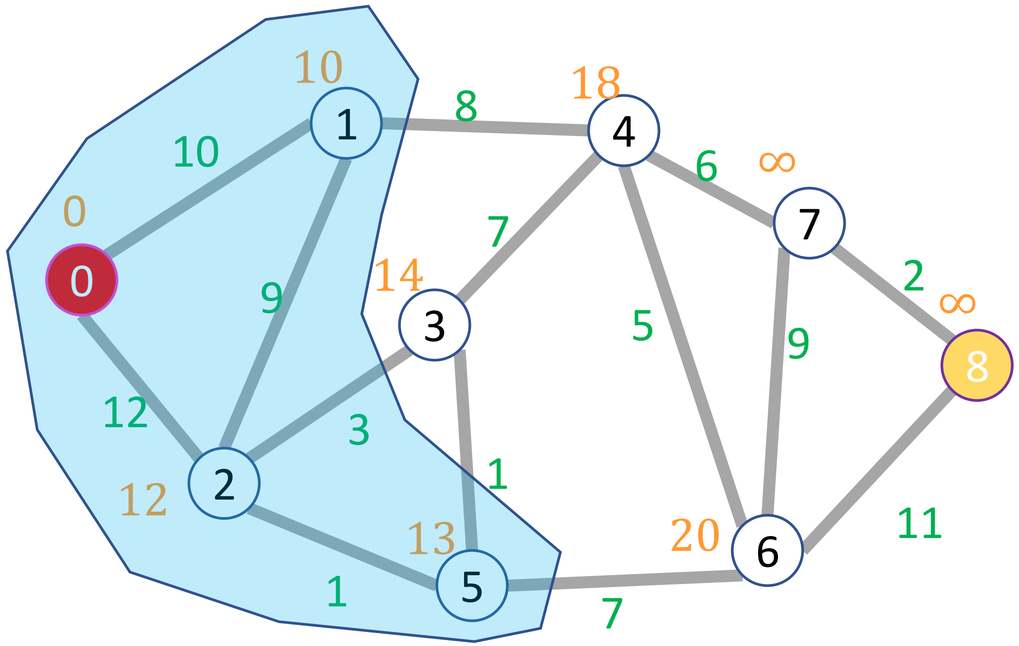
Start: 0

End: 8

Idea: When a node is the closest undiscovered thing to the start, we have found its shortest path

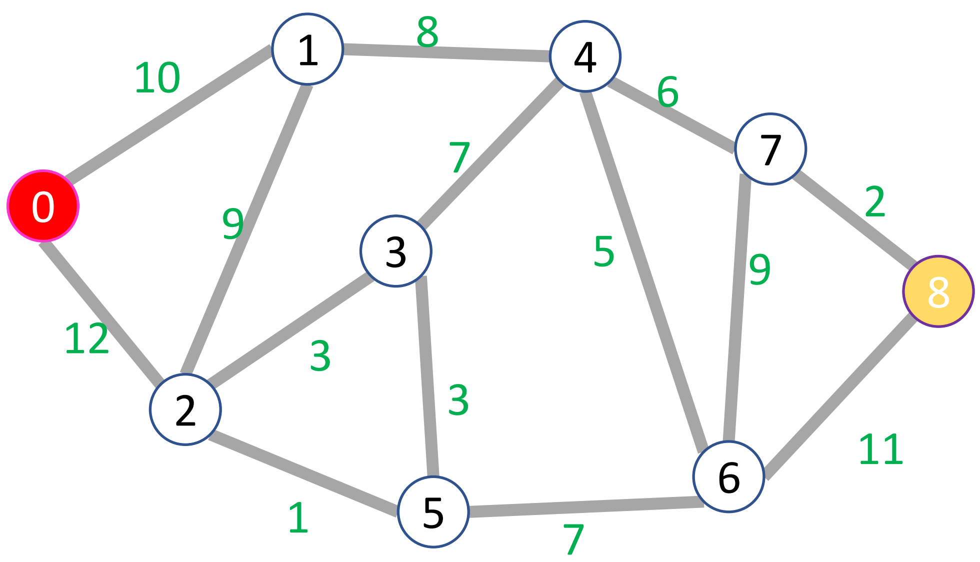
Node	Done?
0	T
1	T
2	T
3	F
4	F
5	T
6	F
7	F
8	F

Node	Distance
0	0
1	10
2	12
3	14
4	18
5	13
6	∞
7	20
8	∞



Dijkstra’s Algorithm

```
int dijkstras(graph, start, end){
        distances = [∞, ∞, ∞,…];  // one index per node
        done = [False,False,False,…];  // one index per node
        PQ = new minheap();
        PQ.insert(0, start);  // priority=0, value=start
        distances[start] = 0;
        while (!PQ.isEmpty){
                current = PQ.deleteMin();
                done[current] = true;
                for (neighbor : current.neighbors){
                        if (!done[neighbor]){
                                new_dist = distances[current]+weight(current,neighbor);
                                if(distances[neighbor] == ∞){
                                        distances[neighbor] = new_dist;
                                        PQ.insert(new_dist, neighbor);
                                }
                                if (new_dist < distances[neighbor]){
                                        distances[neighbor] = new_dist;
                                        PQ.decreaseKey(new_dist,neighbor); }
                        }
                }
        }
        return distances[end]
}
```

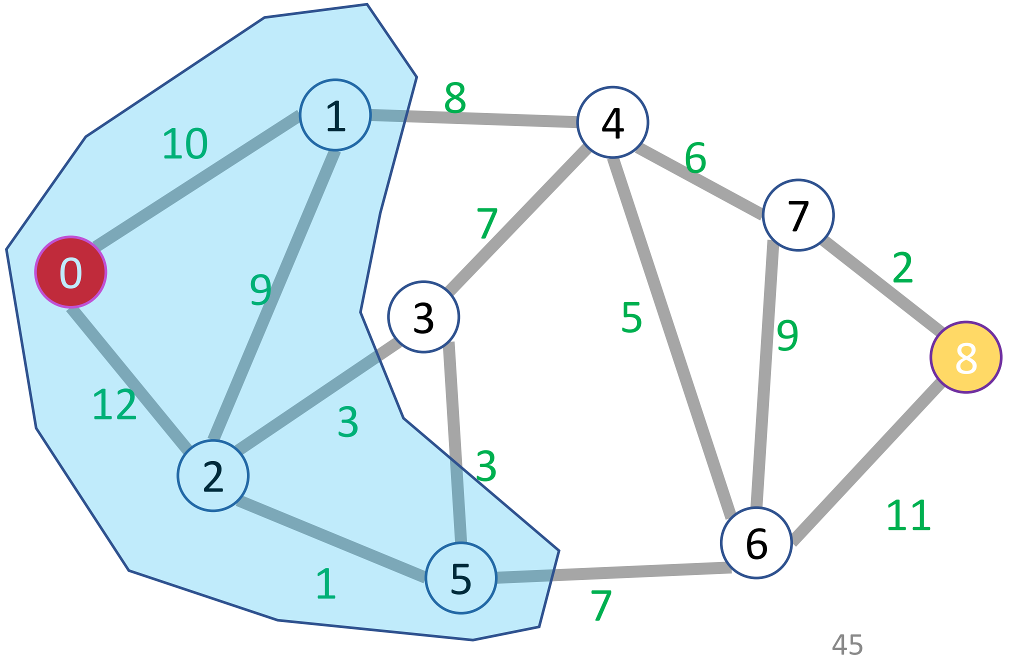


Dijkstra’s Algorithm: Running Time

- How many total priority queue operations are necessary?
  - How many times is each node added to the priority queue?
  - How many times might a node’s priority be changed?
- What’s the running time of each priority queue operation?
- Overall running time:
  - $\Theta(|E| \log|V|)$

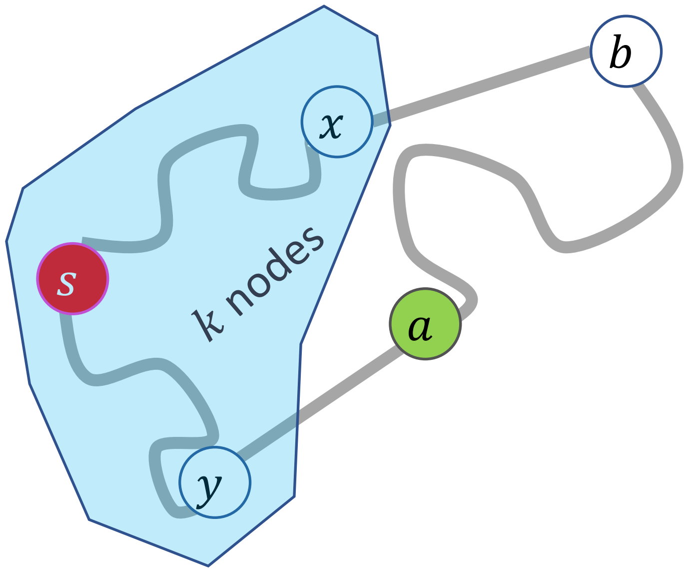
Dijkstra's Algorithm: Correctness

- Claim: when a node is removed from the priority queue, we have found its shortest path
- Induction over number of completed nodes
- Base Case:
- Inductive Step:



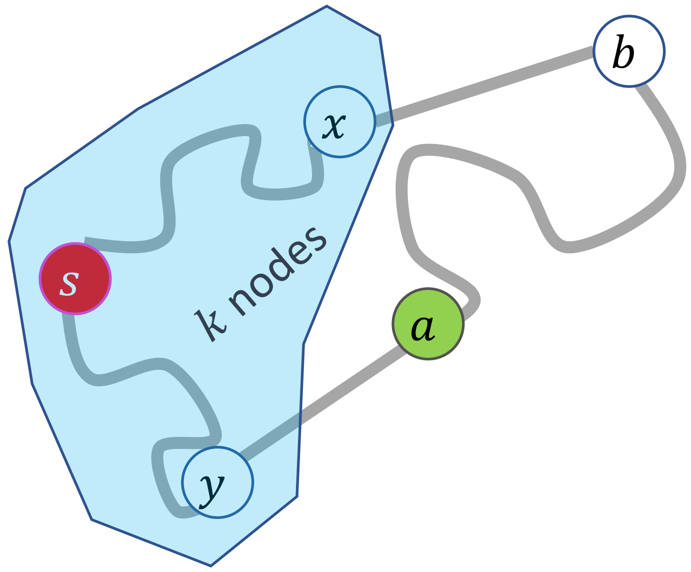
Dijkstra's Algorithm: Correctness

- Claim: when a node is removed from the priority queue, its distance is that of the shortest path
- Induction over number of completed nodes
- Base Case: Only the start node removed
  - It is indeed 0 away from itself
- Inductive Step:
  - If we have correctly found shortest paths for the first k nodes, then when we remove node $k+1$ we have found its shortest path



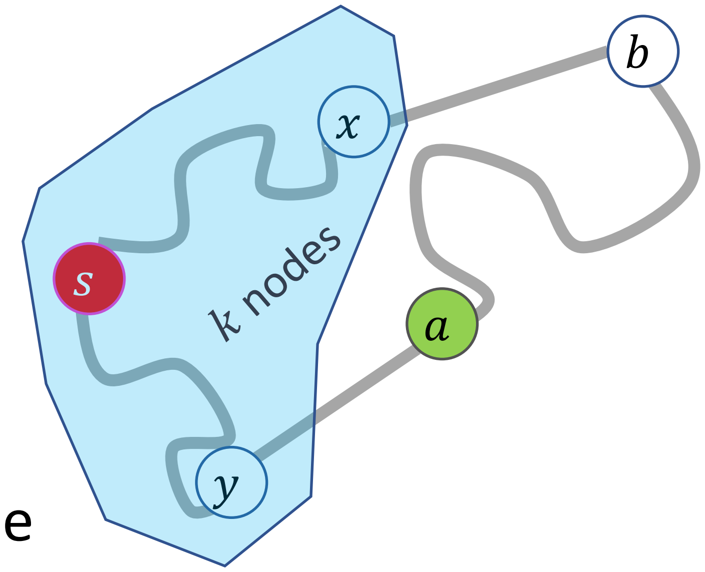
Dijkstra’s Algorithm: Correctness

- Suppose a is the next node removed from the queue. What do we know bout a?



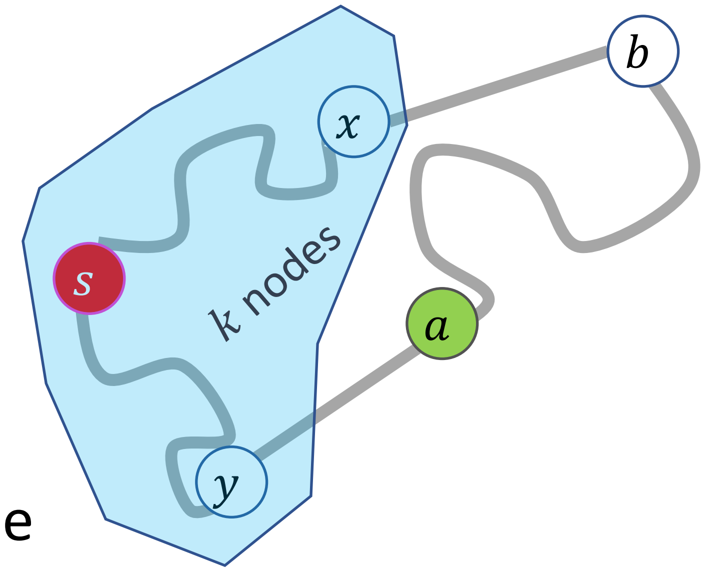
Dijkstra's Algorithm: Correctness

- Suppose a is the next node removed from the queue.
  - No other node incomplete node has a shorter path discovered so far
- Claim: no undiscovered path to a could be shorter
  - Consider any other incomplete node b that is 1 edge away from a complete node
  - a is the closest node that is one away from a complete node
  - Thus no path that includes b can be a shorter path to a
  - Therefore the shortest path to a must use only complete nodes, and therefore we have found it already!



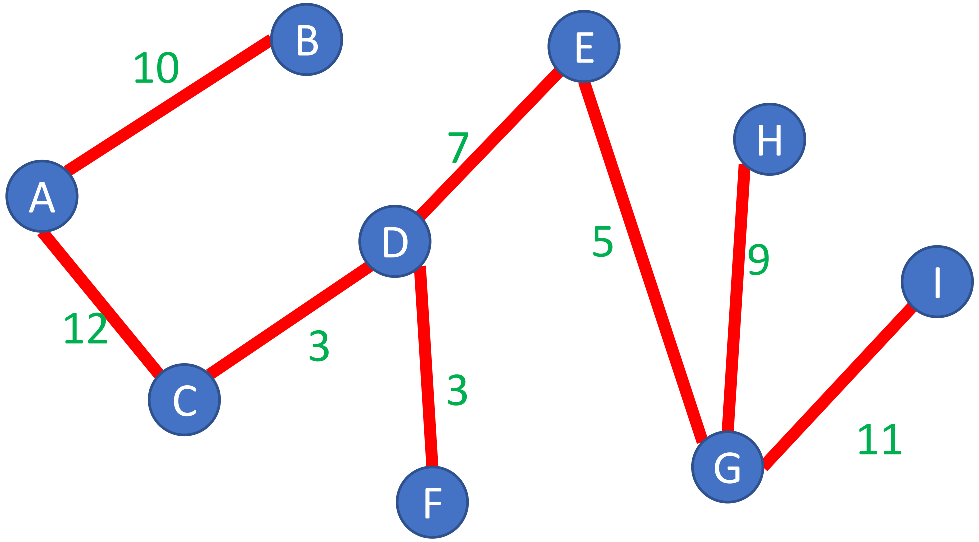
Dijkstra's Algorithm: Correctness

- Suppose a is the next node removed from the queue.
  - No other node incomplete node has a shorter path discovered so far
- Claim: no undiscovered path to a could be shorter
  - Consider any other incomplete node b that is 1 edge away from a complete node
  - a is the closest node that is one away from a complete node
  - No path from b to a can have negative weight
  - Thus no path that includes b can be a shorter path to a
  - Therefore the shortest path to a must use only complete nodes, and therefore we have found it already!



Definition: Tree

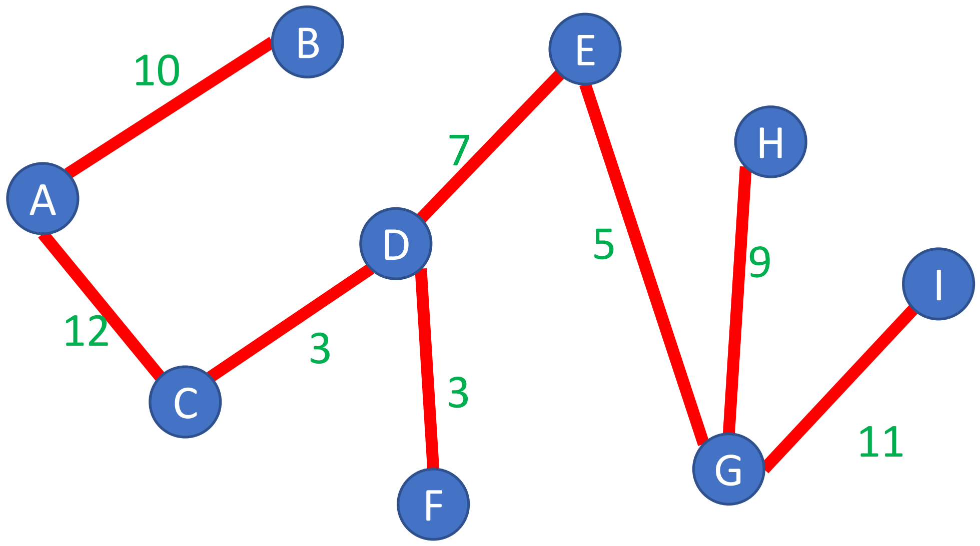
A connected graph with no cycles



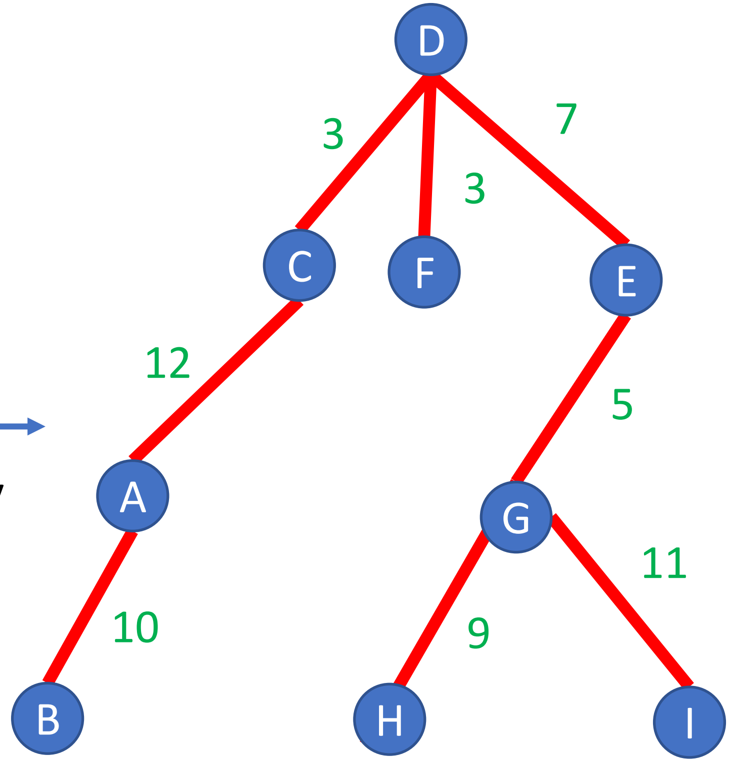
Note: A tree does not need a root, but they often do!

