# CSE 332 Summer 2024
# Lecture 10: Hashing 2

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# Next topic: Hash Tables

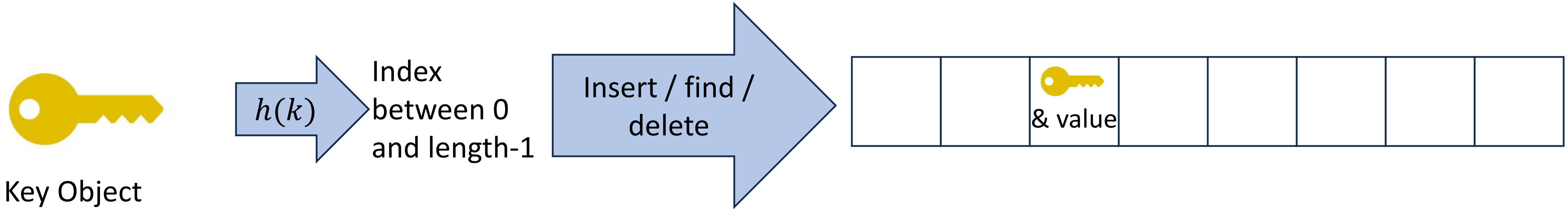
| Data Structure | Time to insert | Time to find | Time to delete |
|---|---|---|---|
| Unsorted Array | $\Theta(n)$ | $\Theta(n)$ | $\Theta(n)$ |
| Unsorted Linked List | $\Theta(n)$ | $\Theta(n)$ | $\Theta(n)$ |
| Sorted Array | $\Theta(n)$ | $\Theta(\log n)$ | $\Theta(n)$ |
| Sorted Linked List | $\Theta(n)$ | $\Theta(n)$ | $\Theta(n)$ |
| Binary Search Tree | $\Theta(\text{height})$ | $\Theta(\text{height})$ | $\Theta(\text{height})$ |
| AVL Tree | $\Theta(\log n)$ | $\Theta(\log n)$ | $\Theta(\log n)$ |
| Hash Table (Worst case) | $\Theta(n)$ | $\Theta(n)$ | $\Theta(n)$ |
| Hash Table (Expected and amortized) | $\Theta(1)$ | $\Theta(1)$ | $\Theta(1)$ |

# Dictionary (Map) ADT

- Contents:
  - Sets of key+value pairs
  - ~~Keys must be comparable~~
- Operations:
  - insert(key, value)
    - Adds the (key,value) pair into the dictionary
    - If the key already has a value, overwrite the old value
      - Consequence: Keys cannot be repeated
  - find(key)
    - Returns the value associated with the given key
  - delete(key)
    - Remove the key (and its associated value)

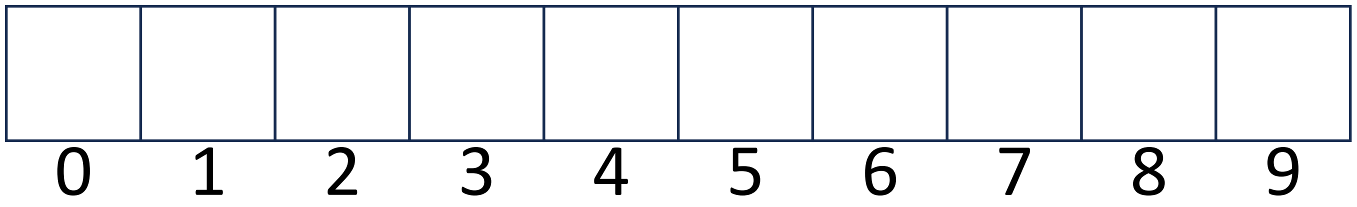
# Hash Tables

- Idea:
  - Have a small array to store information
  - Use a **hash function** to convert the key into an index
    - Hash function should “scatter” the keys, behave as if it randomly assigned keys to indices
  - Store key at the index given by the hash function
  - Do something if two keys map to the same place (should be very rare)
    - Collision resolution



# Collision Resolution

- A Collision occurs when we want to insert something into an already-occupied position in the hash table
- 2 main strategies:
  - Separate Chaining
    - Use a secondary data structure to contain the items
      - E.g. each index in the hash table is itself a linked list
  - Open Addressing
    - Use a different spot in the table instead
      - Linear Probing
      - Quadratic Probing
      - Double Hashing



# Rehashing

- If your load factor $\lambda$ gets too large, copy everything over to a larger hash table
  - To do this: make a new, larger array
  - Re-insert all items into the new hash table by reapplying the hash function
    - We need to reapply the hash function because items should map to a different index
  - New array should be "roughly" double the length (but probably still want it to be prime)
- What does "too large" mean?
  - For separate chaining, typically we want $\lambda < 2$
  - For open addressing, typically we want $\lambda < \frac{1}{2}$

# Linear Probing: Insert Procedure

- To insert $k, v$
  - Calculate $i = h(k)\ \%\ length$
  - If $table[i]$ is occupied then try $(i+1)\%\ length$
  - If that is occupied try $(i+2)\%\ length$
  - If that is occupied try $(i+3)\%\ length$
  - …
  - $h(k) = k\%10$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | 2 | 32 | 24 | 62 | 54 | | | |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

# Linear Probing: Find

- To find key $k$
  - Calculate $i = h(k)\ \%\ length$
  - If $table[i]$ is occupied and does not contain $k$ then look at $(i + 1)\ \%\ length$
  - If that is occupied and does not contain $k$ then look at $(i + 2)\ \%\ length$
  - If that is occupied and does not contain $k$ then look at $(i + 3)\ \%\ length$
  - Repeat until you either find $k$ or else you reach an empty cell in the table

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

# Linear Probing: Delete

- To delete key $k$, where $h(k) = i$
  - Assume it is present
- Beginning at index $i$, probe until we find $k$ (call this location index $j$)
- Mark $j$ as empty (e.g. null), then continue probing while doing the following until you find another empty index
  - If you come across a key which hashes to a value $\leq j$ then move that item to index $j$ and update $j$.

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

# Linear Probing: Delete

- Option 1: Fill in with items that hashed to before the empty slot
- Option 2: “Tombstone” deletion. Leave a special object that indicates an object was deleted from there
  - The tombstone does not act as an open space when finding (so keep looking after its reached)
  - When inserting you can replace a tombstone with a new item

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| $k, v$ | | $k, v$ | $k, v$ |  | $k, v$ | | | | |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

# Downsides of Linear Probing

- What happens when $\lambda$ approaches 1?
  - Get longer and longer contiguous blocks
  - A collision is guaranteed to grow a block
    - Larger blocks experience more collisions
    - Feedback loop!
- What happens when $\lambda$ exceeds 1?
  - Impossible!
  - You can’t insert more stuff

# Quadratic Probing: Insert Procedure

- To insert $k, v$
  - Calculate $i = h(k)\ \%\ size$
  - If $table[i]$ is occupied then try $(i + 1^2)\%\ size$
  - If that is occupied try $(i + 2^2)\%\ size$
  - If that is occupied try $(i + 3^2)\%\ size$
  - If that is occupied try $(i + 4^2)\%\ size$
  - …

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

# Quadratic Probing: Example

- Insert:
  - 76
  - 40
  - 48
  - 5
  - 55
  - 47

| | | | | | | |
|---|---|---|---|---|---|---|
| | | | | | | |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 |

# Using Quadratic Probing

- If you probe $tablesize$ times, you start repeating the same indices
- If $tablesize$ is prime and $\lambda < \frac{1}{2}$ then you're guaranteed to find an open spot in at most $tablesize/2$ probes
- Helps with the clustering problem of linear probing, but does not help if many things hash to the same value

# Double Hashing: Insert Procedure

- Given $h$ and $g$ are both good hash functions
- To insert $k, v$
  - Calculate $i = h(k)\ \%\ size$
  - If $table[i]$ is occupied then try $\big(i + g(k)\big)\ \%\ size$
  - If that is occupied try $\big(i + 2 \cdot g(k)\big)\%\ size$
  - If that is occupied try $\big(i + 3 \cdot g(k)\big)\%\ size$
  - If that is occupied try $\big(i + 4 \cdot g(k)\big)\%\ size$
  - …

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

# Sorting

- Rearrangement of items into some defined sequence
  - Usually: reordering a list from smallest to largest according to some metric
- Why sort things?

