CSE 332
Data Structures & Parallelism

P, NP, NP-Complete (Part 2)

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Course notes

- Course evals
- P3 was due LAST NIGHT
  - Late due date is Saturday night (11:59pm)
- Final review session - Tuesday 6/4, 2:30-5:20pm, GWN 301
- OH in finals week
- Final exam is THURSDAY 6/6 at 8:30am

Agenda (2 lectures)

- A Few (graph) Problems:
  - Euler Circuits
  - Hamiltonian Circuits
- Intractability: P and NP
- NP-Complete
- What now?

A Glimmer of Hope

- If given a candidate solution to a problem, we can **check if that solution is correct in polynomial-time,** then maybe a polynomial-time solution exists?

- Example: can we do this with Hamiltonian Circuit?
    - Given a candidate path, is it a Hamiltonian Circuit?

The Complexity Class NP

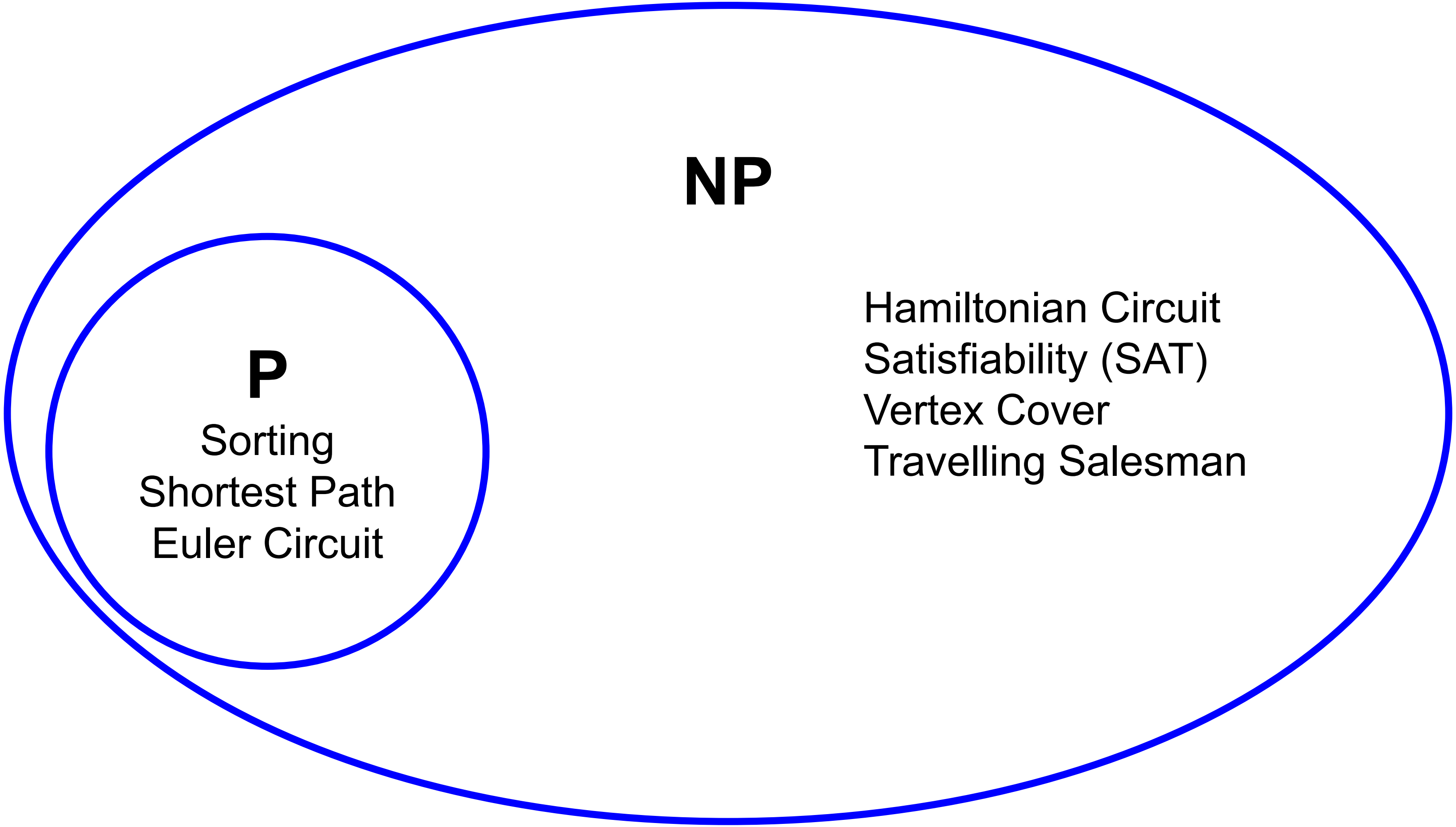
- **Definition**: NP is the set of all problems for which a given *candidate solution* can be *tested* in polynomial time