Definition: Tree

A connected graph with no cycles



Pick some arbitrary root node and rearrange tree

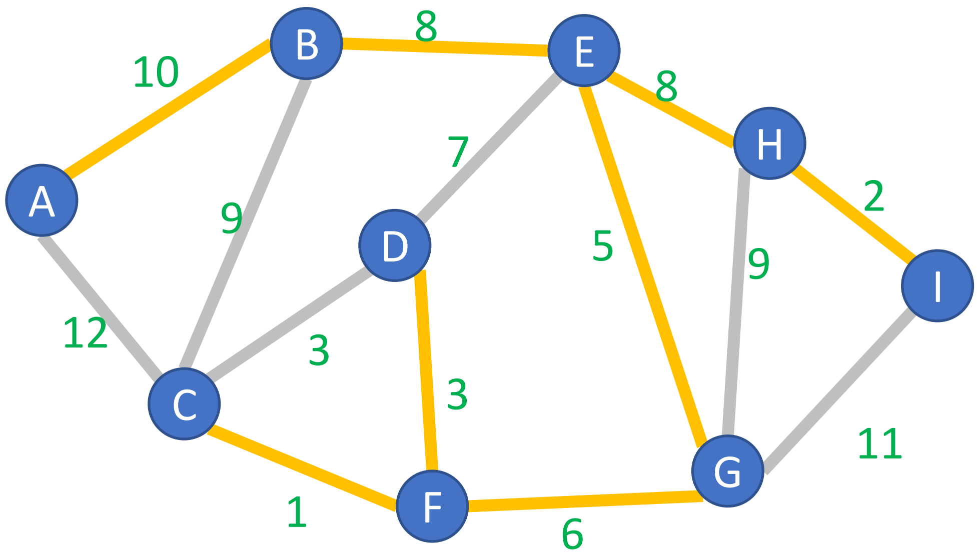


Definition: Spanning Tree

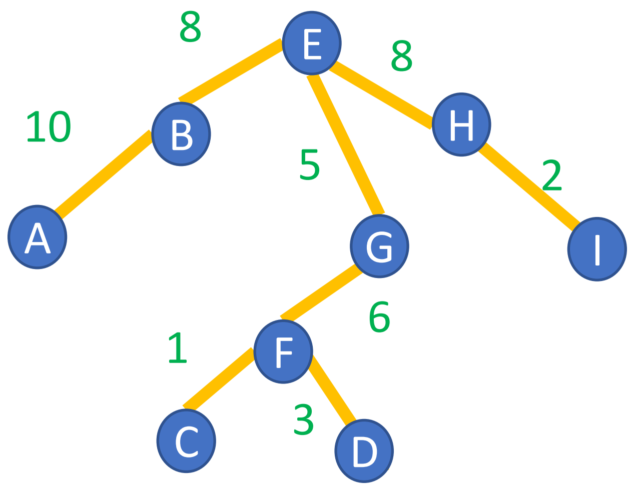
A Tree $T = (V_T, E_T)$ which connects ("spans") all the nodes in a graph $G = (V, E)$

How many edges does T have?

$V - 1$



Pick some arbitrary root node and rearrange tree

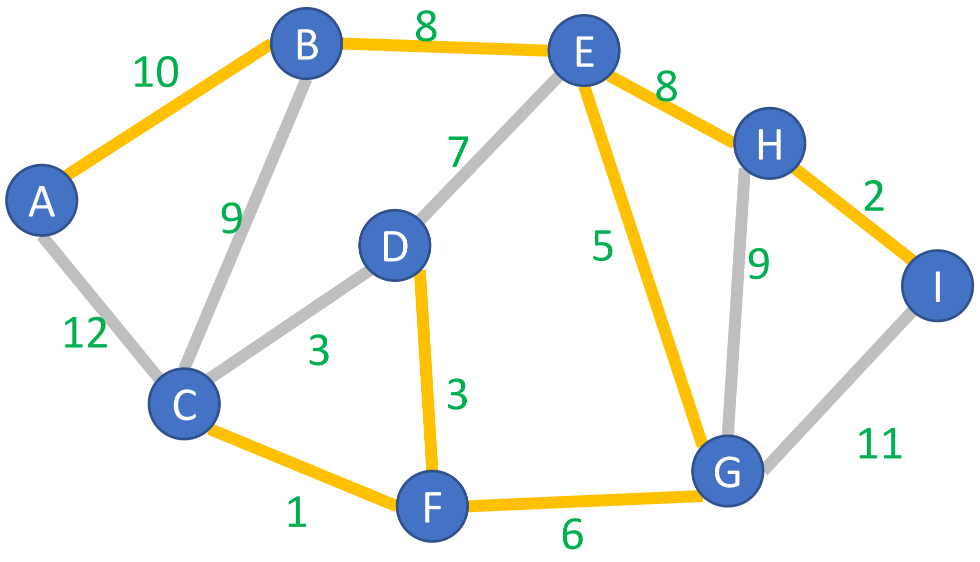


Any set of V-1 edges in the graph that doesn't have any cycles is guaranteed to be a spanning tree!

Any set of V-1 edges that connects all the nodes in the graph is guaranteed to be a spanning tree!

Definition: Minimum Spanning Tree

A Tree $\boldsymbol{T} = (V_T, E_T)$ which connects ("spans") all the nodes in a graph $G = (V, E)$, that has minimal cost

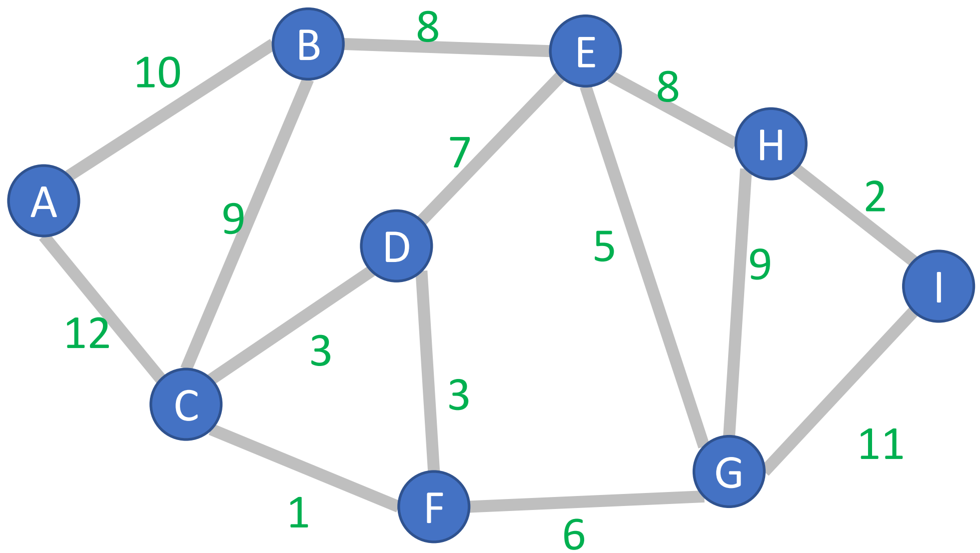


$$Cost(T) = \sum_{e \in E_T} w(e)$$

Kruskal's Algorithm

Start with an empty tree A

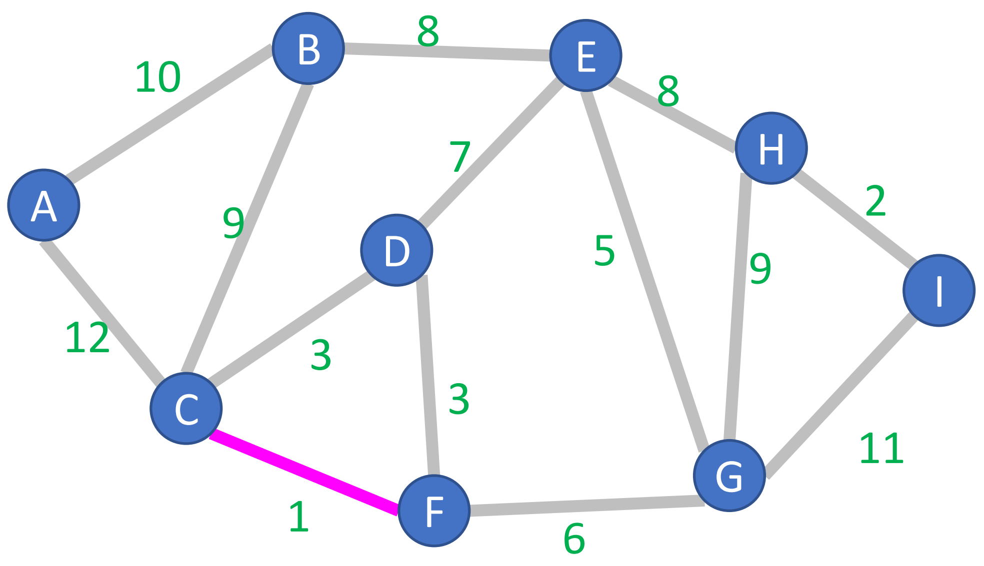
Add to A the lowest-weight edge that does not create a cycle