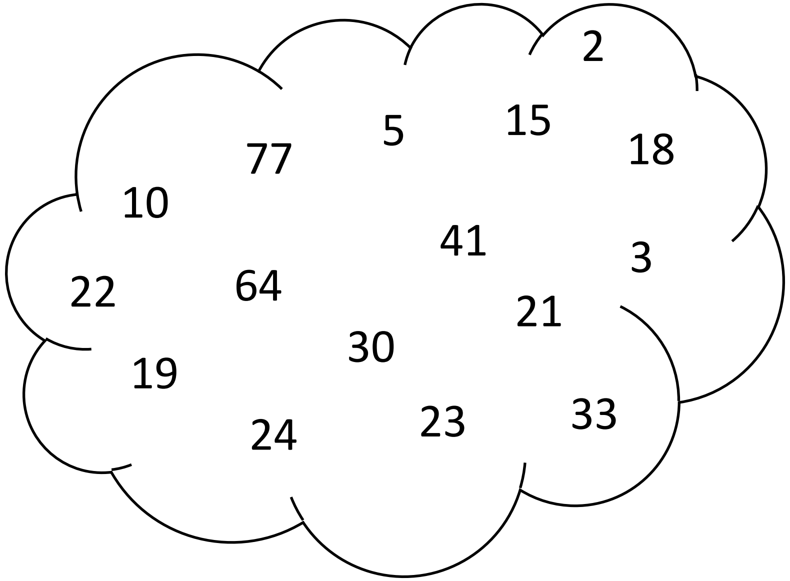
# More Formal Definition

- Input:
  - An array $A$ of items
  - A comparison function for these items
    - Given two items $x$ and $y$, we can determine whether $x < y$, $x > y$, or $x = y$
- Output:
  - A permutation of $A$ such that if $i \leq j$ then $A[i] \leq A[j]$
  - Permutation: a sequence of the same items but perhaps in a different order

# Sorting "Landscape"

- There is no singular best algorithm for sorting
- Some are faster, some are slower
- Some use more memory, some use less
- Some are super extra fast if your data matches particular assumptions
- Some have other special properties that make them valuable
- No sorting algorithm can have only all the "best" attributes

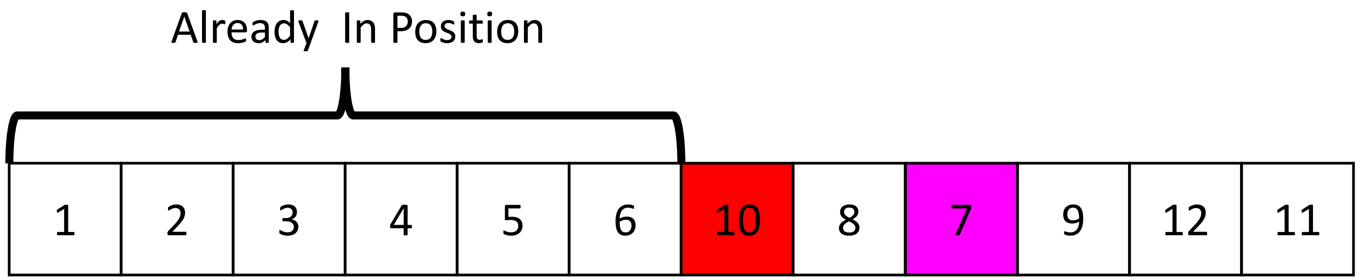
# “Moving Day” Sorting Algorithm

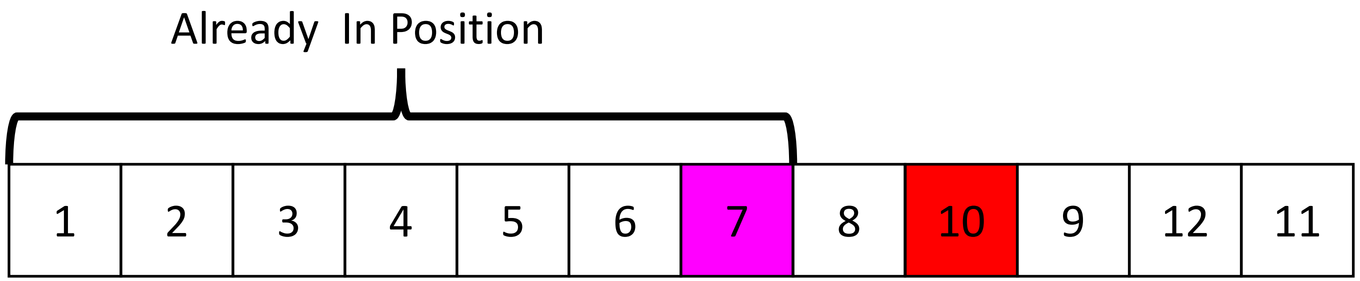



| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |

# Selection Sort

- Idea: Find the next smallest element, swap it into the next index in the array



# Selection Sort

- Swap the thing at index $0$ with the smallest thing in the array
- Swap the thing at index $1$ with the smallest thing after index $0$
- …
- Swap the thing at index $i$ with the smallest thing after index $i - 1$

```
for (i=0; i<a.length; i++){
	smallest = i;
	for (j=i; j<a.length; j++){
		if (a[j]<a[smallest]){ smallest=j;}
	}
	temp = a[i];
	a[i] = a[smallest];
	a[smallest] = a[i];
}
```

Running Time:

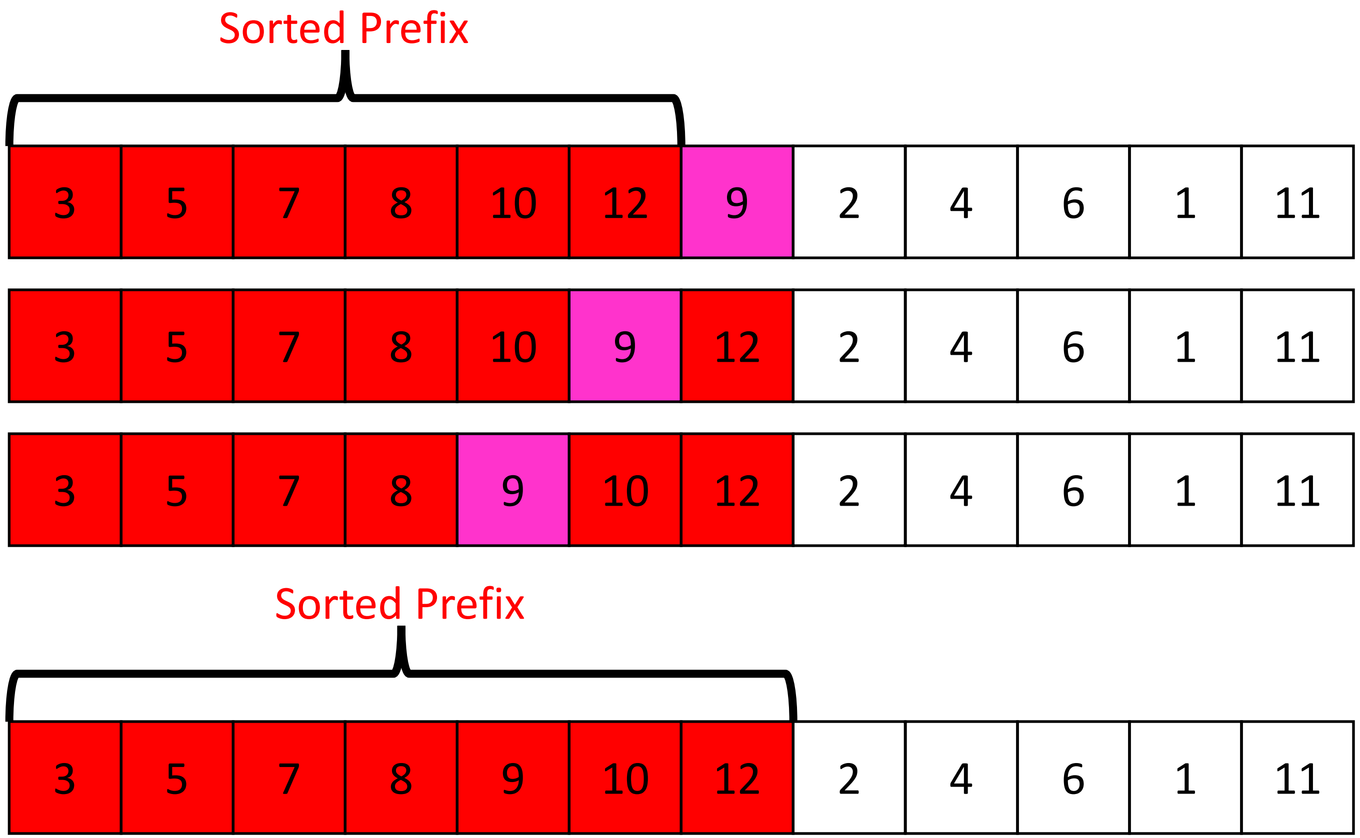
Worst Case: $\Theta(\cdot)$

Best Case: $\Theta(\cdot)$

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 10 | 77 | 5 | 15 | 2 | 22 | 64 | 41 | 18 | 19 | 30 | 21 | 3 | 24 | 23 | 33 |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |

# Insertion Sort

- Idea: Maintain a sorted list prefix, extend that prefix by “inserting” the next element



# Insertion Sort

- If the items at index $0$ and $1$ are out of order, swap them
- Keep swapping the item at index $2$ with the thing to its left as long as the left thing is larger
- …
- Keep swapping the item at index $i$ with the thing to its left as long as the left thing is larger

```
for (i=1; i<a.length; i++){
	prev = i-1;
	while(a[i] < a[prev] && prev > -1){
		temp = a[i];
		a[i] = a[prev];
		a[prev] = a[i];
		i--;
		prev--;
	}
}
```

Running Time:

Worst Case: $\Theta(\cdot)$

Best Case: $\Theta(\cdot)$

| | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 10 | 77 | 5 | 15 | 2 | 22 | 64 | 41 | 18 | 19 | 30 | 21 | 3 | 24 | 23 | 33 |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 |

# Aside: Bubble Sort – we won’t cover it

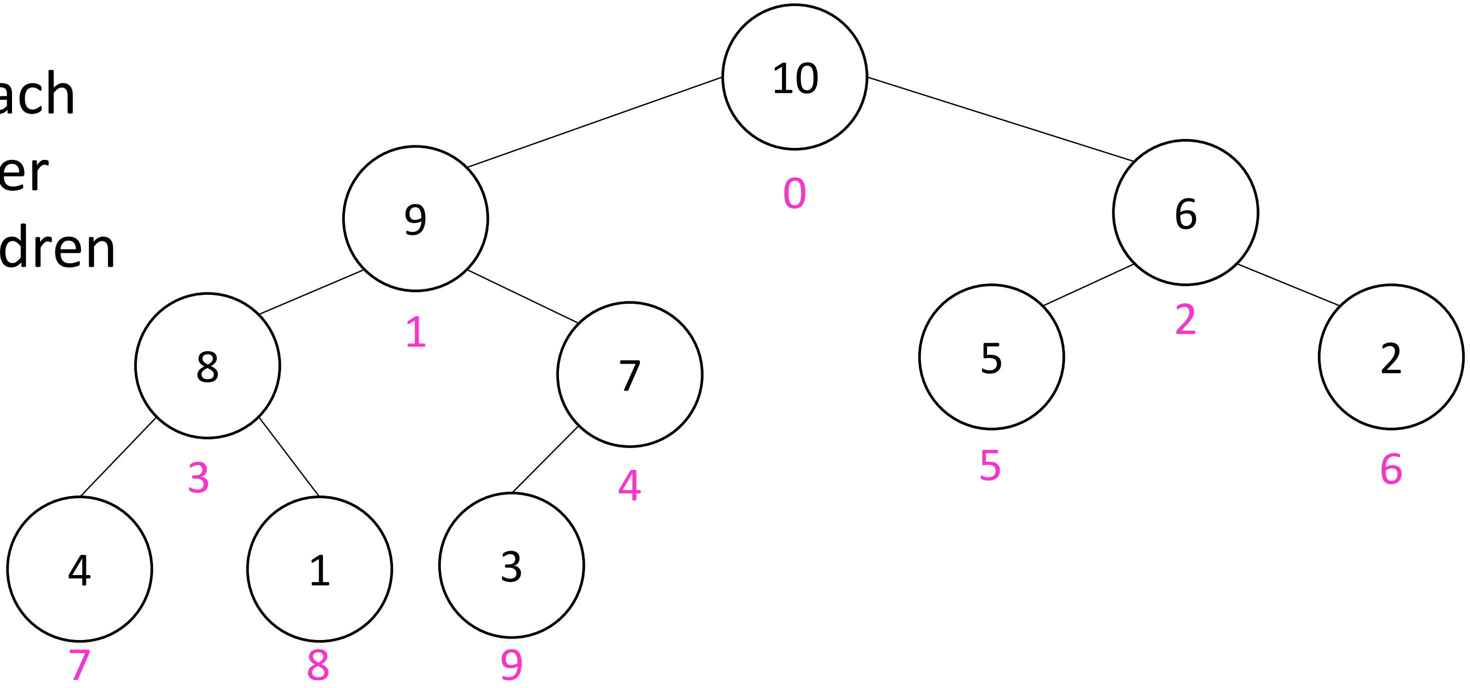
"the bubble sort seems to have nothing to recommend it, except a catchy name and the fact that it leads to some interesting theoretical problems” –Donald Knuth, The Art of Computer Programming

# Heap Sort

- Idea: Build a maxHeap, repeatedly delete the max element from the heap to build sorted list Right-to-Left