- Examples of problems in NP:
  - *Hamiltonian circuit*: Given a candidate path, can test in linear time if it is a Hamiltonian circuit
  - *Vertex Cover*: Given a subset of vertices, do they cover all edges?
  - *All problems that are in P (why?)*

Why do we call it “NP”?

NP stands for **Nondeterministic Polynomial time**

- Why “nondeterministic”? Corresponds to algorithms that can guess a solution (if it exists), the solution is then verified to be correct in polynomial time
- Can also think of as allowing a special operation that allows the algorithm to magically guess the right choice at each branch point.
- The algorithms we’ve discussed are NOT nondeterministic – purely theoretical idea invented to understand how hard a problem could be


NP
P
Sorting
Shortest Path
Euler Circuit
Hamiltonian Circuit
Satisfiability (SAT)
Vertex Cover
Travelling Salesman

Your Chance to Win a Turing Award!

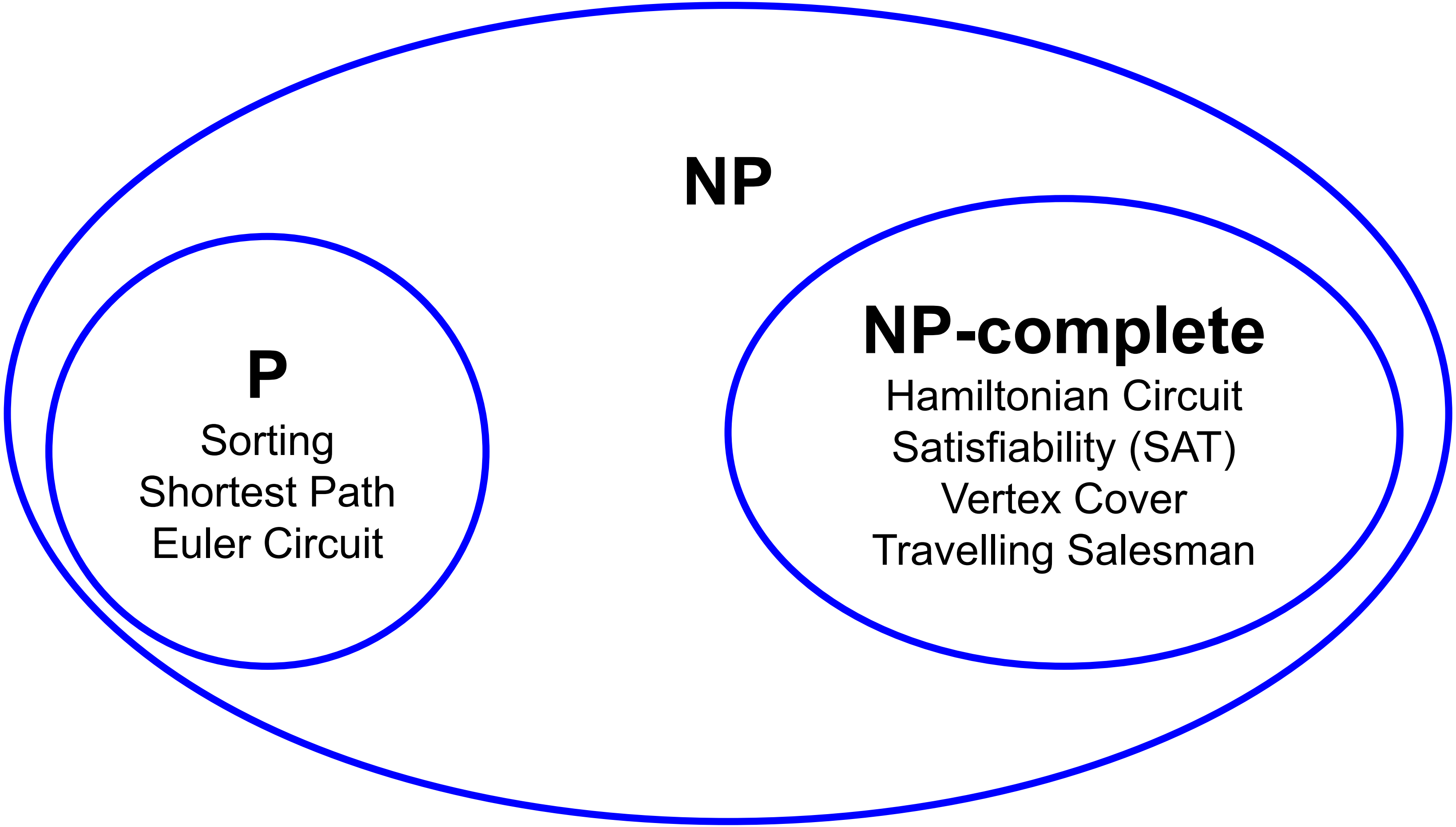
It is generally believed that $P \neq NP$,