Kruskal’s Algorithm

Start with an empty tree A

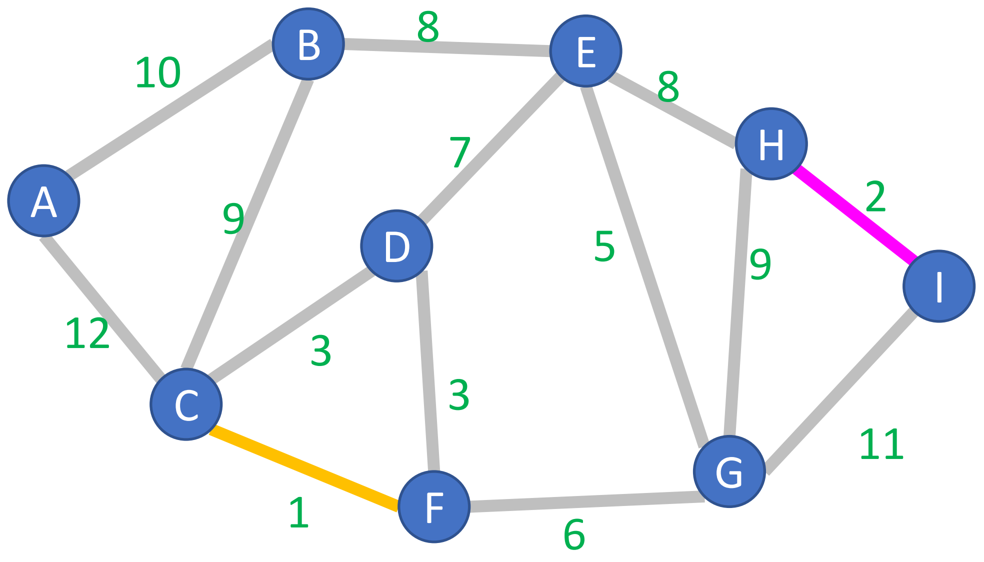
Add to A the lowest-weight edge that does not create a cycle



Kruskal's Algorithm

Start with an empty tree A

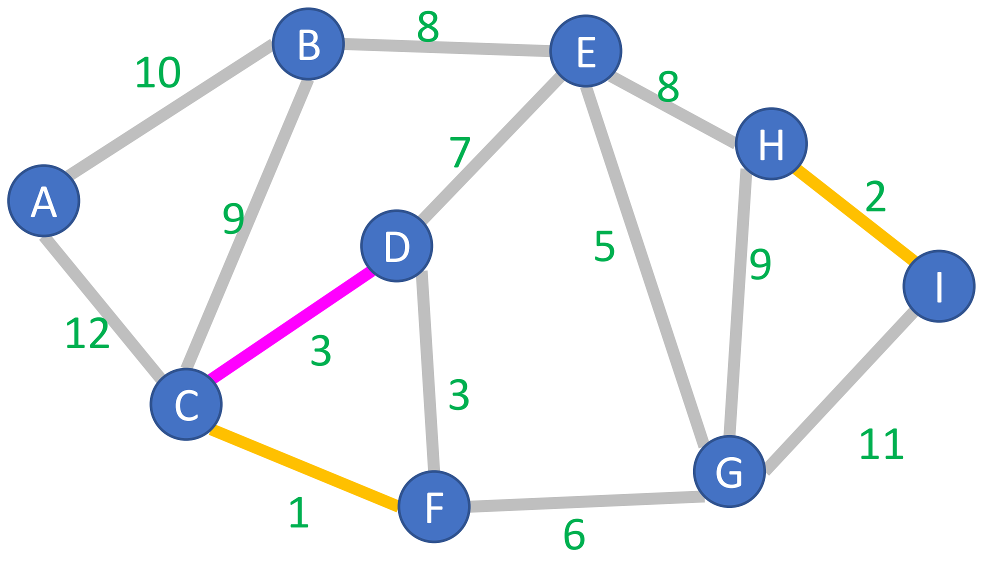
Add to A the lowest-weight edge that does not create a cycle



Kruskal's Algorithm

Start with an empty tree A

Add to A the lowest-weight edge that does not create a cycle



Kruskal's Algorithm

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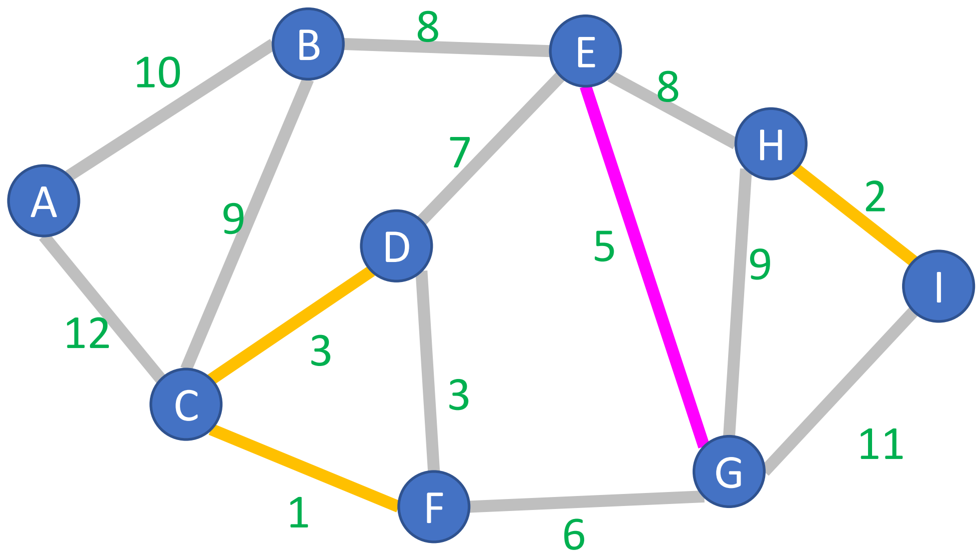

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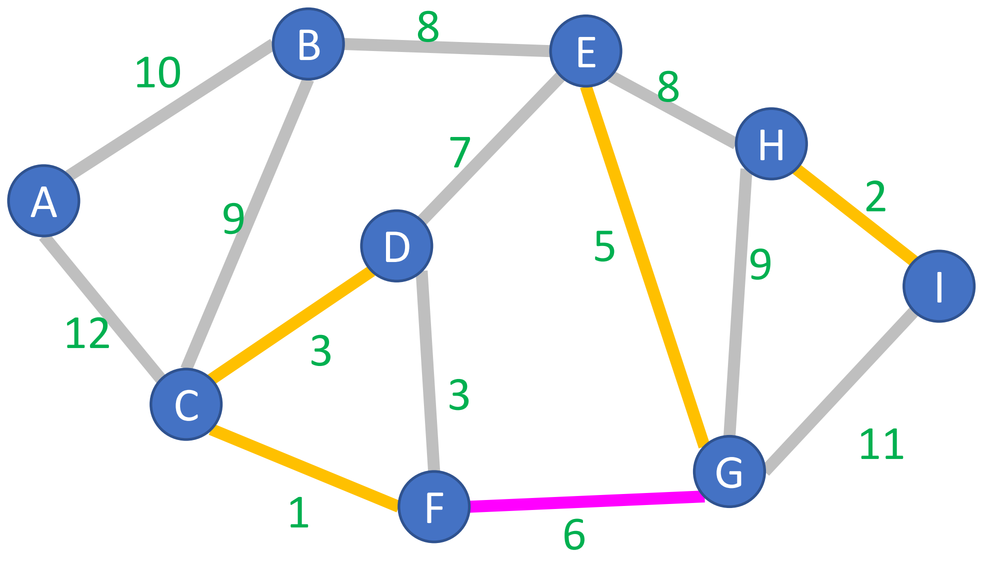
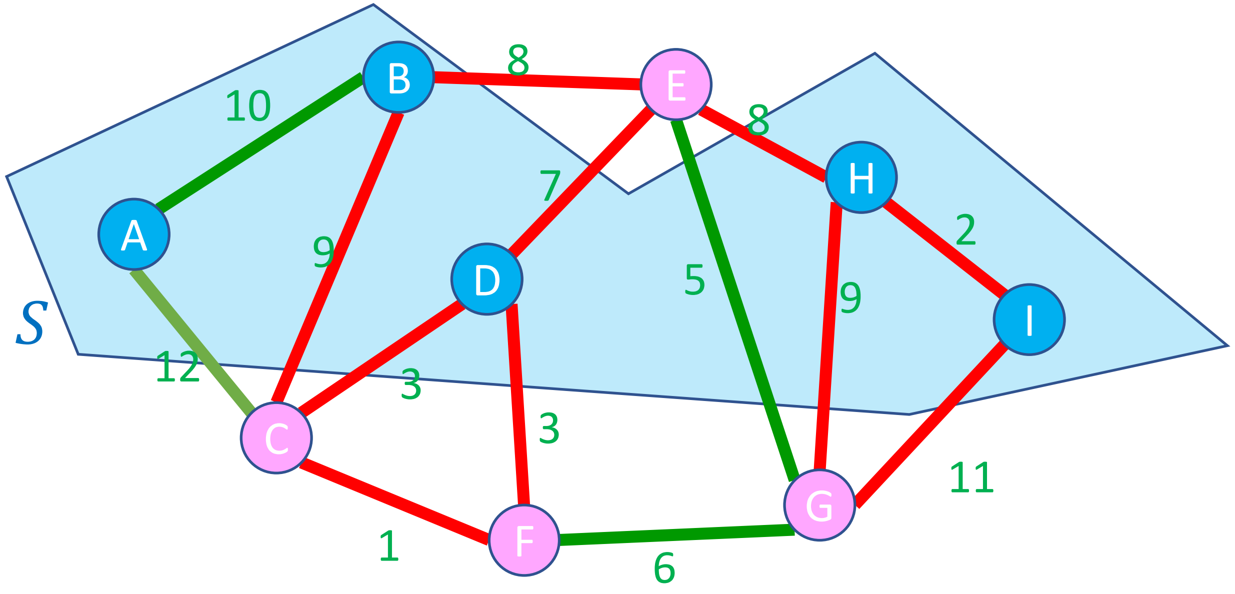
Start with an empty tree A