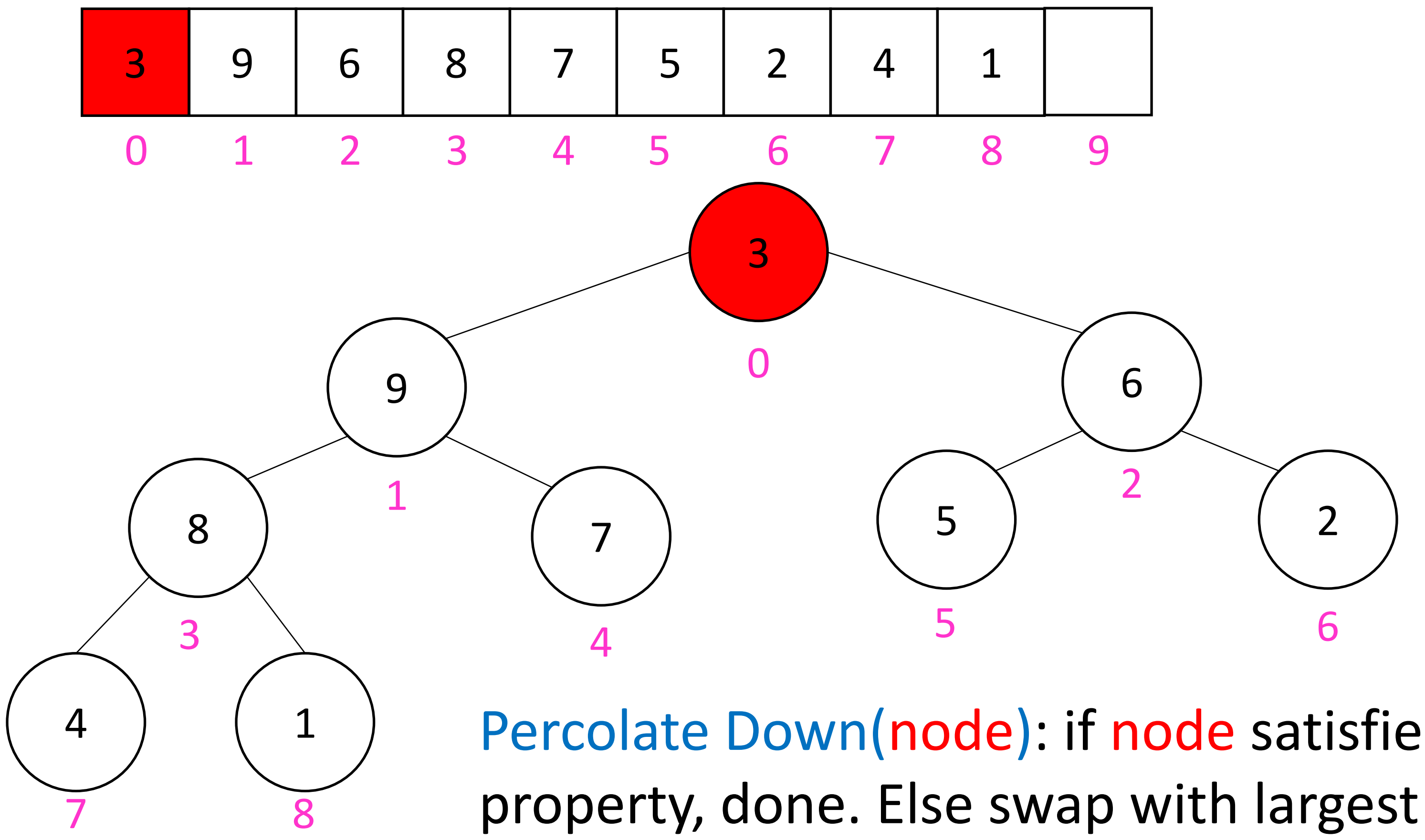
| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 10 | 9 | 6 | 8 | 7 | 5 | 2 | 4 | 1 | 3 |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |

Max Heap Property: Each node is larger than its children



# Heap Sort

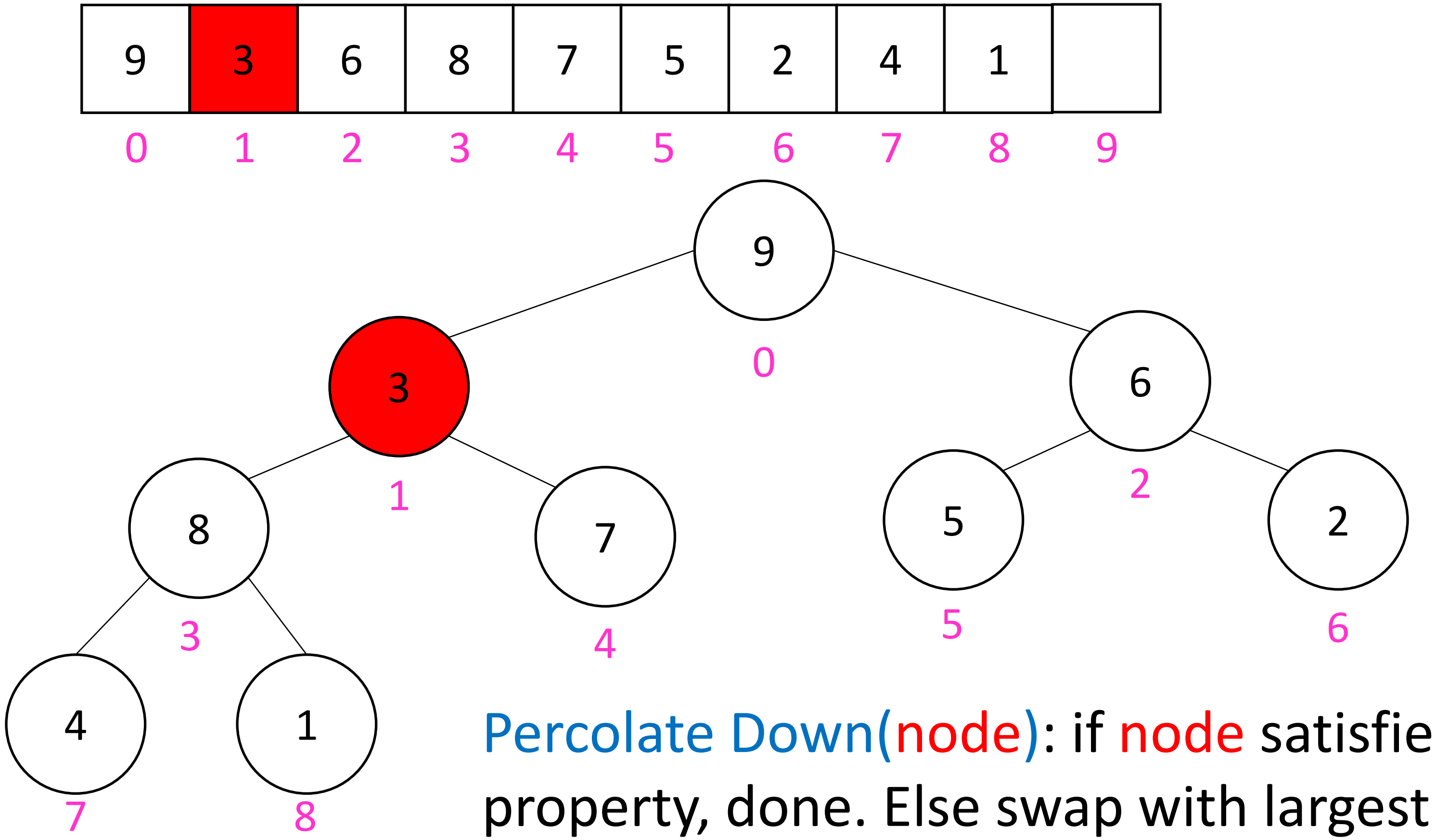
- Remove the Max element (i.e. the root) from the Heap: replace with last element, call percolateDown(root)




Percolate Down(node): if node satisfies heap property, done. Else swap with largest child and repeat on that subtree

# Heap Sort

- Remove the Max element (i.e. the root) from the Heap:
  replace with last element, call percolateDown(root)