i.e. there are problems in NP that are **not** in P

- But no one has been able to show even one such problem!
- This is the fundamental open problem in theoretical computer science
- Nearly everyone has given up trying to prove it. Instead, theoreticians prove theorems about what follows once we assume $P \neq NP$!

NP-completeness

- Set of problems in NP that (we are pretty sure) ***cannot*** be solved in polynomial time.
- These are thought of as the **hardest** problems in the class NP.
- **Interesting fact**: If any one NP-complete problem could be solved in polynomial time, then ***all*** NP-complete problems could be solved in polynomial time.
- Also: If any NP-complete problem is in P, then all of NP is in P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Saving Your Job

- Try as you might, every solution you come up with for the Hamiltonian Circuit problem runs in exponential time…..
- You have to report back to your boss.
- Your options:
    - Keep working
    - Come up with an alternative plan…

Your Third Task

- Your boss buys your story that others couldn't solve the last problem.
- Again, your company has to send someone by car to a set of cities. There is a road between every pair of cities.
- The primary cost is distance traveled (which translates to fuel costs).
- Your boss wants you to figure out *how to drive to each city exactly once*, then return to the first city, while *staying within a fixed mileage budget k*.

Travelling Salesperson Problem (TSP)

- Your third task is basically TSP:
  - Given complete weighted graph G, integer k.
  - Is there a cycle that visits all vertices with cost <= k?
- One of the canonical problems.

- Note difference from Hamiltonian cycle:
  - graph is complete
  - we care about weight.

In general, what to do with a Hard Problem

- Your problem seems really hard.

- If you can transform a known NP-complete problem into the one you're trying to solve, then you can stop working on your problem!