Add to A the lowest-weight edge that does not create a cycle



Definition: Cut

A Cut of graph $G = (V, E)$ is a partition of the nodes into two sets, S and $V - S$



Edge $(v_1, v_2) \in E$ crosses a cut if $v_1 \in S$ and $v_2 \in V - S$ (or opposite), e.g. (A, C)

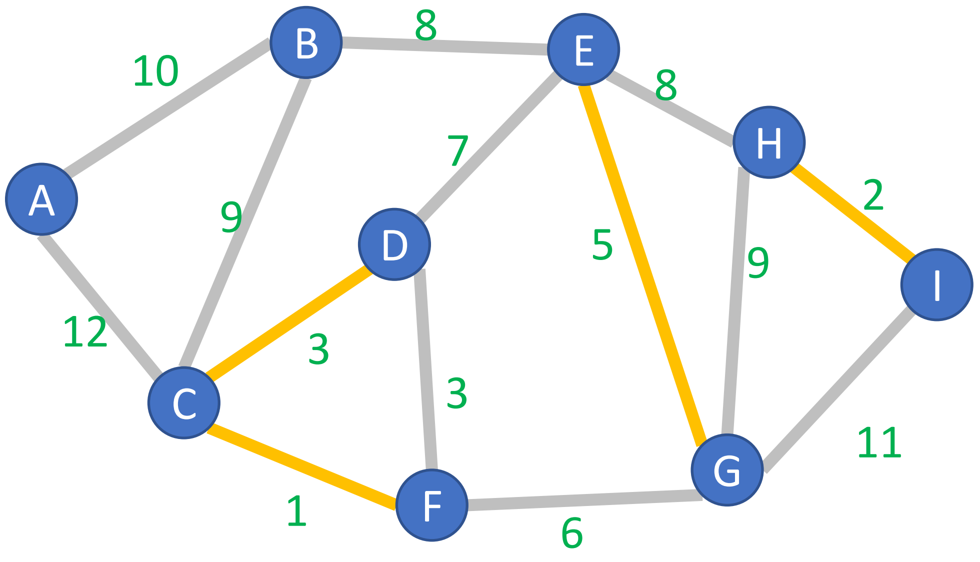
A set of edges R Respects a cut if no edges cross the cut e.g. $R = \{(A, B), (E, G), (F, G)\}$

Cut Theorem

If a set of edges A is a subset of a minimum spanning tree T, let $(S, V - S)$ be any cut which A respects. Let e be the least-weight edge which crosses $(S, V - S)$. $A \cup \{e\}$ is also a subset of a minimum spanning tree.

Cut Theorem

If a set of edges A is a subset of a minimum spanning tree T, let $(S, V - S)$ be any cut which A respects. Let e be the least-weight edge which crosses $(S, V - S)$. $A \cup \{e\}$ is also a subset of a minimum spanning tree.



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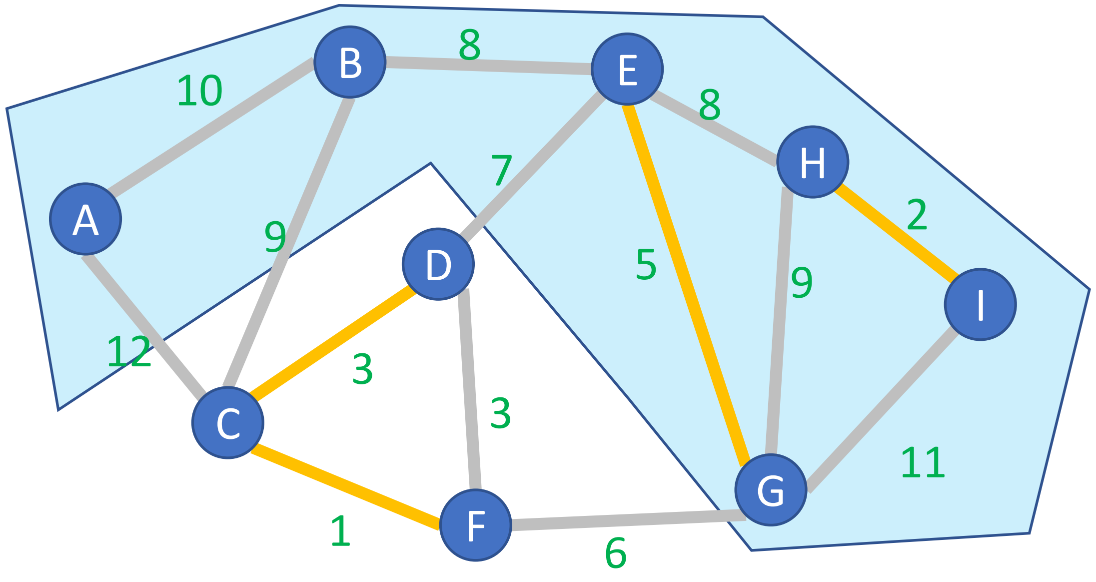

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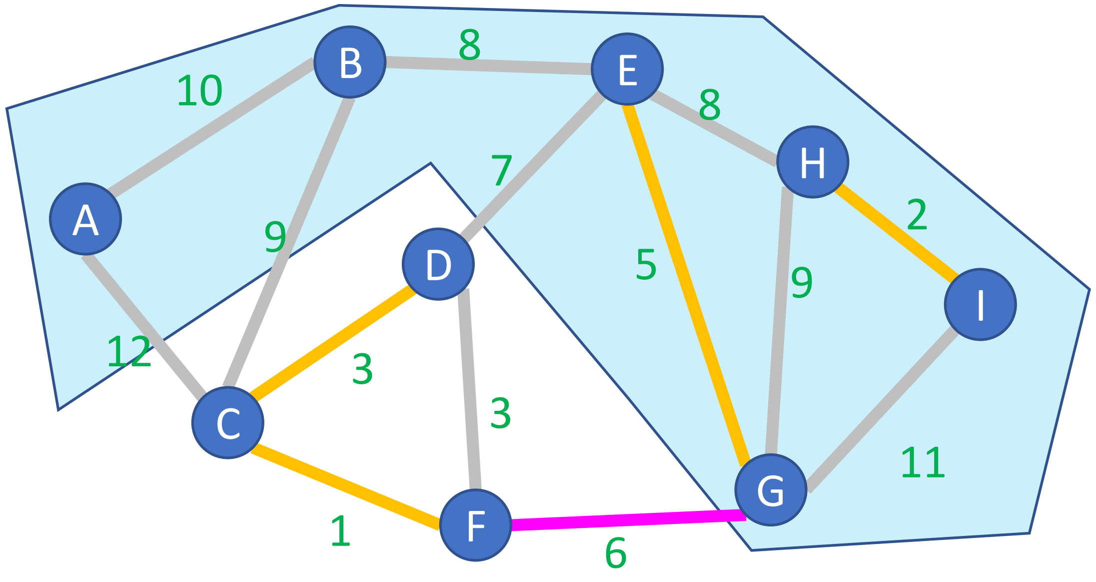

Cut Theorem

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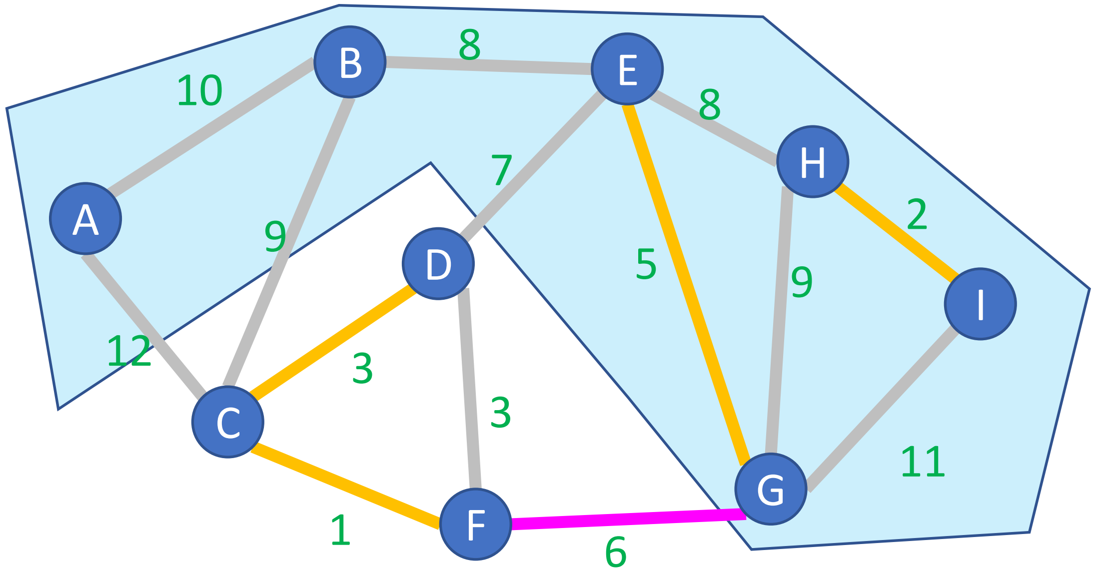
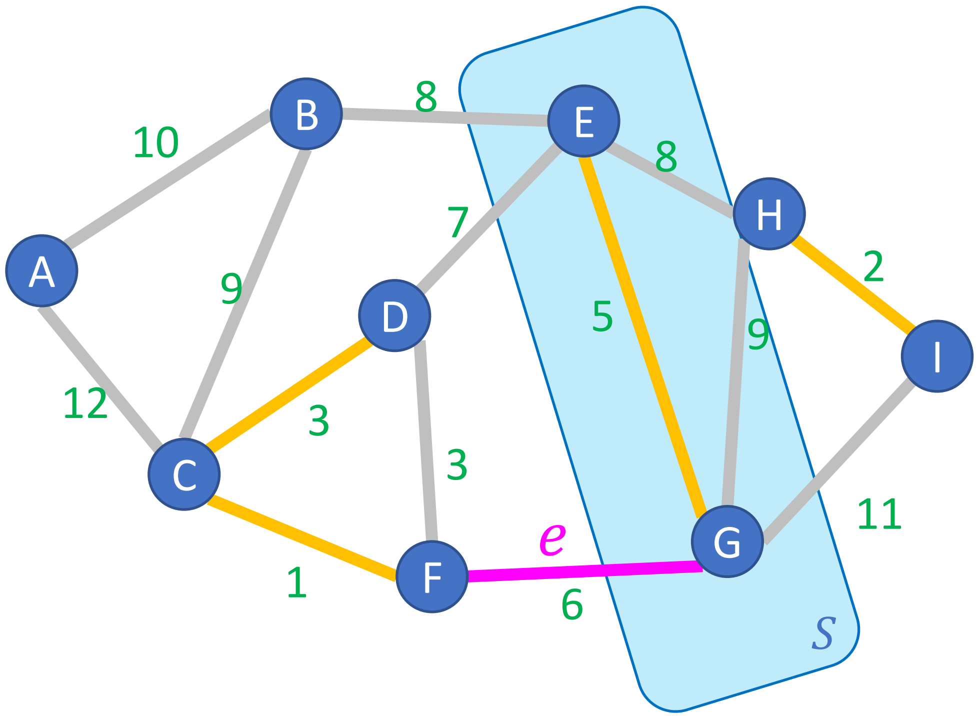