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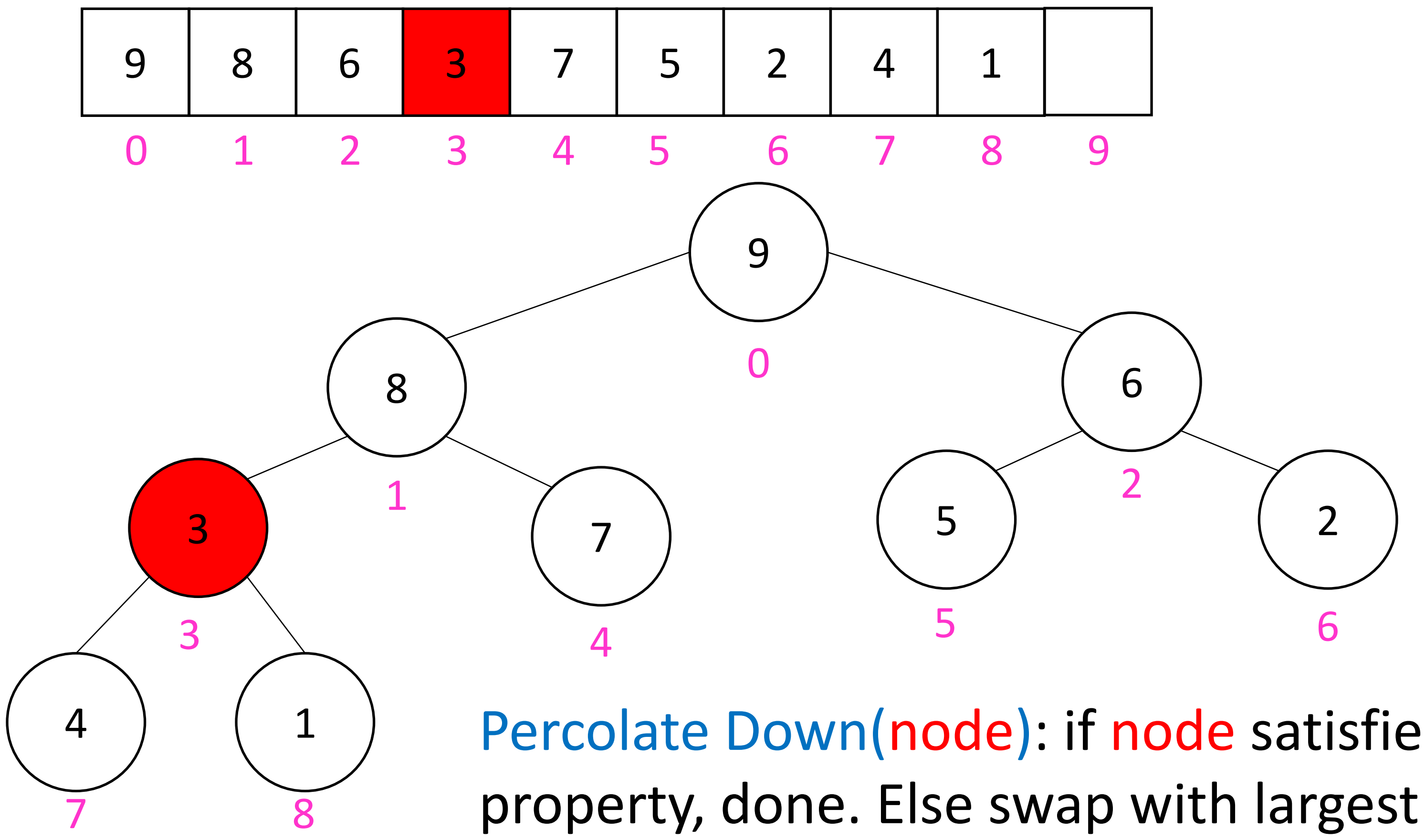
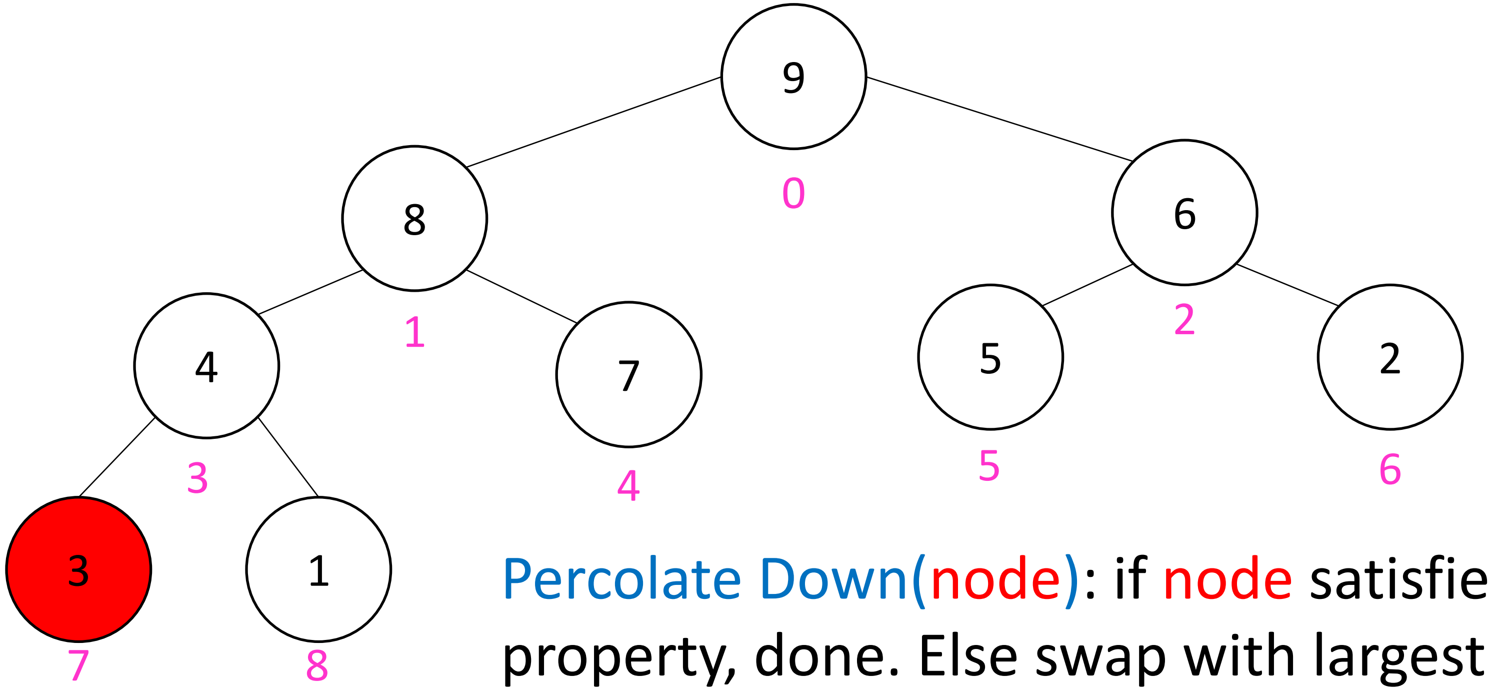



Percolate Down(node): if node satisfies heap property, done. Else swap with largest child and repeat on that subtree

# Heap Sort

- Remove the Max element (i.e. the root) from the Heap: replace with last element, call percolateDown(root)

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 9 | 8 | 6 | 4 | 7 | 5 | 2 | 3 | 1 | |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |




Percolate Down(node): if node satisfies heap property, done. Else swap with largest child and repeat on that subtree

# Heap Sort

- Build a heap
- Call deleteMax
- Put that at the end of the array

```
myHeap = buildHeap(a);
for (int i = a.length-1; i>=0; i--){
	item = myHeap.deleteMax();
	a[i] = item;
}
```

Running Time:

Worst Case: $\Theta(\cdot)$

Best Case: $\Theta(\cdot)$

# “In Place” Sorting Algorithm

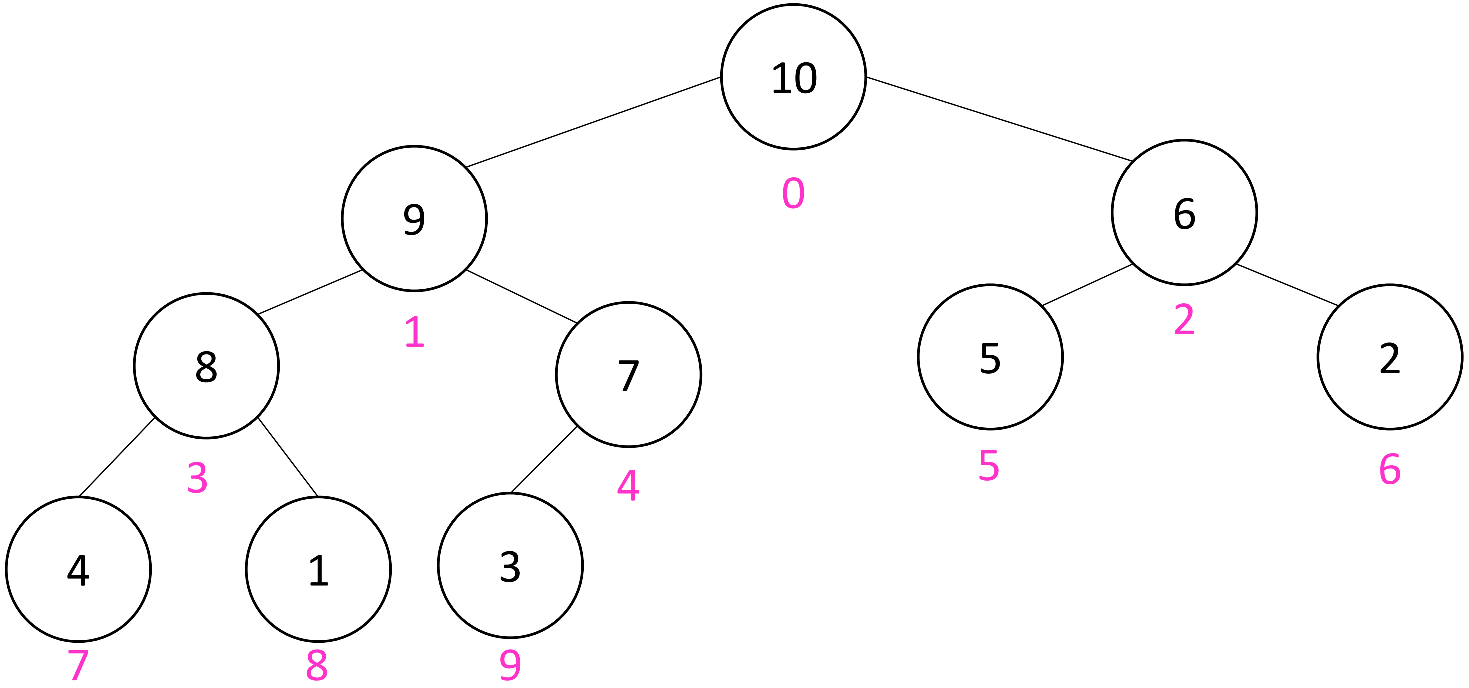
- A sorting algorithm which requires no extra data structures
- Idea: It sorts items just by swapping things in the same array given
- Definition: it only uses $\Theta(1)$ extra space

- Selection sort: In Place!
- Insertion sort: In Place!
- Heap sort: Not In Place!
    - But we can fix that!

# In Place Heap Sort

- Idea: When “removing” an element from the heap, swap it with the last item of the heap then “pretend” the heap is one item shorter

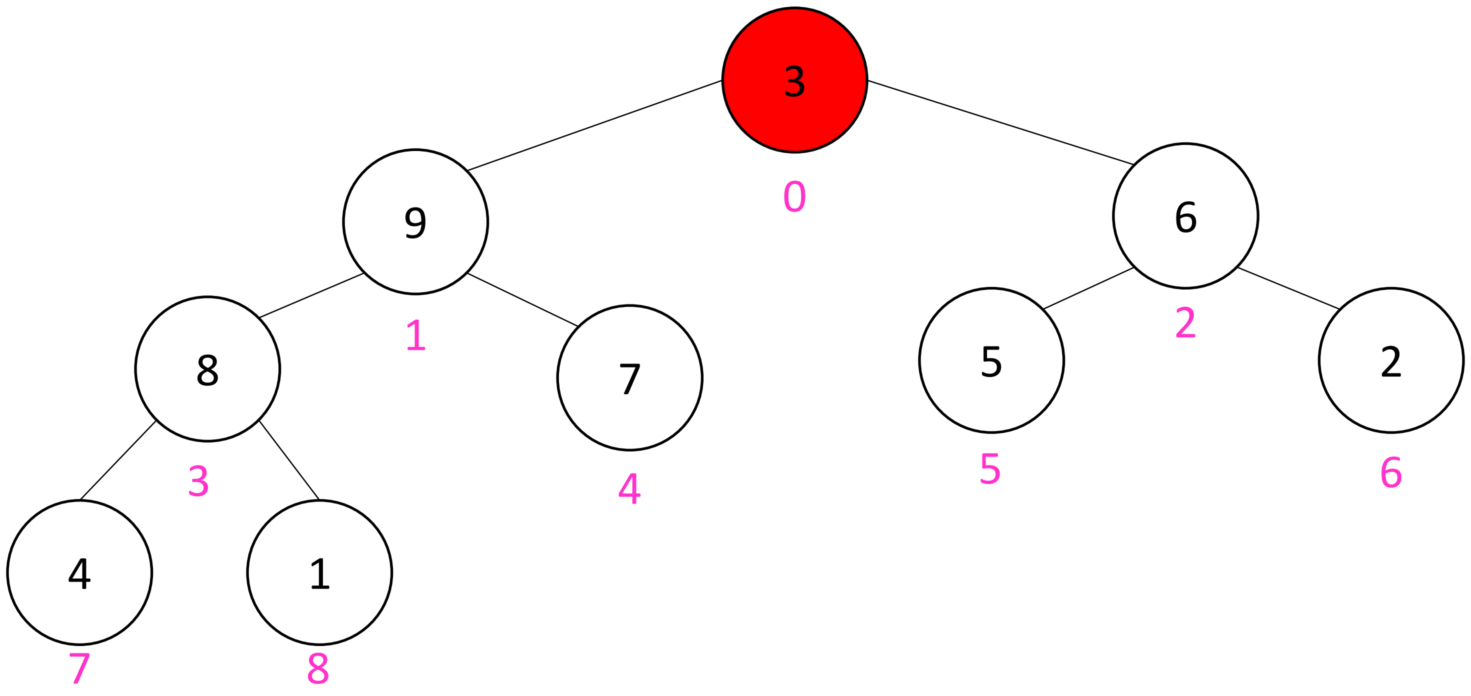
| 10 | 9 | 6 | 8 | 7 | 5 | 2 | 4 | 1 | 3 |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |



# Heap Sort

- Idea: When "removing" an element from the heap, swap it with the last item of the heap then "pretend" the heap is one item shorter

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 3 | 9 | 6 | 8 | 7 | 5 | 2 | 4 | 1 | 10 |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |



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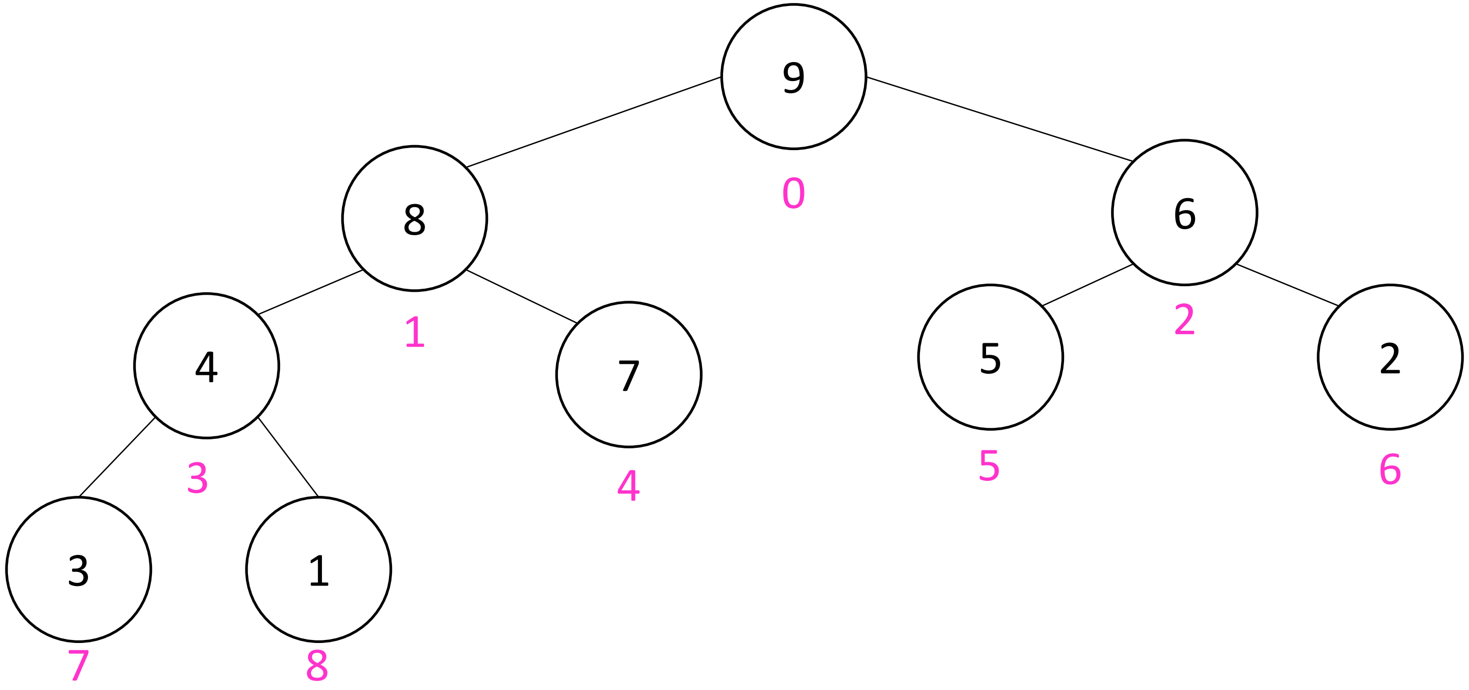
- Idea: When "removing" an element from the heap, swap it with the last item of the heap then "pretend" the heap is one item shorter

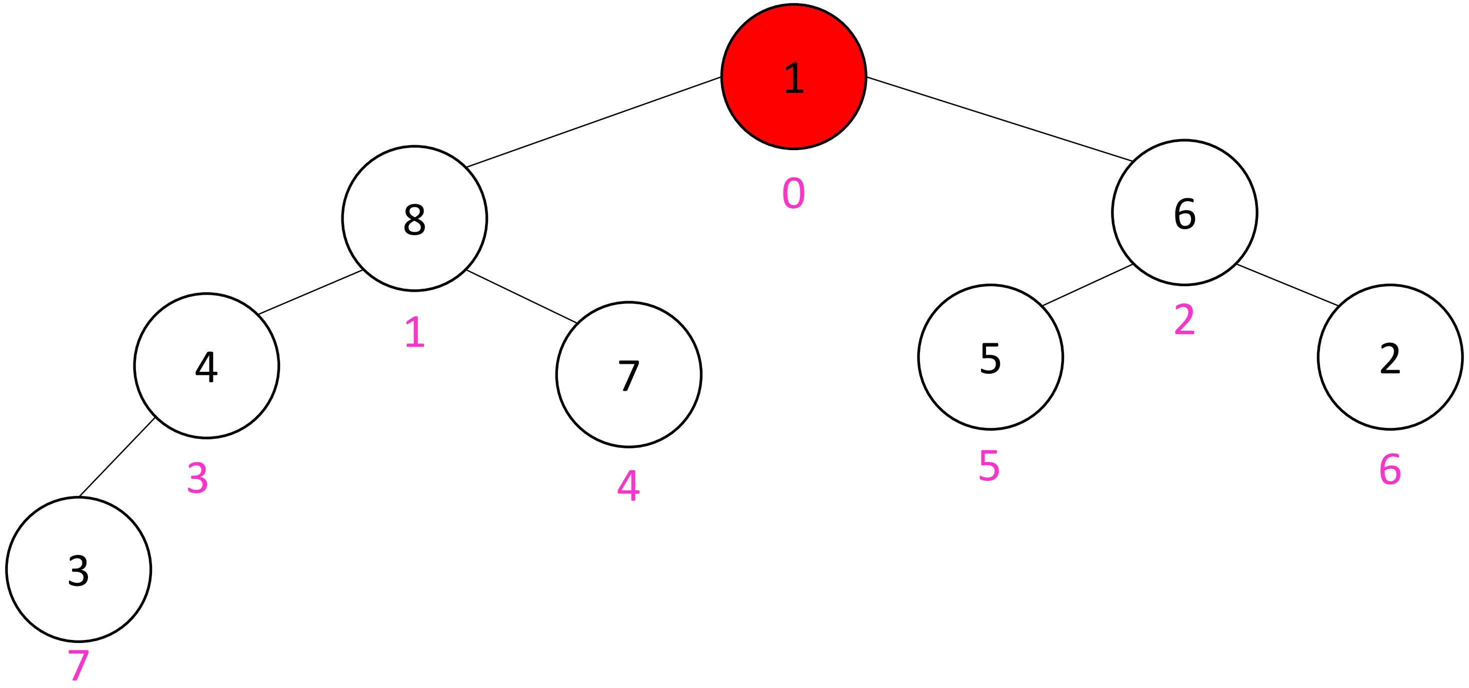
| 9 | 8 | 6 | 4 | 7 | 5 | 2 | 3 | 1 | 10 |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |



# Heap Sort

- Idea: When “removing” an element from the heap, swap it with the last item of the heap then “pretend” the heap is one item shorter