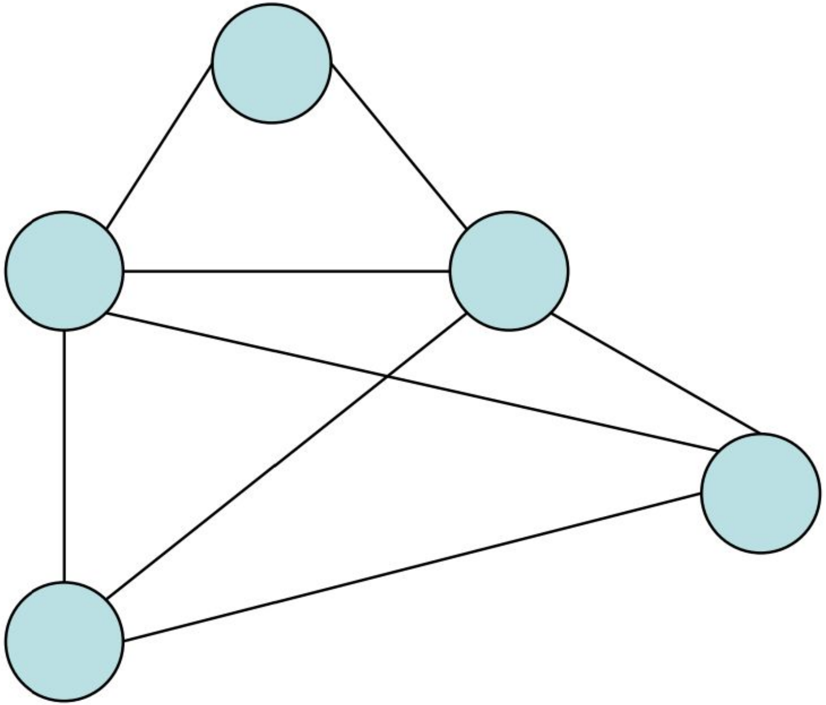
Transforming Hamiltonian Cycle to TSP

- We can "reduce" Hamiltonian Cycle to TSP.
- Given graph G=(V, E):
    - Assign weight of 1 to each edge
    - Augment the graph with edges until it is a complete graph G'=(V, E')
    - Assign weights of 2 to the new edges
    - Let k = |V|.

Notes:

- The transformation must take polynomial time
- You reduce the known NP-complete problem into your problem (not the other way around)
- In this case we are assuming Hamiltonian Cycle is our known NP-complete problem (in reality, both are known NP-complete)