Proof of Kruskal's Algorithm

Start with an empty tree A
Repeat $V - 1$ times:
  Add the min-weight edge that doesn't cause a cycle



Proof: Suppose we have some arbitrary set of edges A that Kruskal's has already selected to include in the MST. $e = (F, G)$ is the edge Kruskal's selects to add next

We know that there cannot exist a path from F to G using only edges in A because e does not cause a cycle

We can cut the graph therefore into 2 disjoint sets:

- nodes reachable from G using edges in A
- All other nodes

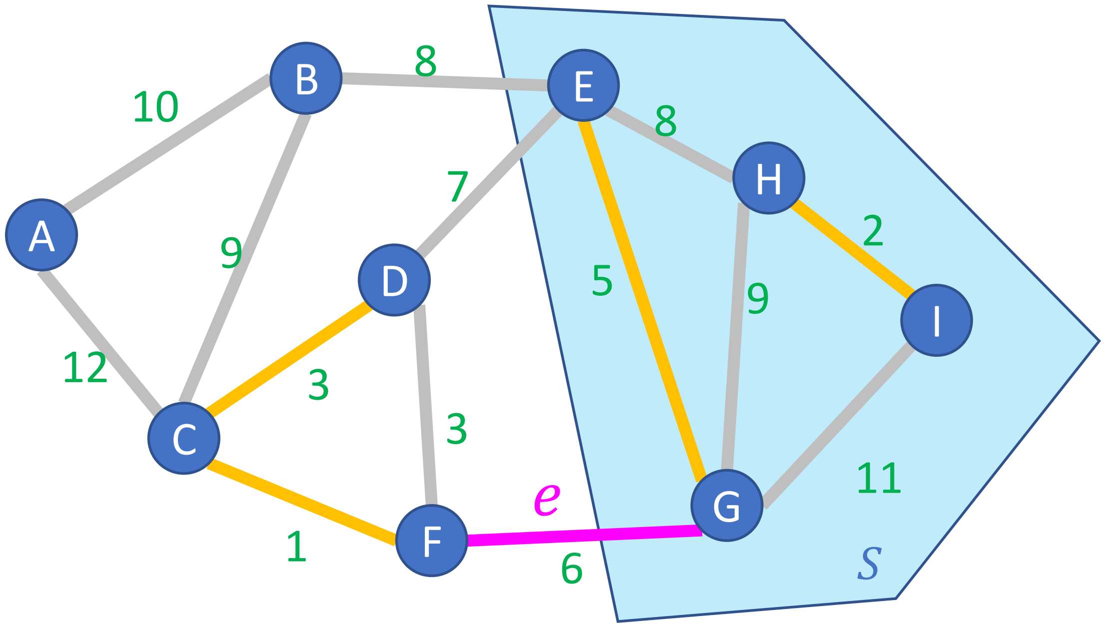
e is the minimum cost edge that crosses this cut, so by the Cut Theorem, Kruskal's is optimal!

Kruskal’s Algorithm Runtime

Start with an empty tree A

Repeat $V - 1$ times:

Add the min-weight edge that doesn’t cause a cycle



Keep edges in a Disjoint-set data structure (very fancy)

$O(E \log V)$

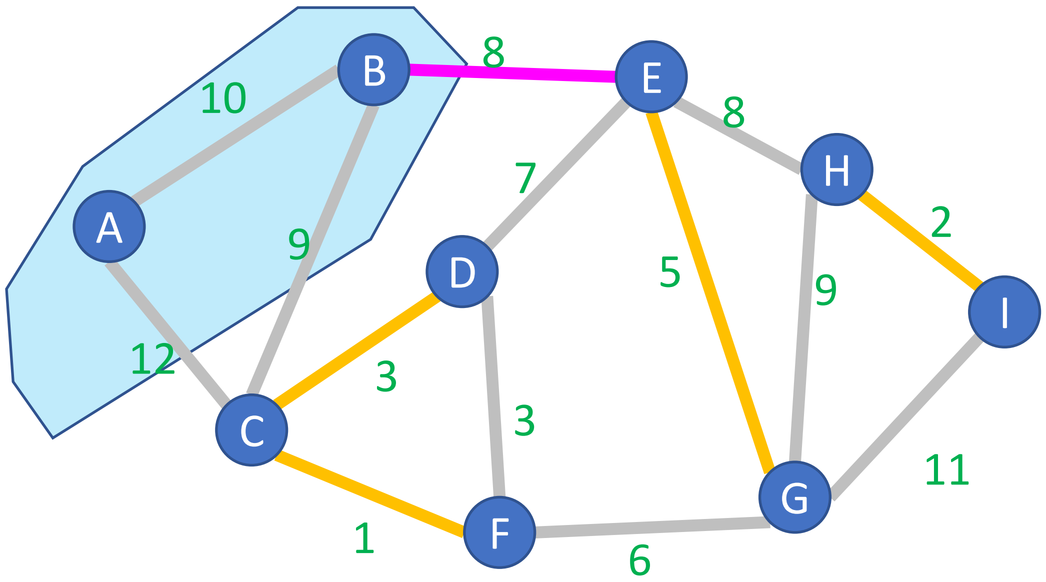
General MST Algorithm

Start with an empty tree A

Repeat $V - 1$ times:

Pick a cut $(S, V - S)$ which A respects (typically implicitly)

Add the min-weight edge which crosses $(S, V - S)$



Prim's Algorithm

Start with an empty tree A

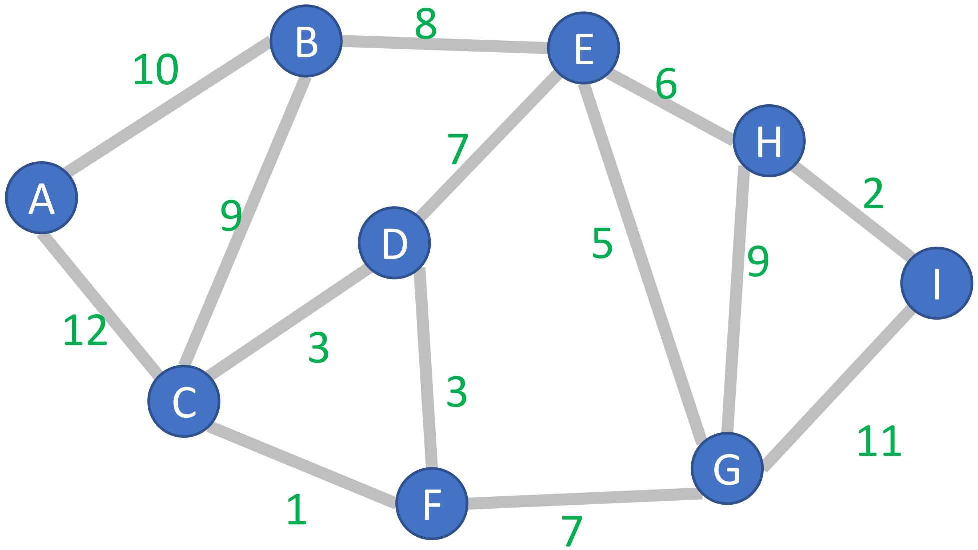
Repeat $V - 1$ times:

Pick a cut $(S, V - S)$ which A respects

Add the min-weight edge which crosses $(S, V - S)$

S is all endpoint of edges in A

e is the min-weight edge that grows the tree



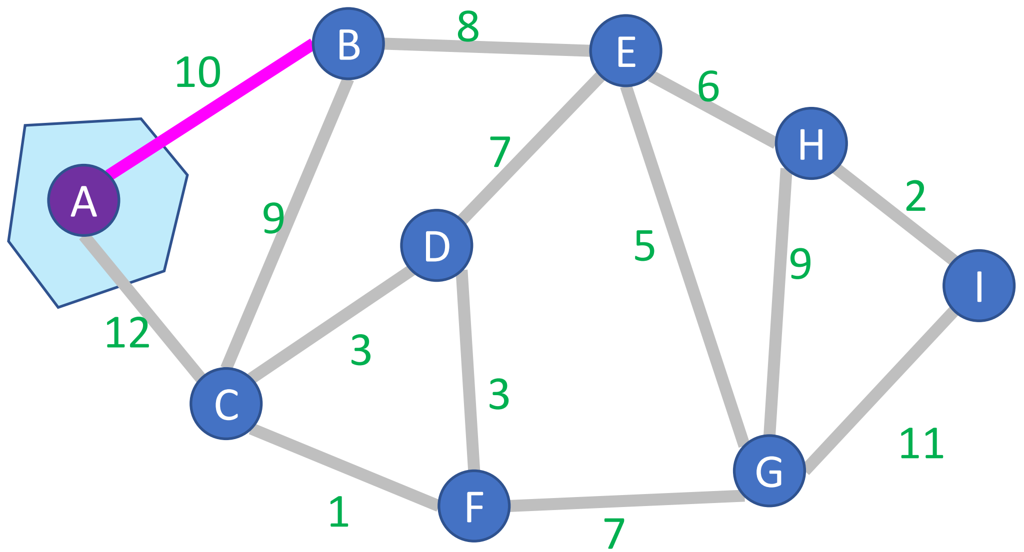
Prim's Algorithm

Start with an empty tree A

Pick a start node

Repeat $V - 1$ times:

Add the min-weight edge which connects to node in A with a node not in A



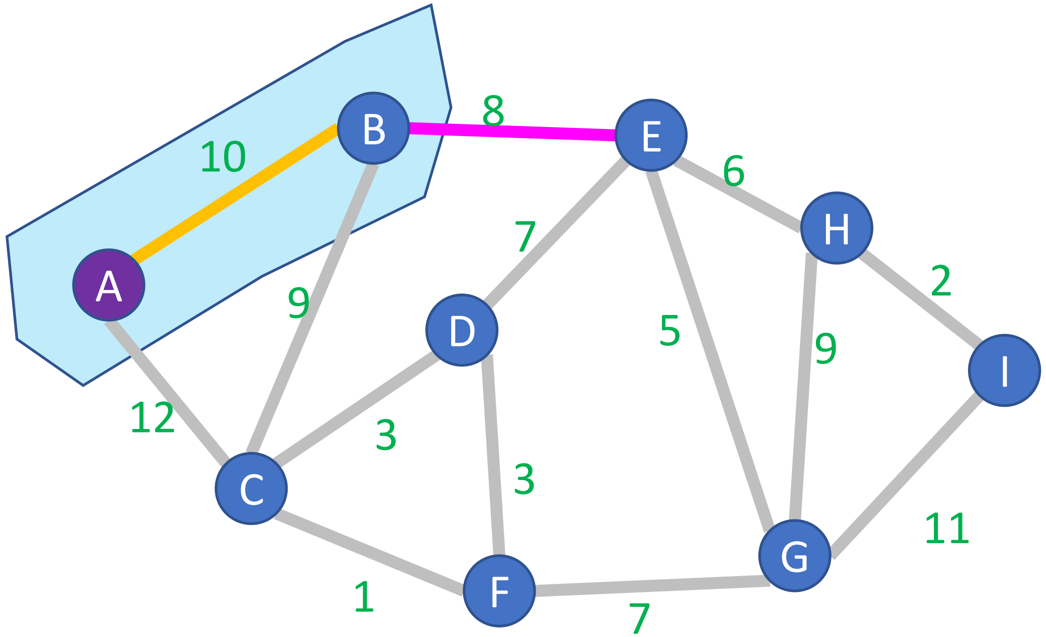
Prim's Algorithm

Start with an empty tree A

Pick a start node

Repeat $V - 1$ times:

Add the min-weight edge which connects to node in A with a node not in A



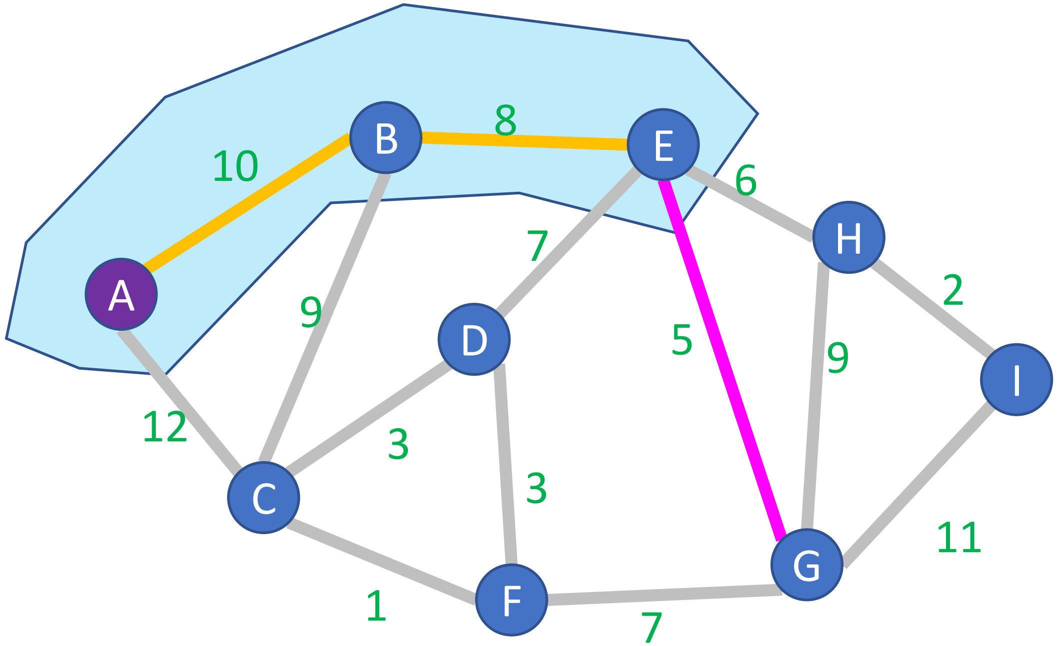
Prim's Algorithm

Start with an empty tree A

Pick a start node

Repeat $V - 1$ times:

Add the min-weight edge which connects to node in A with a node not in A



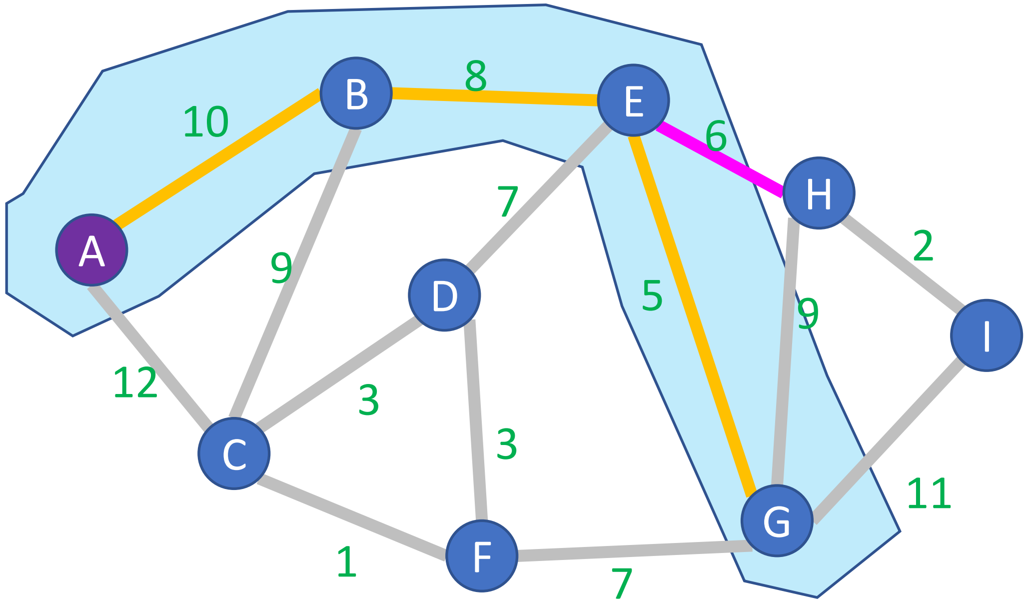
Prim's Algorithm

Start with an empty tree A

Pick a start node

Repeat $V - 1$ times:

Add the min-weight edge which connects to node in A with a node not in A



Prim's Algorithm

Start with an empty tree A

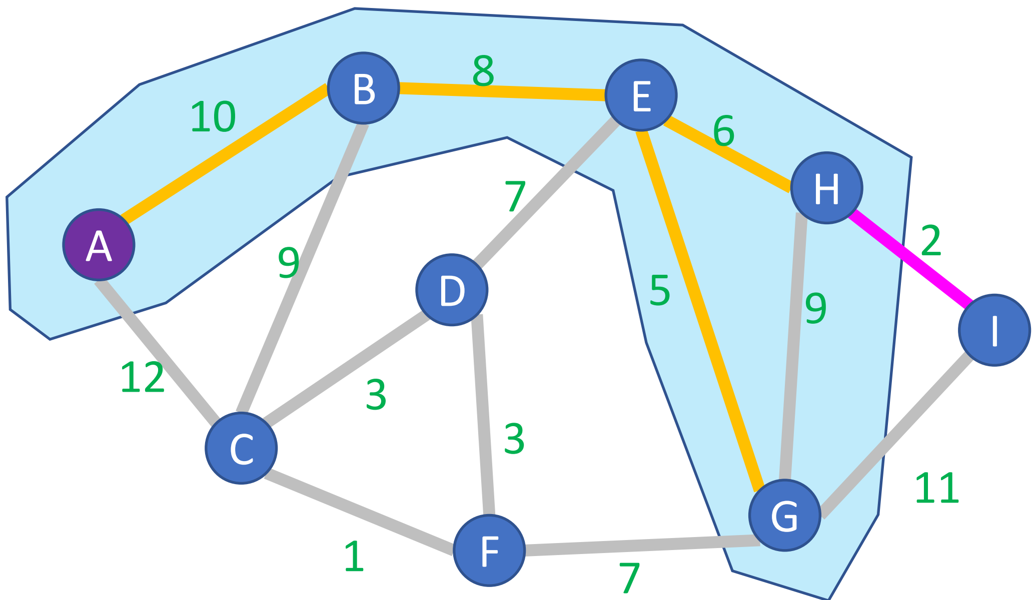
Pick a start node

Repeat $V - 1$ times:

Add the min-weight edge which connects to node in A with a node not in A

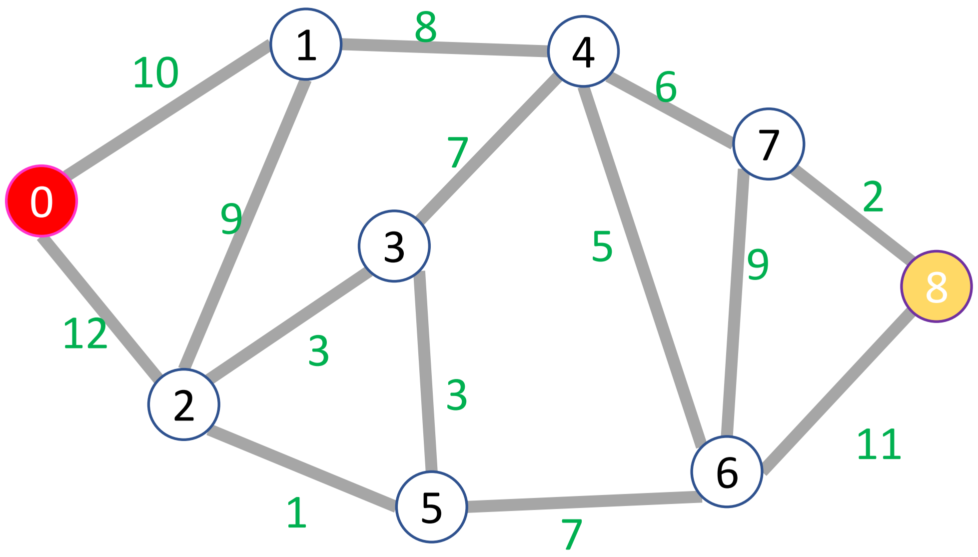
Keep edges in a Heap

$O(E \log V)$



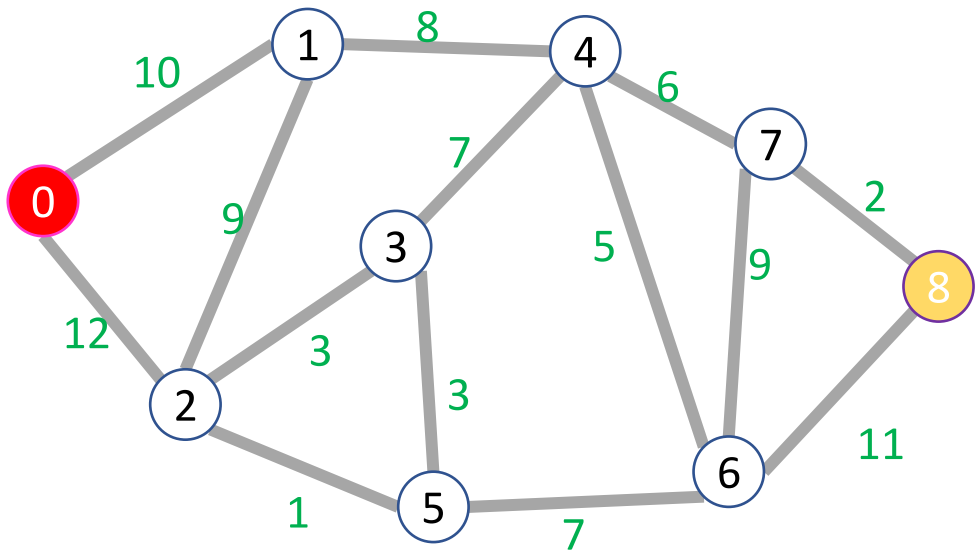
Dijkstra’s Algorithm

```
int dijkstras(graph, start, end){
        distances = [∞, ∞, ∞,…];  // one index per node
        done = [False,False,False,…];  // one index per node
        PQ = new minheap();
        PQ.insert(0, start);  // priority=0, value=start
        distances[start] = 0;
        while (!PQ.isEmpty){
                current = PQ.deleteMin();
                done[current] = true;
                for (neighbor : current.neighbors){
                        if (!done[neighbor]){
                                new_dist = distances[current]+weight(current,neighbor);
                                if(distances[neighbor] == ∞){
                                        distances[neighbor] = new_dist;
                                        PQ.insert(new_dist, neighbor);
                                }
                                if (new_dist < distances[neighbor]){
                                        distances[neighbor] = new_dist;
                                        PQ.decreaseKey(new_dist,neighbor); }
                        }
                }
        }
        return distances[end]
}
```



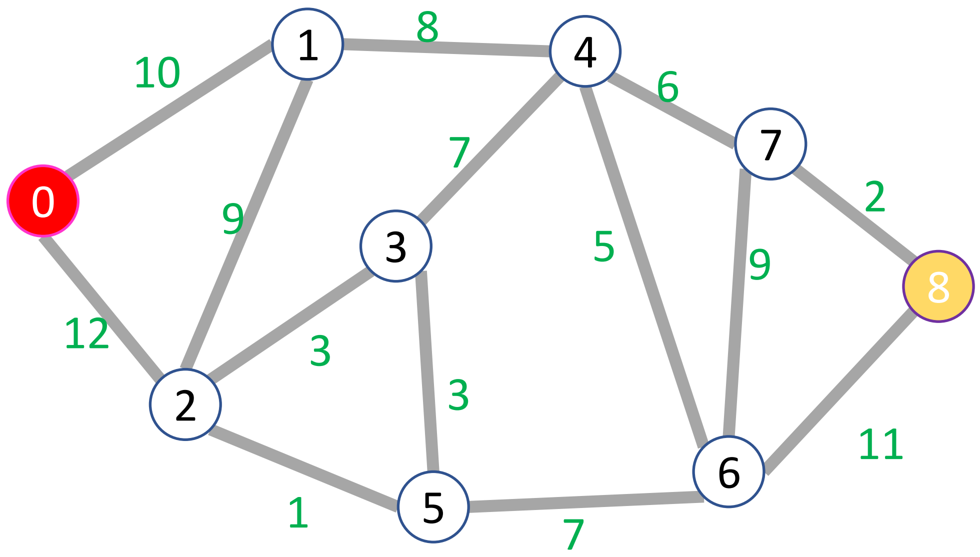
Prims's Algorithm

```
int primss(graph, start, end){
        distances = [∞, ∞, ∞,…];  // one index per node
        done = [False,False,False,…];  // one index per node
        PQ = new minheap();
        PQ.insert(0, start);  // priority=0, value=start
        distances[start] = 0;
        while (!PQ.isEmpty){
                current = PQ.deleteMin();
                done[current] = true;
                for (neighbor : current.neighbors){
                        if (!done[neighbor]){
                                new_dist = weight(current,neighbor);
                                if(distances[neighbor] == ∞){
                                        distances[neighbor] = new_dist;
                                        PQ.insert(new_dist, neighbor);
                                }
                                if (new_dist < distances[neighbor]){
                                        distances[neighbor] = new_dist;
                                        PQ.decreaseKey(new_dist,neighbor); }
                        }
                }
        }
        return distances[end]
}
```



Dijkstra’s Algorithm

```
int dijkstras(graph, start, end){
        distances = [∞, ∞, ∞,…];  // one index per node
        done = [False,False,False,…];  // one index per node
        PQ = new minheap();
        PQ.insert(0, start);  // priority=0, value=start
        distances[start] = 0;
        while (!PQ.isEmpty){
                current = PQ.deleteMin();
                done[current] = true;
                for (neighbor : current.neighbors){
                        if (!done[neighbor]){
                                new_dist = distances[current]+weight(current,neighbor);
                                if(distances[neighbor] == ∞){
                                        distances[neighbor] = new_dist;
                                        PQ.insert(new_dist, neighbor);
                                }
                                if (new_dist < distances[neighbor]){
                                        distances[neighbor] = new_dist;
                                        PQ.decreaseKey(new_dist,neighbor); }
                        }
                }
        }
        return distances[end]
}
```



Prims's Algorithm

```
int primss(graph, start, end){
        distances = [∞, ∞, ∞,…];  // one index per node
        done = [False,False,False,…];  // one index per node
        PQ = new minheap();
        PQ.insert(0, start);  // priority=0, value=start
        distances[start] = 0;
        while (!PQ.isEmpty){
                current = PQ.deleteMin();
                done[current] = true;
                for (neighbor : current.neighbors){
                        if (!done[neighbor]){
                                new_dist = weight(current,neighbor);
                                if(distances[neighbor] == ∞){
                                        distances[neighbor] = new_dist;
                                        PQ.insert(new_dist, neighbor);
                                }
                                if (new_dist < distances[neighbor]){
                                        distances[neighbor] = new_dist;
                                        PQ.decreaseKey(new_dist,neighbor); }
                        }
                }
        }
        return distances[end]
}
```