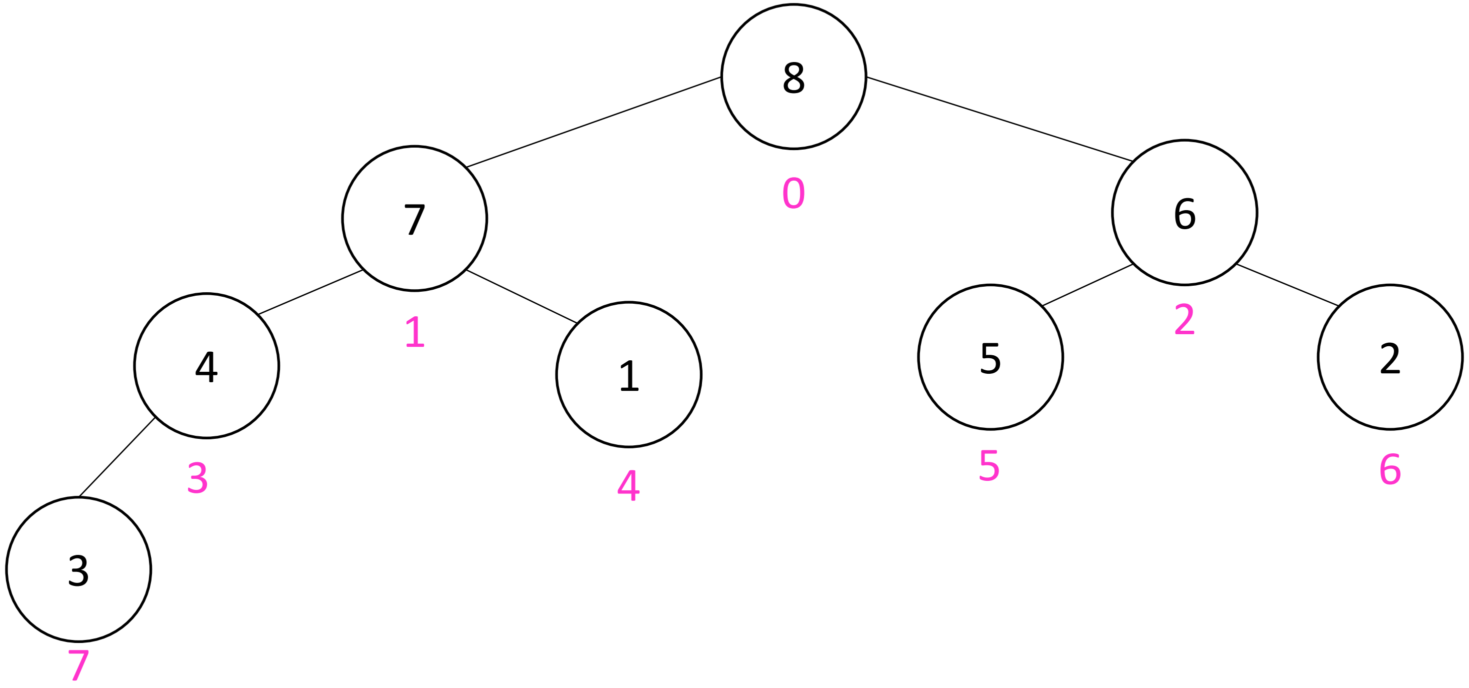
| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 8 | 6 | 4 | 7 | 5 | 2 | 3 | 9 | 10 |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |



# Heap Sort

- Idea: When “removing” an element from the heap, swap it with the last item of the heap then “pretend” the heap is one item shorter

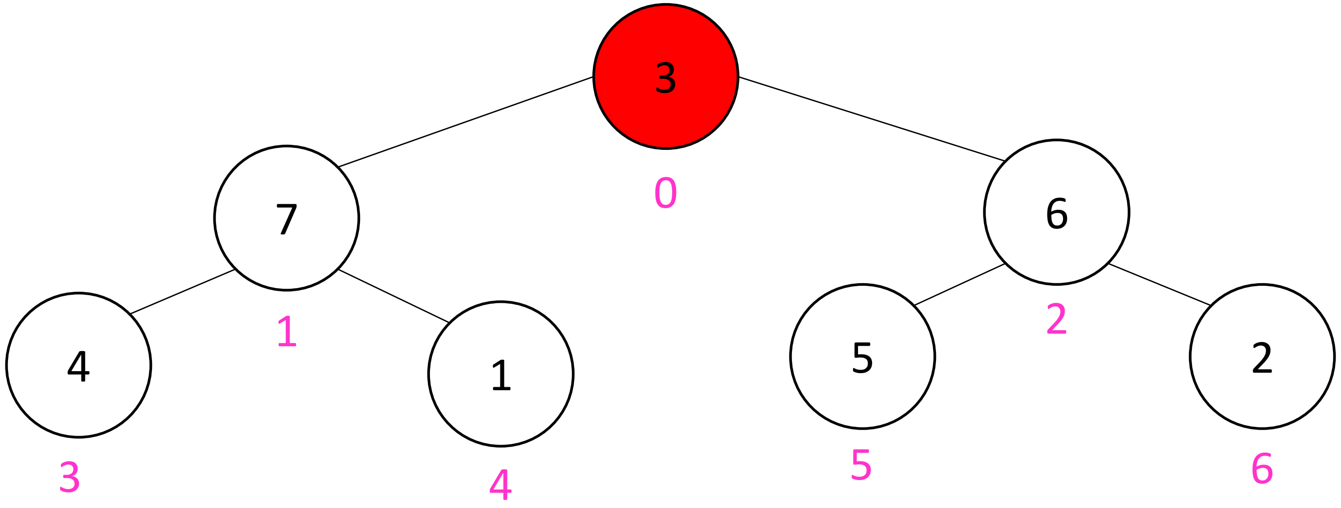
| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 8 | 6 | 4 | 7 | 5 | 2 | 3 | 9 | 10 |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |



# Heap Sort

- Idea: When “removing” an element from the heap, swap it with the last item of the heap then “pretend” the heap is one item shorter

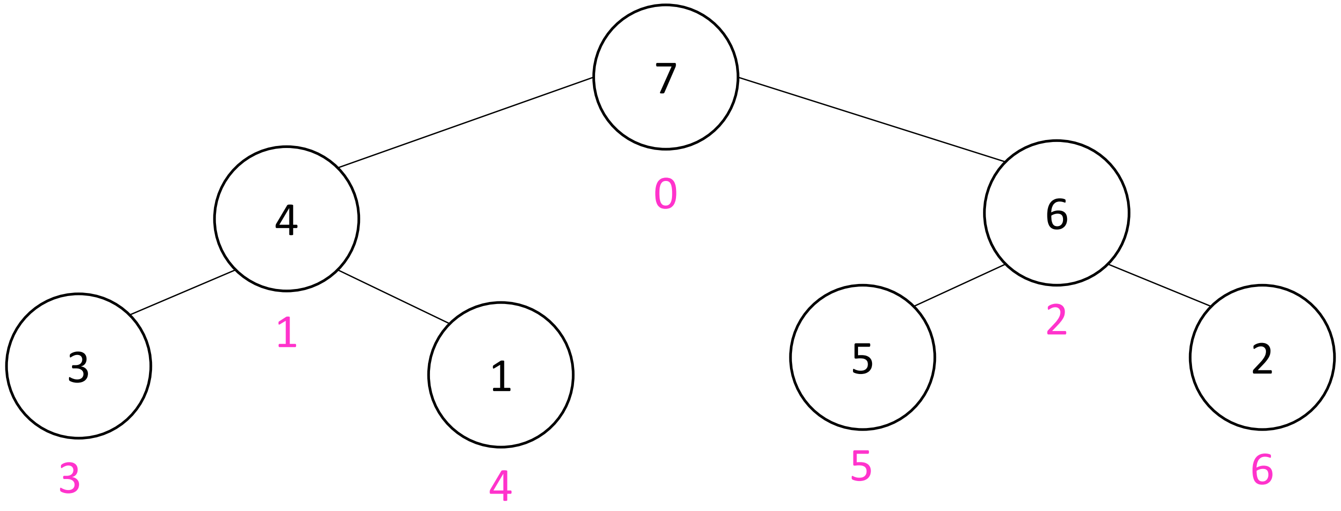
| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 3 | 8 | 6 | 4 | 7 | 5 | 2 | 8 | 9 | 10 |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |



# Heap Sort

- Idea: When "removing" an element from the heap, swap it with the last item of the heap then "pretend" the heap is one item shorter

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 3 | 8 | 6 | 4 | 7 | 5 | 2 | 8 | 9 | 10 |
| 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |



# In Place Heap Sort

- Build a heap using the same array (Floyd's build heap algorithm works)
- Call deleteMax
- Put that at the end of the array

```
buildHeap(a);
for (int i = a.length-1; i>=0; i--){
	temp=a[i]
	a[i] = a[0];
	a[0] = temp;
	percolateDown(0);
}
```

Running Time:

Worst Case: $\Theta(\cdot)$

Best Case: $\Theta(\cdot)$

# Floyd’s buildHeap method

- Working towards the root, one row at a time, percolate down

```
buildHeap(){
    for(int i = size; i>0; i--){
        percolateDown(i);
    }
}
```