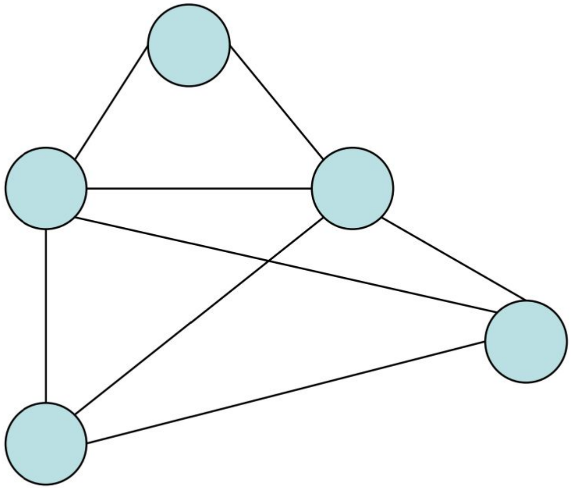
Example

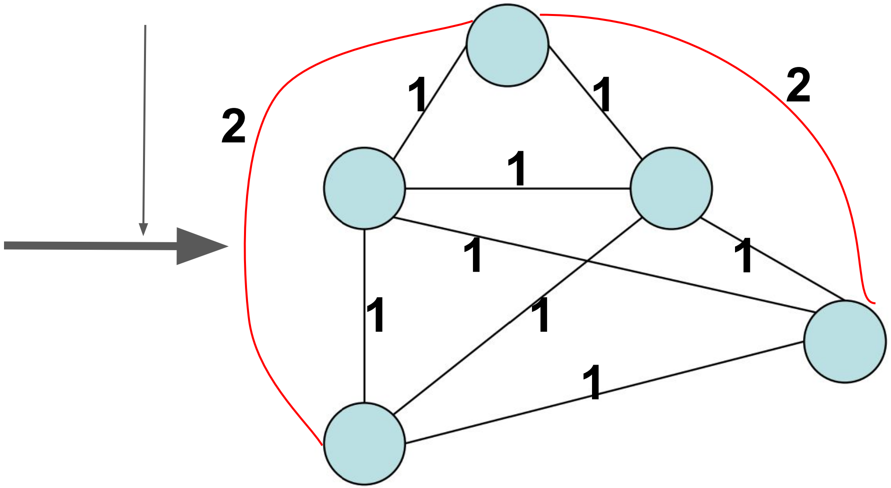


G

Input to Hamiltonian Circuit Problem

Example





Polynomial time transformation

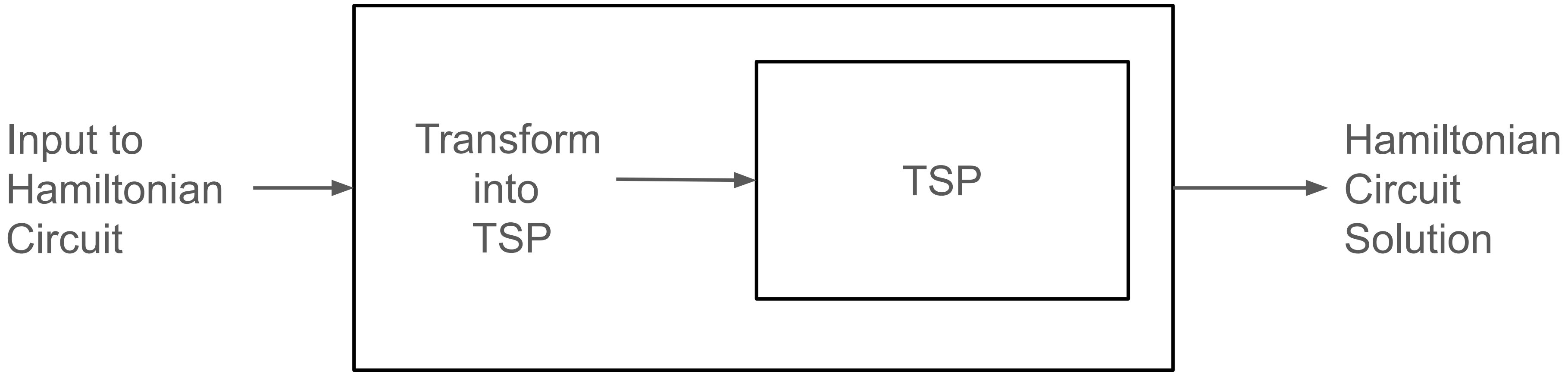
G

Input to Hamiltonian Circuit Problem

G'

Input to Traveling Salesperson Problem

Hamiltonian Circuit -> Traveling Salesperson



Polynomial-time transformation

- G' has a TSP tour of weight |V| iff G has a Hamiltonian Cycle.
- What was the cost of transforming HC into TSP?

- In the end, because there is a polynomial time transformation from HC to TSP, we say *TSP is "at least as hard as" Hamiltonian cycle*.

What do we do about it?

- Approximation Algorithm:
  - Can we get an efficient algorithm that guarantees something *close* to optimal? (e.g. Answer is guaranteed to be within 1.5x of Optimal, but solved in polynomial time).
- Restrictions:
  - Many hard problems are easy for restricted inputs (e.g. graph is always a tree, degree of vertices is always 3 or less).
- Heuristics:
  - Can we get something that seems to work well (good approximation/fast enough) *most* of the time? (e.g. In practice, n is small-ish)

Great Quick Reference

Computers and Intractability: A Guide to the Theory of NP-Completeness, by Michael S. Garey and David S. Johnson